Coherent Spontaneous-Superradiance and Stimulated-Superradiance of Bunched Electron Beams

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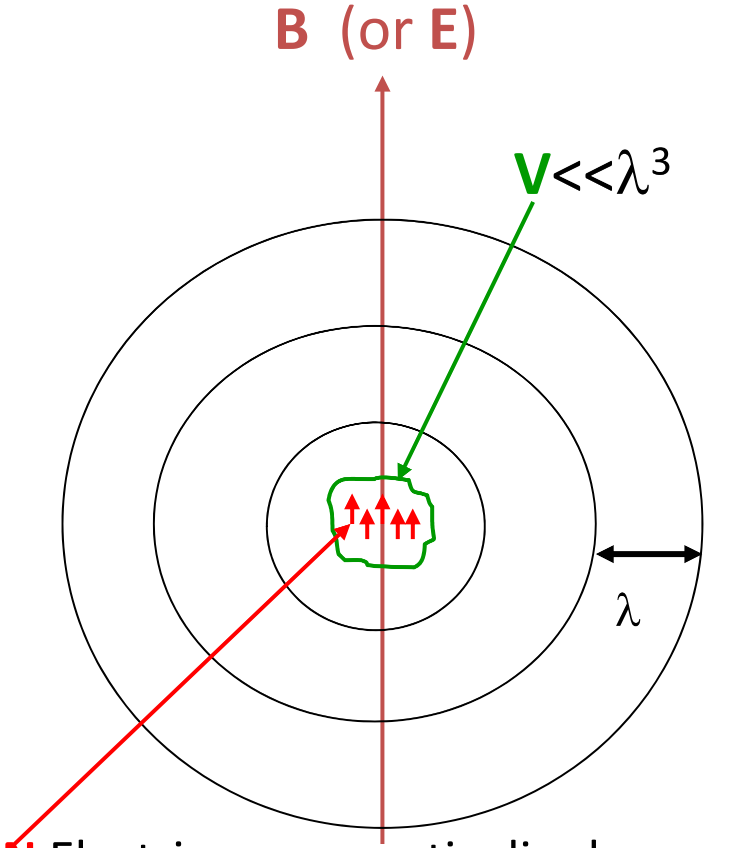
Rev. Mod. Phys. 91, 035003 – Published 19 August 2019

Tutorial - FEL International conference Hamburg, August 2019

CONCEPTS TO BE CONSIDERED

- Fundamental processes of coherent radiation emission from bunched beam current:
  - -Spontaneous superradiant emission (SP-SR)
  - -Stimulated Superradiant emission (ST-SR).
- Nonlinear SR, ST-SR dynamics of a trapped bunched beam.
- Tapering Enhanced Superradiant (TES) and Stimulated Superradiance (TESSA)
- Bunched-beam/radiation Self-interaction.
- Review of applications.

DICKE'S SUPERRADIANCE



N Electric or magnetic dipoles
prepared by means of a "$\pi/2$ pulse"

$$\boxed{I = (r+m)(r-m+1)I_0}$$

$r = 1/2, 1, 3/2\ldots\ldots N/2$

$m = -r\ldots\ldots 0\ldots\ldots r$

Spontaneous emission (quantum)

$N=1, \quad r = m = 1/2 \quad \Rightarrow \quad I = I_0$

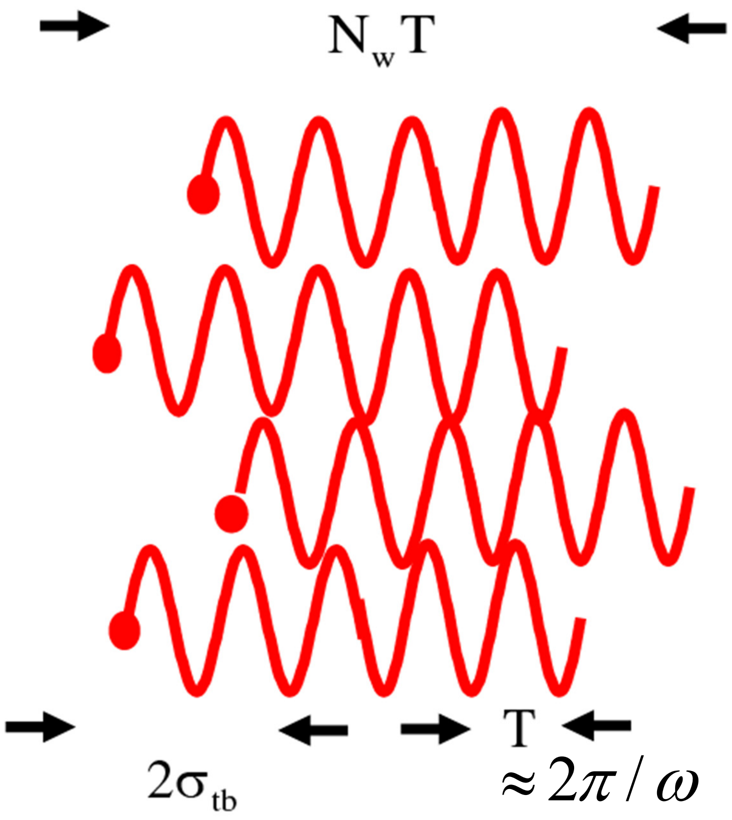
Superradiant emission (classical)

$r = N/2 \gg 1, \quad m = 0 \quad \Rightarrow \quad I = \frac{1}{4}N^2 I_0$

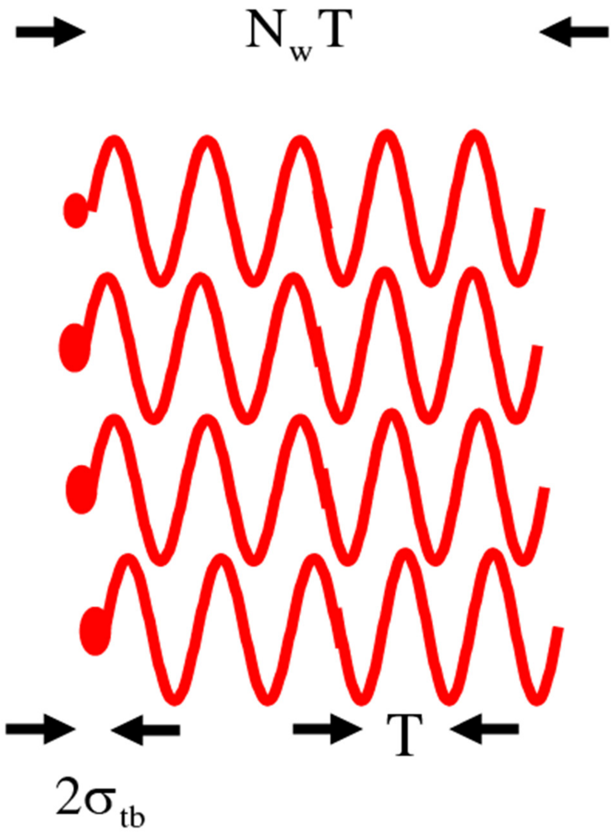
R. H. Dicke, PR, **93**, 99 (1954)

COHERENT vs RANDOM SUPERPOSITION OF RADIATION WAVEPACKETS

Spontaneous Emission
(Shot-Noise Radiation)



Spontaneous Superradiance
SP-SR



Formulation of Radiation mode Expansion

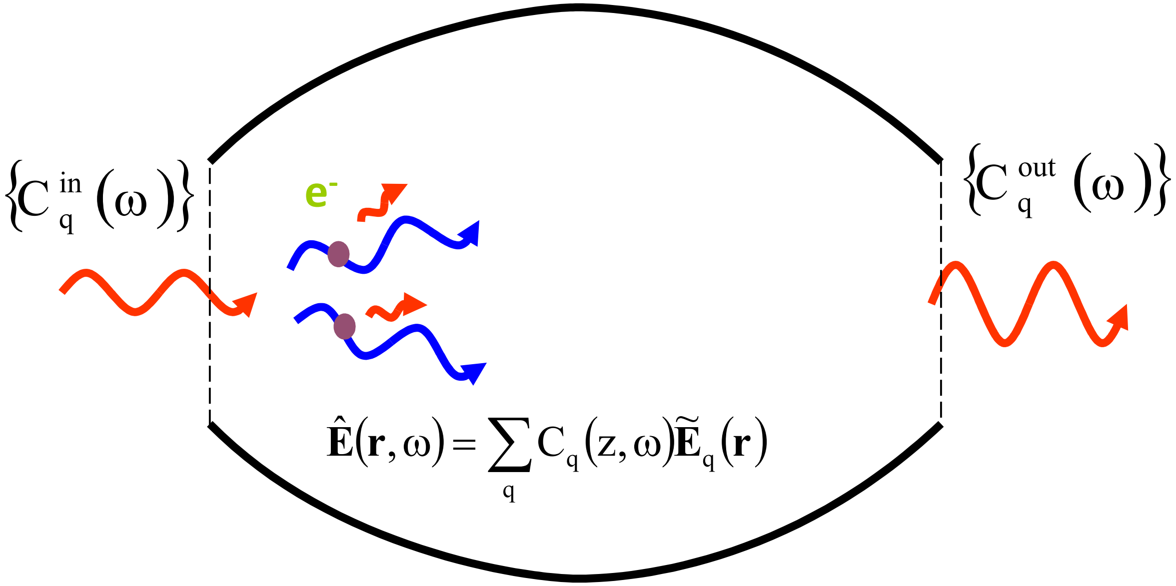
Spectral (Fourier)
Frequency domain

$$\breve{\boldsymbol{E}}(\boldsymbol{r},\omega)=\sum_{\pm q}\check{C}_q(z,\omega)\widetilde{\boldsymbol{E}}_q(\boldsymbol{r})$$

$$\breve{\boldsymbol{H}}(\boldsymbol{r},\omega)=\sum_{\pm q}\check{C}_q(z,\omega)\widetilde{\boldsymbol{H}}_q(\boldsymbol{r})$$

$$\frac{d\check{C}_q(z,\omega)}{dz}=-\frac{1}{4\mathcal{P}_q}\int\breve{\boldsymbol{J}}(\boldsymbol{r},\omega)\cdot\widetilde{\boldsymbol{E}}_q^*(\boldsymbol{r})\mathrm{dA}$$

$$\check{C}_q^{out}(\omega)-\check{C}_q^{in}(\omega)=-\frac{1}{4\mathcal{P}_q}\int\breve{\boldsymbol{J}}(\boldsymbol{r},\omega)\cdot\widetilde{\boldsymbol{E}}_q^*(\boldsymbol{r})\mathrm{dV}$$



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WAVEPACKETS EMISSION BY PARTICULATE CHARGES

For particulate current: $$\mathbf{J}(\mathbf{r},t)=\sum_{j=1}^{N}-e\mathbf{v}_j(t)\delta\left(\mathbf{r}-\mathbf{r}_j(t)\right)$$ ➡

Radiation wavepackets

$$C_q^{out}(\omega)=C_q^{in}(\omega)-\frac{1}{4\mathcal{P}_q}\sum_{j=1}^{N}\Delta\breve{\mathcal{W}}_{qj}$$

$$\Delta\breve{\mathcal{W}}_{qj}=-e\int_{-\infty}^{\infty}\tilde{\boldsymbol{v}}_j\cdot\tilde{\boldsymbol{E}}_q^{*}\left(\boldsymbol{r}_{\boldsymbol{j}}(t)\right)e^{i\omega t}dt$$

Zero order: $$\Delta\breve{\mathcal{W}}_{qj}^{(0)}=\Delta\breve{\mathcal{W}}_{qe}^{(0)}e^{i\omega t_{j0}}$$

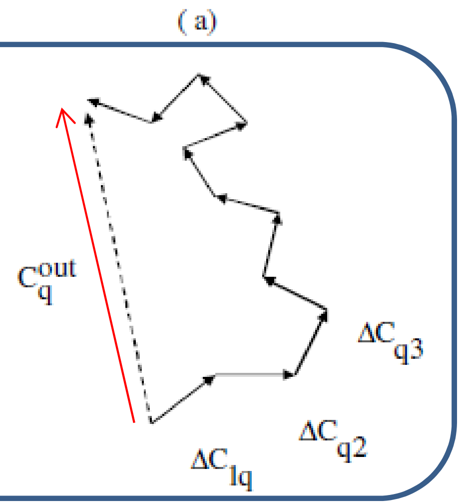
Spectral radiant energy:

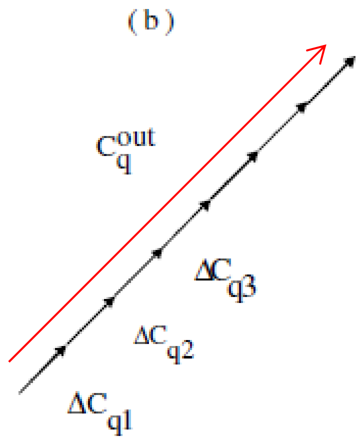
$$\boxed{\begin{gathered}\frac{dW_q}{d\omega}=\frac{2}{\pi}P_q\left|C_q^{out}(\omega)\right|^2=\\=\left(\frac{dW_q}{d\omega}\right)_{in}+\left(\frac{dW_q}{d\omega}\right)_{SP/SR}+\left(\frac{dW_q}{d\omega}\right)_{ST-SR}\end{gathered}}$$

COMPLEX PLANE REPRESENTATION OF SINGLE MODE EXCITATION AMPLITUDES

$$\breve{C}_q^{out} = \breve{C}_q^{in} + \sum_j \Delta\breve{C}_{qj}$$

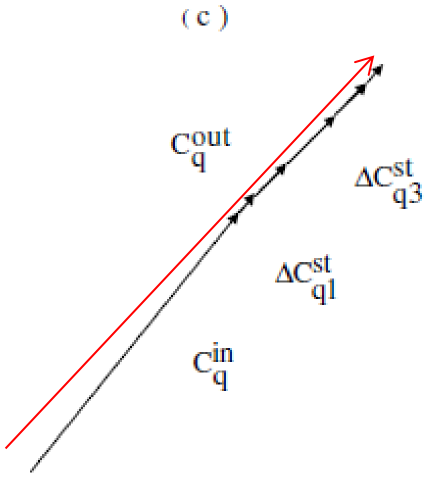
Spontaneous Emission

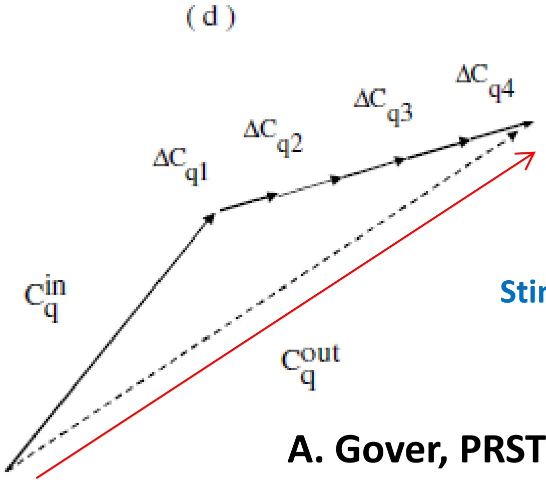




Superradiance

Stimulated Emission
(Laser Amplifier)





Stimulated – Super radiance

A. Gover, PRST-AB 8, (030701) 2005

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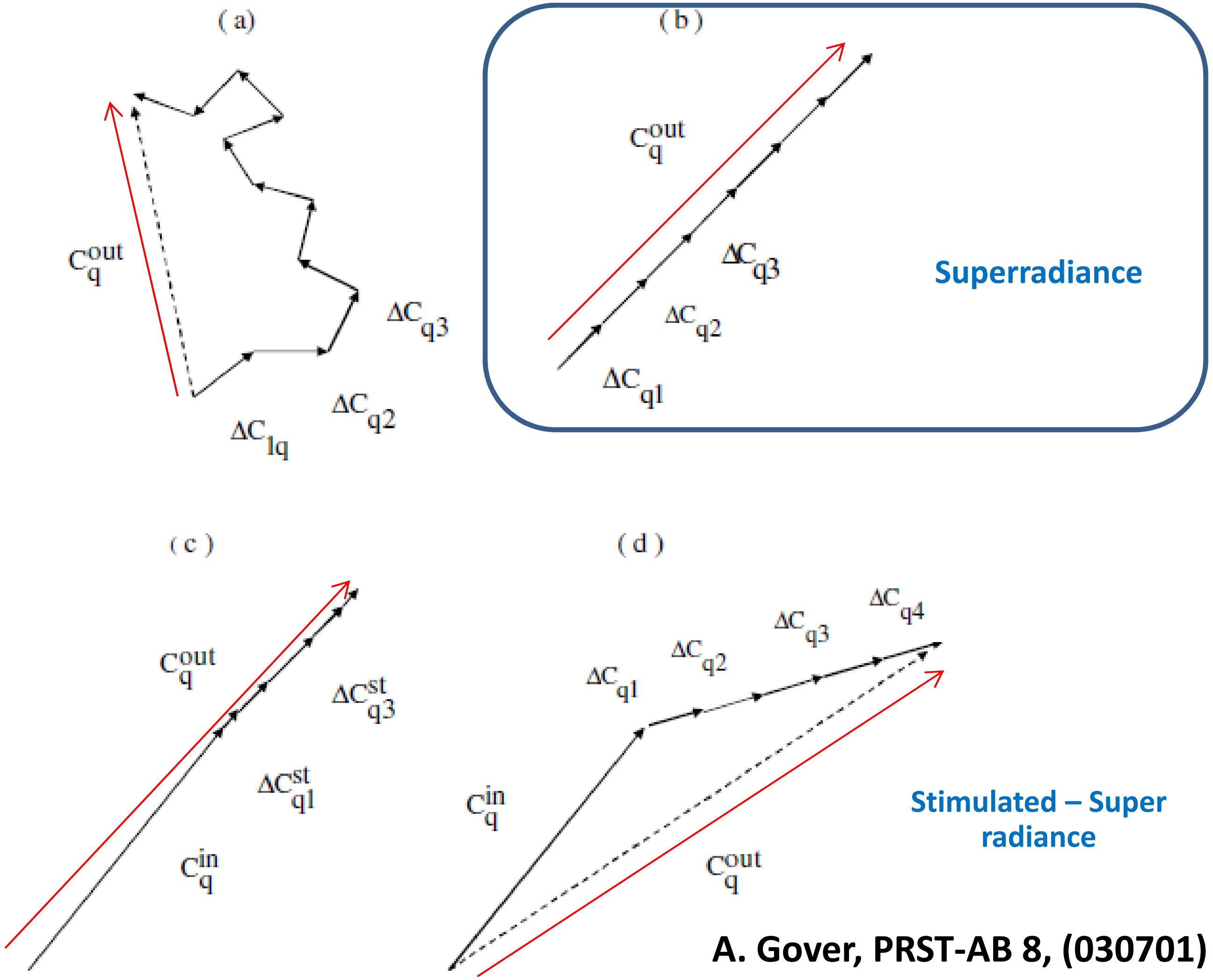
COMPLEX PLANE REPRESENTATION OF SINGLE MODE EXCITATION AMPLITUDES

$$\breve{C}_q^{out} = \breve{C}_q^{in} + \sum_j \Delta\breve{C}_{qj}$$



Spontaneous Emission

Superradiance

Stimulated Emission (Laser Amplifier)

Stimulated – Super radiance

A. Gover, PRST-AB 8, (030701) 2005

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COMPLEX PLANE REPRESENTATION OF SINGLE MODE EXCITATION AMPLITUDES

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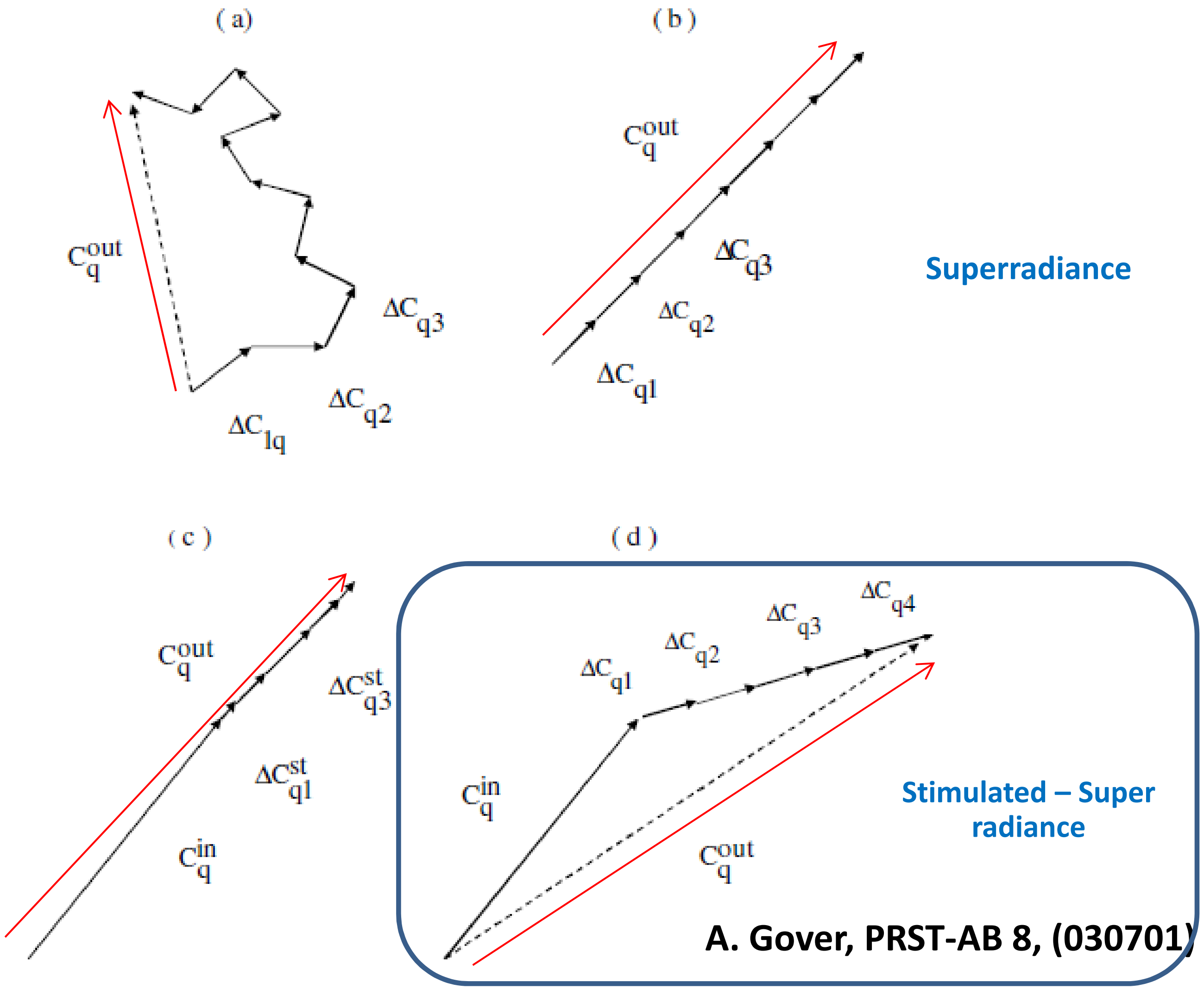


Spontaneous Emission

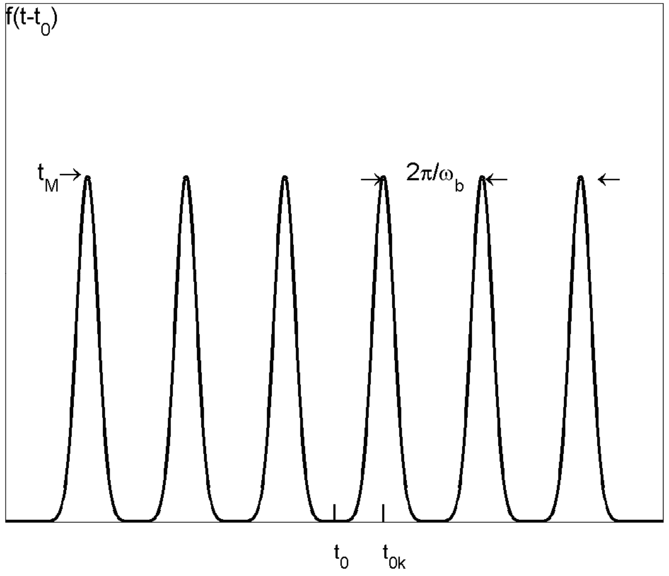
Superradiance

Stimulated Emission (Laser Amplifier)

Stimulated – Super radiance

A. Gover, PRST-AB 8, (030701) 2005

A TRAIN OF PULSES (MACRO-PULSE)



$$\sum_{j=1}^{N} e^{i\omega t_{0\,j}} = \sum_{k=1}^{N_M}\sum_{j=1}^{N_{bk}} e^{i\omega t_{0\,j}} =$$

$$\boxed{= N \cdot M_b\left(\omega\right)\cdot M_M\left(\omega\right)\cdot e^{i\omega t_0}}$$

Bunch form factor:

$$M_b\left(\omega\right) = \frac{1}{N_M}\sum_{J=1}^{N_M} e^{i\omega t_{0\,j}} = \int_{-\pi/\omega_b}^{\pi/\omega_b} f\left(t_0'\right)e^{i\omega t_0'}dt_0' \approx \mathrm{F}\left\{f\left(t_0'\right)\right\}$$

Macro-pulse form factor:

$$\mathrm{M}_{\mathrm{M}}\left(\omega\right) = \frac{1}{\mathrm{N}_M}\sum_{k=1}^{N_M} e^{i\omega t_{0_k}} = \frac{\sin\left(N_M\pi\omega/\omega_b\right)}{N_M\sin\left(\pi\omega/\omega_b\right)}$$

SR and ST-SR of UNDULATOR RADIATION

For a finite train of periodic bunches:

$$\left(\frac{dW_q}{d\omega}\right)_{SP-SR} = \frac{N^2 e^2 Z_q}{16\pi}\left(\frac{\overline{a}_w}{\beta_z\gamma}\right)^2 \frac{L^2}{A_{em}} |M_b(\omega)|^2 |M_M(\omega)|^2 sinc^2(\theta L/2)$$

$$\left(\frac{dW_q}{d\omega}\right)_{ST-SR} = \left|\check{C}_q^{in}(\omega)\right| \frac{Ne}{2\pi}\left(\frac{\overline{a}_w}{\beta_z\gamma}\right)\sqrt{\frac{2Z_q\mathcal{P}_q}{A_{emq}}}\, L|M_b(\omega)||M_M(\omega)| sinc(\theta L/2) cos(\varphi - \theta L/2)$$

Detuning parameter: $\theta(\omega)L = \left(\frac{\omega}{v_z} - k_{zq}(\omega) - k_w\right)L \cong 2\pi\frac{\omega - \omega_0}{\Delta\omega}$

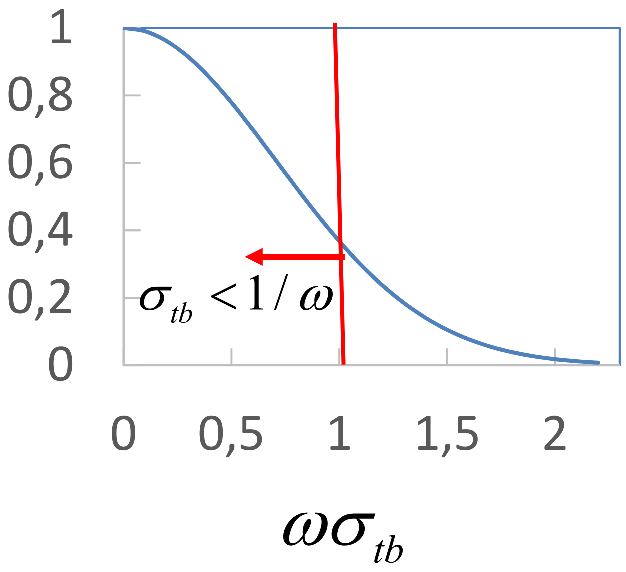
$\Delta\omega = \omega_0/N_w$, $\omega_0 = 2\gamma_{z0}^2\lambda_w$ $\gamma_z = \gamma / \sqrt{1 + \overline{a}_w^2}$, $\overline{a}_w = \overline{B}_w / k_w mc$

A. Gover, PRST-AB 8, (030701) 2005

Bunching coefficient:

$$\left(\text{for} \quad f(t_0) = e^{-t_0^2/2\sigma_{tb}^2} / \sqrt{2\pi}\sigma_{tb}\right)$$

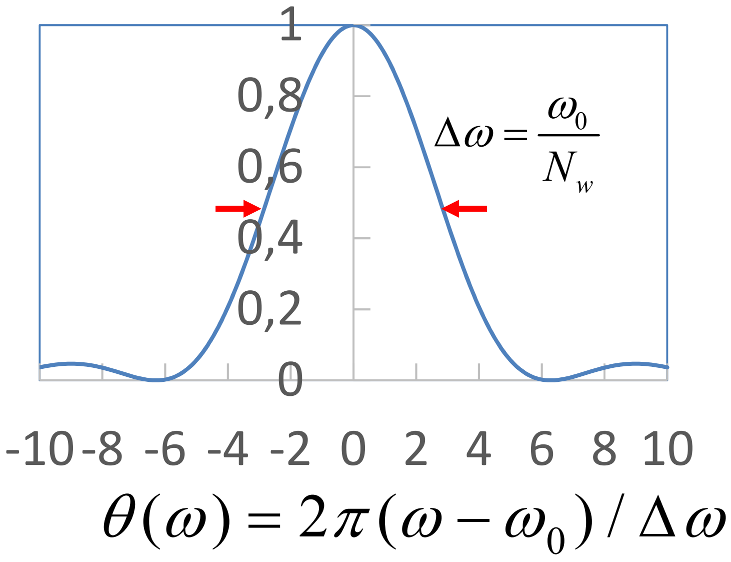
$$|M_b|^2 = e^{-\omega^2\sigma_{tb}^2}$$



Finite interaction length frequency line-shape

(N$_w$=# of wiggler periods)

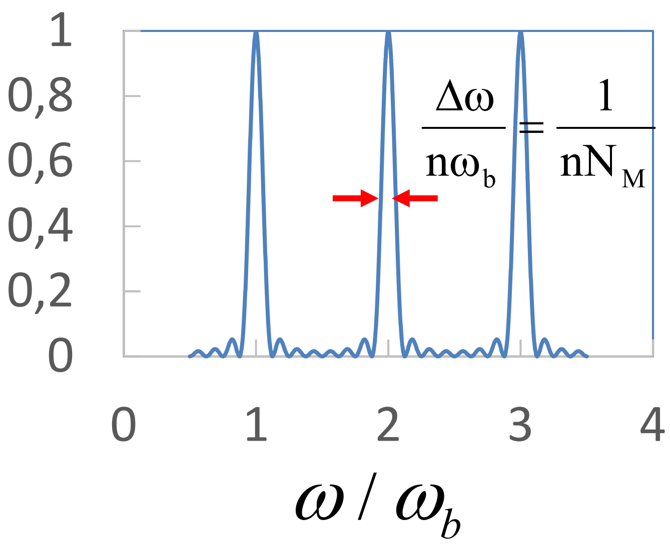
$$\operatorname{sin}c^2\left(\theta(\omega)L/2\right)$$



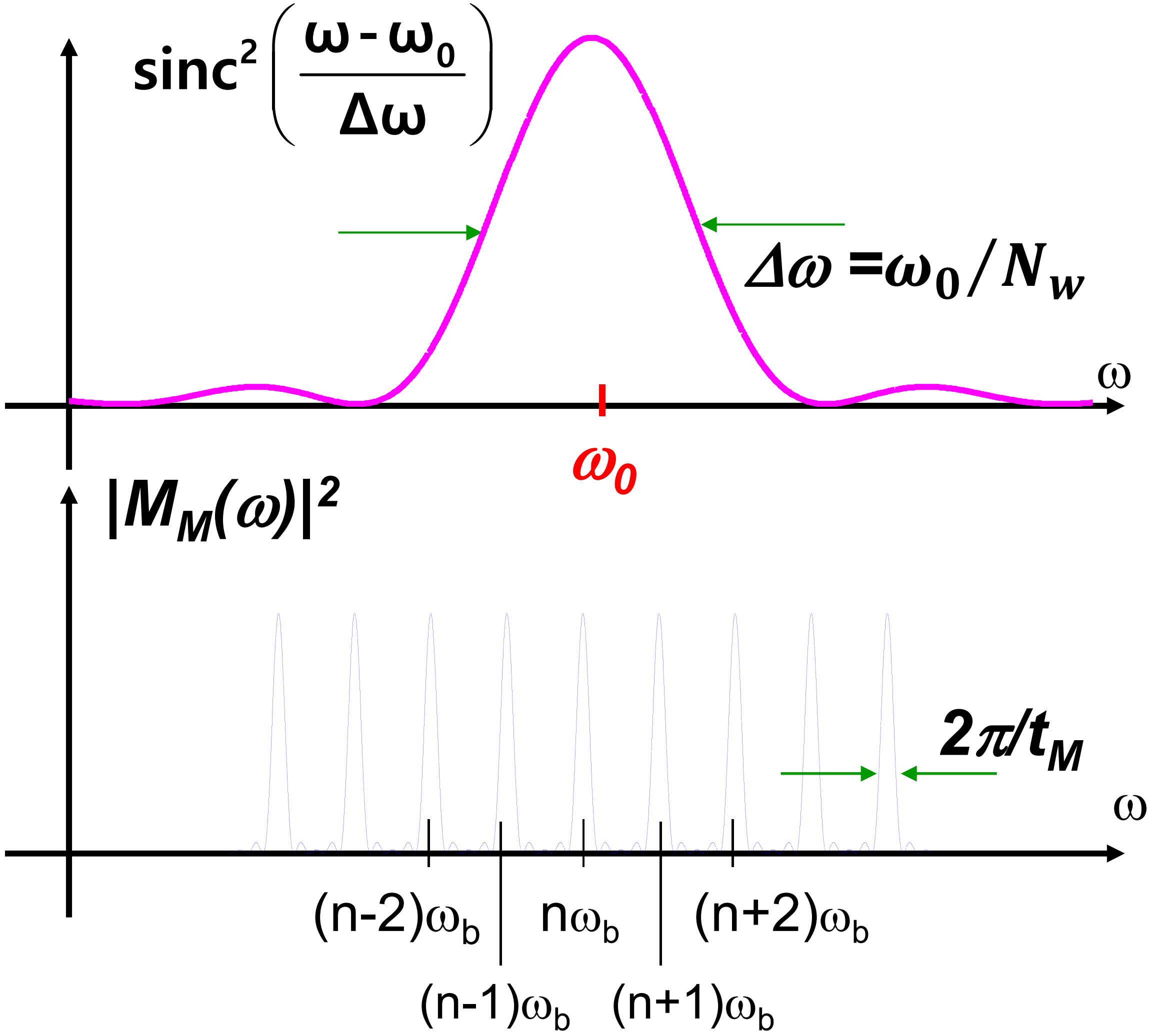
Pulse train form-factor:

(N$_M$=# of bunches in macropulse)

$$|M_M(\omega)|^2 = \frac{sin^2(N_M\pi\omega/\omega_b)}{N_M^2 sin^2(\pi\omega/\omega_b)}$$



Superradiant Emission from a Pulse Composed of a Train of Bunches



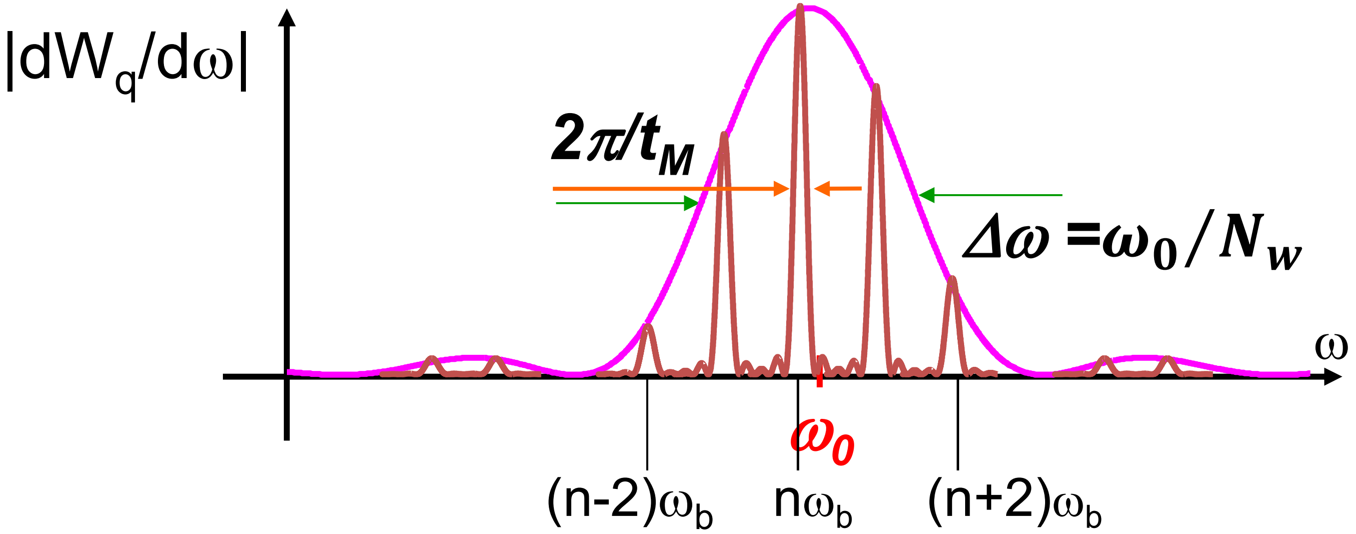
Single harmonic:

$$\omega_0 \approx n\omega_b$$

$$\omega_0 / N_w = n\omega_b / N_w < \omega_b \Rightarrow$$

$$n < N_w$$

Superradiant Emission from a Pulse Composed of a Train of Bunches



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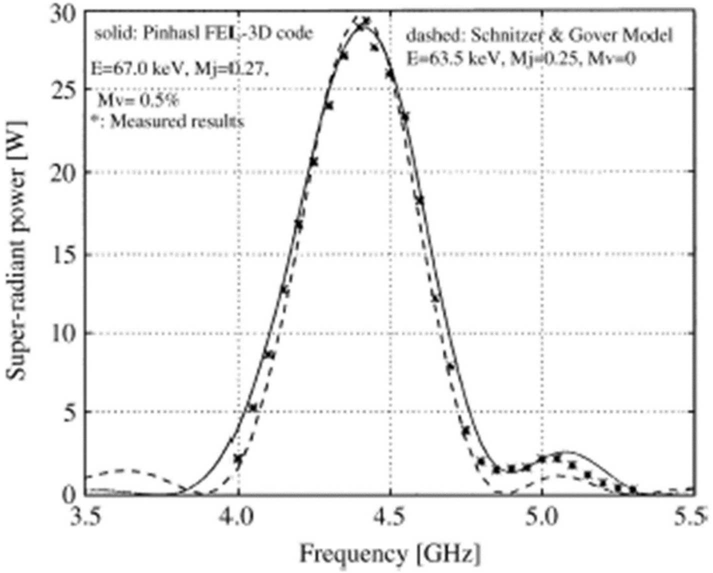
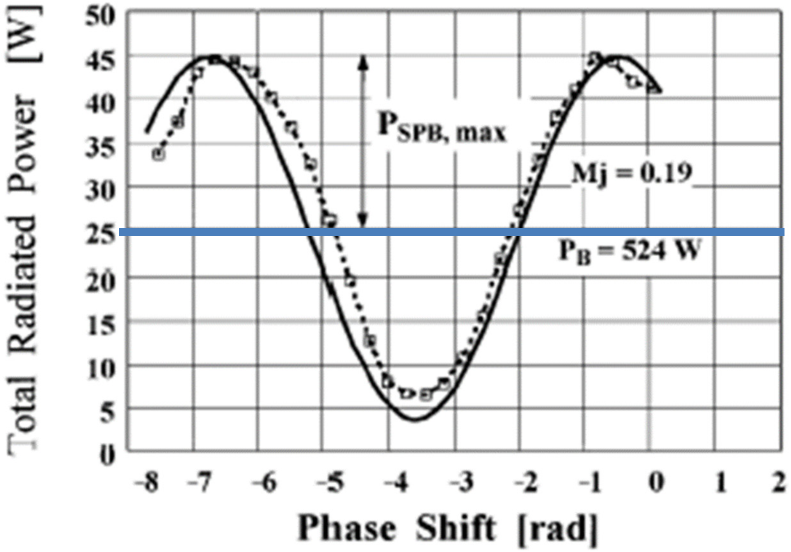
PREBUNCHED – FEM SUPERRADIANCE MEASUREMENT



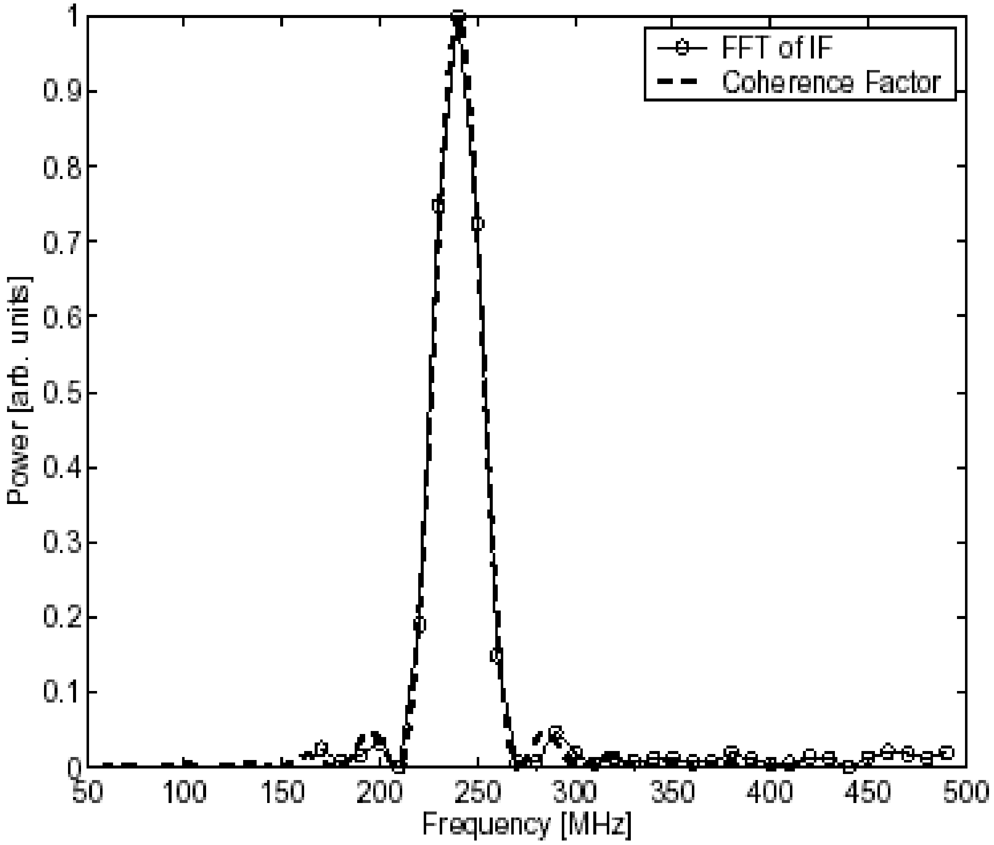
A. Cohen et al, PRL, **74**, 3812 (1995) ;
M. Arbel et al, NIM **A445** (2000)

PREBUNCHED-FEM STIMULATED SUPERRADIANCE MEASUREMENT



M. Arbel et al, PRL, 86, 256 (2001)

Mesured multi-bunch coherent Smith-Purcell linewidth (MIT - S.E. Korbly et al PRL 2005)



$f_n = n f_b = 240 GHz$

14 1.7 GHz

$\Delta f / f_n = 1 / n N_M = 1.3 \cdot 10^{-4}$

14 550

SP-SR HARMONIC POWER EMISSION of A LONG PERIODICALLY BUNCHED e-BEAM $(N_M >> N_w)$

Discrete harmonics: $\omega_n = n\omega_b = 2\gamma_{z0}^2\lambda_w$

Spectral radiant energy:

$$\left(\frac{dW_q}{d\omega}\right)_{SR} = \frac{N^2 e^2 Z_q}{16\pi}\left(\frac{\bar{a}_w}{\beta_z\gamma}\right)^2 \frac{L^2}{A_{em}} |M_b(\omega)|^2 |M_M(\omega)|^2 sinc^2(\theta L/2)$$

Integrate over frequencies → Power of harmonic n:

$$P_{q,n} = \frac{dW_{qn}(\omega=\omega_n)}{d\omega}\left(\frac{\omega_b}{N_M}\right) \Big/ t_M$$

$$\boxed{P_{q,n} = \frac{I_0^2}{8}\sqrt{\frac{\mu_0}{\varepsilon_0}}\left(\frac{\bar{a}_w}{\beta_z\gamma}\right)^2 |b_n|^2 \frac{L_w^2}{A_{em,q}(\omega_n)} \sin c^2\left(\theta(\omega_n)/2\right)}$$

[Bunching parameter :

$b_n = M_b(\omega=\omega_n) = e^{-\omega_n^2\sigma_{tb}^2/2}$]

ELECTRON BEAM BUNCHING SCHEMES

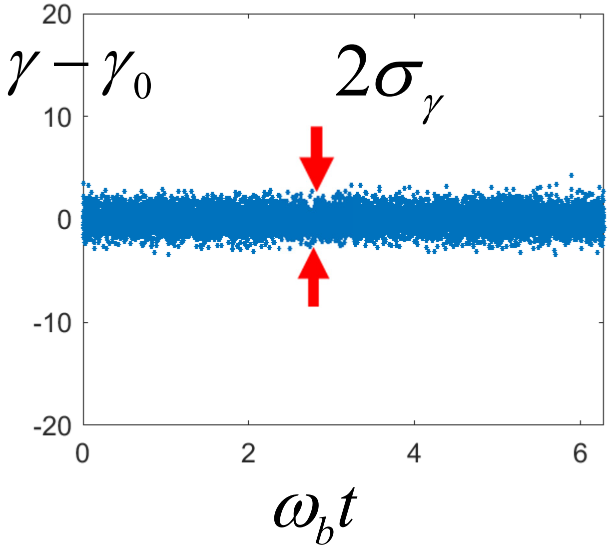
- Photo-cathode emission (sub-pSec)
- Klystron (RF cavity) (Superradiance)
- Optical klystron oscillator (Stimulated-Superradiance)(Vinokurov and Skrinsky in 1977)
- Ultrafast laser bunching:
  - HGHG (Li Hua Yu)
  - EEHG (Stupakov)
  - E-SASE (Zholent)

Extensive reference list: Gover et al Rev. of Mod. Phys. Vol/EID: 91/035003 (August 2019)

BUNCHING BY LASER MODULATION AND DISPERSIVE SECTION

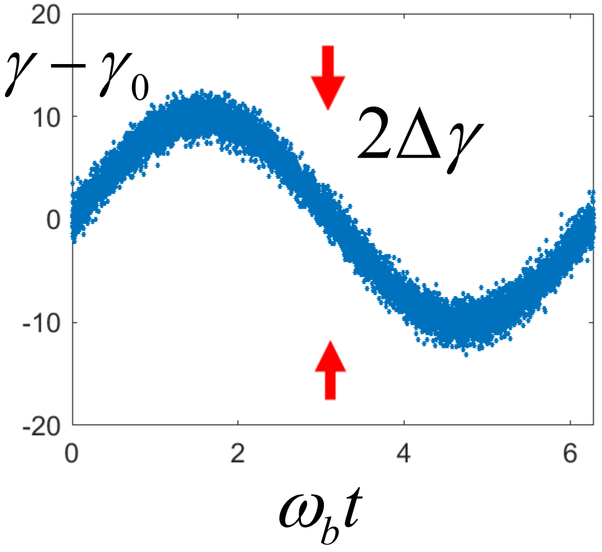
Energy distribution:

$$f\left(\gamma_j-\gamma_0\right)=\frac{1}{\sqrt{2\pi}\sigma_{\gamma 0}}e^{-\left(\gamma_j-\gamma_0\right)^2/2\sigma_{\gamma 0}^2}$$



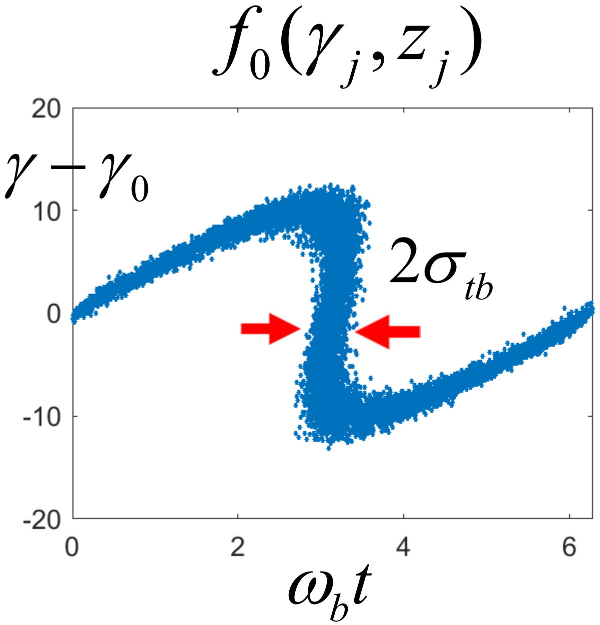
Energy modulation:

$$\gamma_i=\gamma_0+\Delta\gamma_{\text{mod}}\sin\omega_b t_j$$



Dispersive section:

$$t_j'=t_j+\frac{R_{56}}{c}\left(\gamma_j-\gamma_0\right)$$



BUNCHING OPTIMIZATION

Normalized parameters *:

$$p_j = \frac{\gamma_j - \gamma_0}{\sigma_{\gamma 0}}, \qquad A = \frac{\Delta\gamma_{\text{mod}}}{\sigma_{\gamma 0}}, \qquad B = \omega_b \frac{R_{56}}{c}\frac{\sigma_{\gamma 0}}{\gamma_0}$$

$$f_0\left(p_j, t_j\right) = \frac{1}{\sqrt{2\pi}} e^{-\left[p_j - A\sin\omega_b t_j - Bp_j\right]^2/2}$$

$$b_n = \left\langle e^{i\omega_n \Delta t_j{}'\left(t_j, \gamma_j\right)} \right\rangle = \int_{-T_b/2}^{T_b/2} dt_j \int_{-\infty}^{\infty} dp_j f_0\left(t_j, p_j\right)$$

$$\boxed{b_n = J_n\left(nAB\right) e^{-\omega_n^2 \sigma_{tb}{}^2/2}} \qquad \left(\sigma_{tb} = \frac{R_{56}}{c}\frac{\sigma_{\gamma 0}}{\gamma_0}\right)$$

Maximal bunching for $AB \approx 1$:

$$b_n \approx \frac{0.67}{n^{1/3}} e^{-\omega_n^2 \sigma_{tb}^2/2}$$

*Hemsing E., Stupakov G., Xiang D., & Zholents A., Rev. Mod. Phys., 86(3), 897 (2014)

Phase-merging enhanced harmonic generation

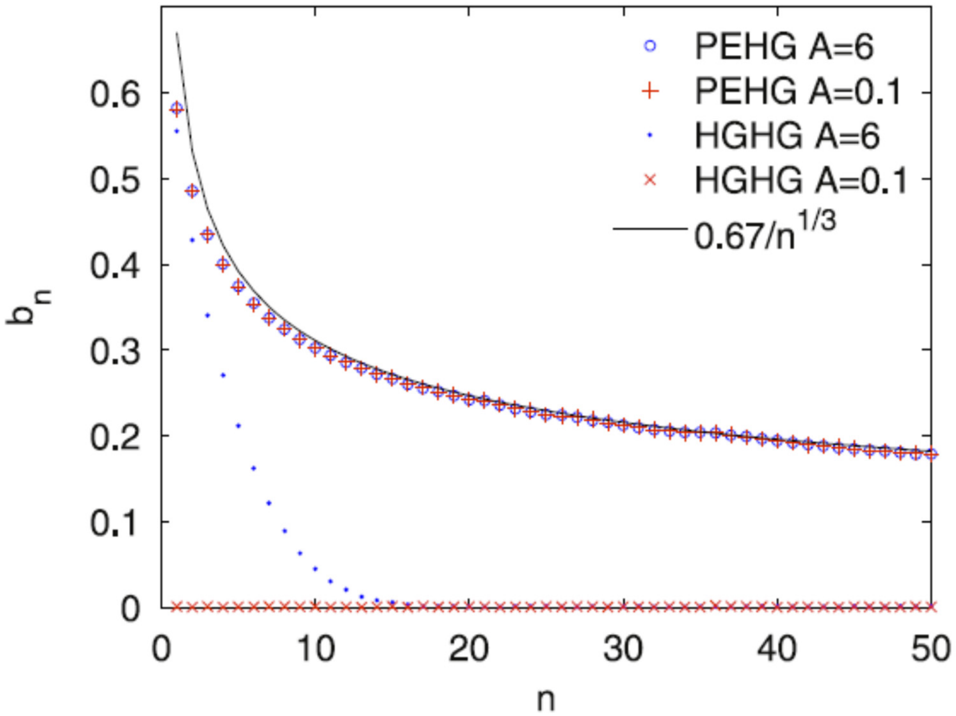
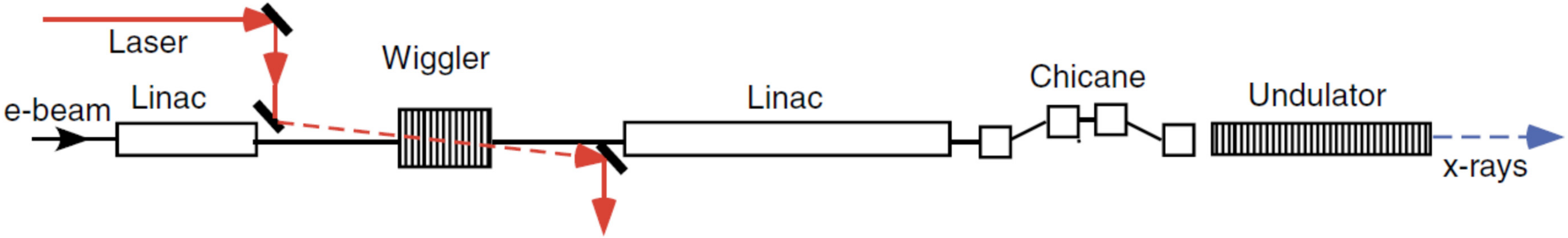


Figure 3. Comparison of the bunching factor of PEHG and standard HGHG with different energy modulation amplitudes. The black line is the theoretical prediction of the maximal bunching factor of PEHG.

Feng, C., Deng, H., Wang, D., & Zhao, Z. (2014). Phase-merging enhanced harmonic generation free-electron laser. *New Journal of Physics*, *16*(4), 043021.

ULTRA-COMPACT X-RAY FEL BASED ON e-SASE*



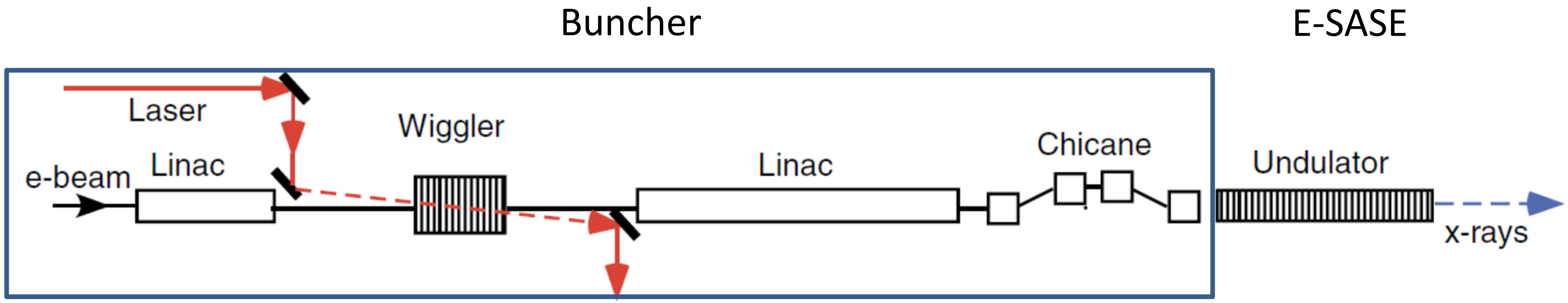
Towards Ultra-Compact X-ray FEL, J. Rosenzweig, UCLA Moore Foundation Workshop 1/22-25/2019

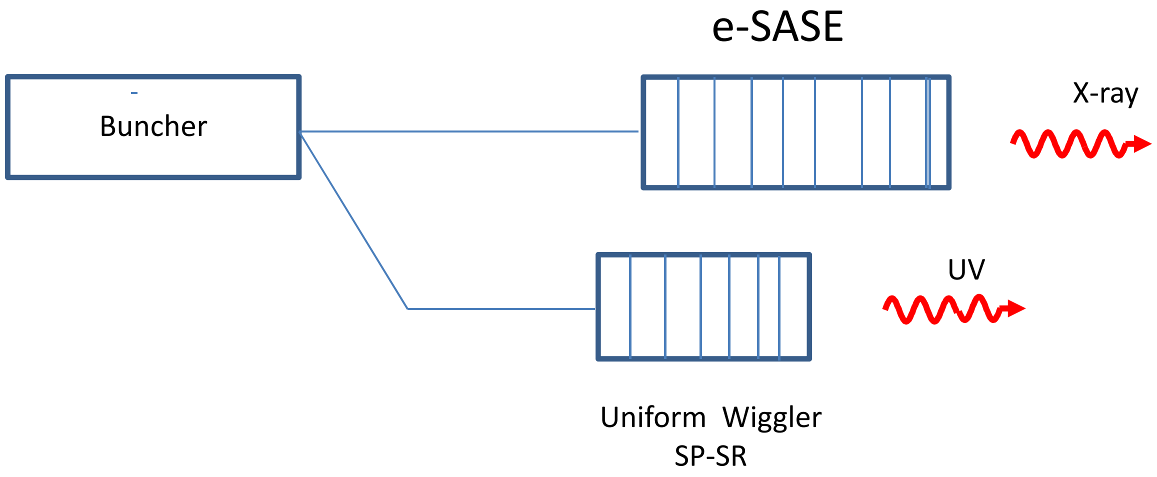
$$E=1GeV\ \ (\gamma=2000)\qquad \lambda_b=3.2\mu\qquad Q_M=200\,pC\qquad I_M=800\,A$$

$$with\ \lambda_w=1.2mm\quad\Rightarrow\ \lambda=1.57\text{Å}$$

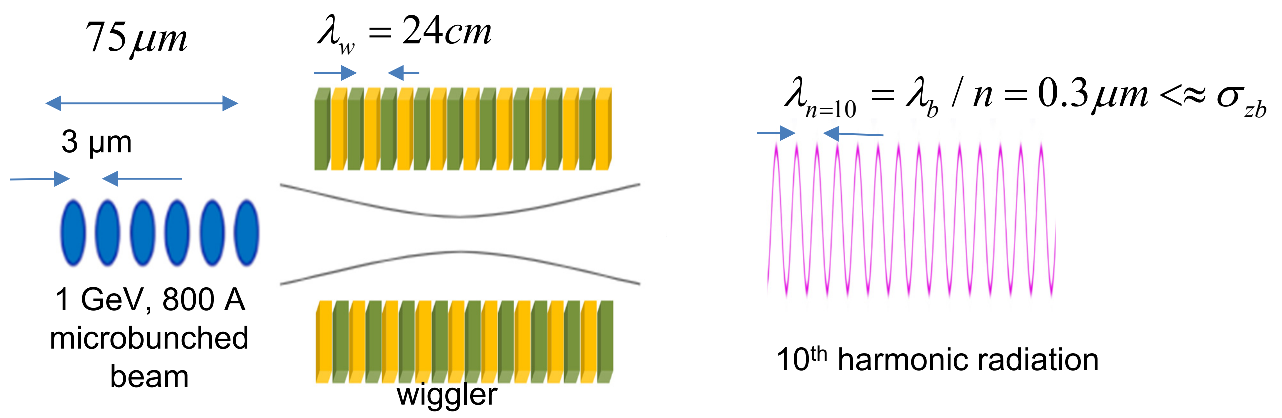
A. A. ZHOLENTS Phys. Rev. ST Accel. Beams 8, 040701 (2005)

EXERSIZE # 1: HIGH HARMONIC UV SUPERRADIANT SOURCE





A Model Problem with a SAMURAI beam: 10th harmonic SR



$$\bar{a}_w = 10, \quad N_w = 10, \quad L_w = 2.4m \Rightarrow \lambda_w = 2\frac{\gamma^2}{\cancel{1+}\,\bar{a}_w^2}\lambda_{n=10} = 24cm$$

$$L_w = 2z_R(\lambda_{n=10}) \Rightarrow A_{em,q}(\omega_n) = \frac{z_R\lambda_n}{2} = \frac{L_w\lambda_{10}}{4}$$

$$\Rightarrow \quad P_{q=TE_{0,0},n=10} = \frac{I_0^2}{8}\sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{L_w^2}{A_{em}} = 2.4GW$$

***RIVER ROBLES - STUDENT UCLA PHYSICS DEPT.**

Steady-state (single frequency) phasor formulation for periodic pre-bunching

SINGLE-MODE PHASOR MODEL for SP-SR /ST-SR $(\omega_0 = n\omega_b)$

$$\tilde{C}_q(z) = \tilde{C}_q(0) - \frac{1}{4P_q}\iiint_0^z \tilde{\boldsymbol{J}}_\perp(\boldsymbol{r},\omega_0)\cdot\varepsilon^*_{q\perp}(r_\perp)\,e^{-ik_{zq}z}\,d^2r_\perp\,dz$$

$$P_q(z) = P_q\left|\tilde{C}_q(z)\right|^2 \qquad\qquad P_q(z) = P_q(0) + P_{SR}(z) + P_{ST-SR}(z)$$

$$P_{SP-SR} = \frac{1}{32} Z_q \left|\bar{I}_m\right|^2 F^2 \frac{z^2}{A_{em}} \operatorname{sin}c^2\left(\frac{\theta_0 z}{2}\right)$$

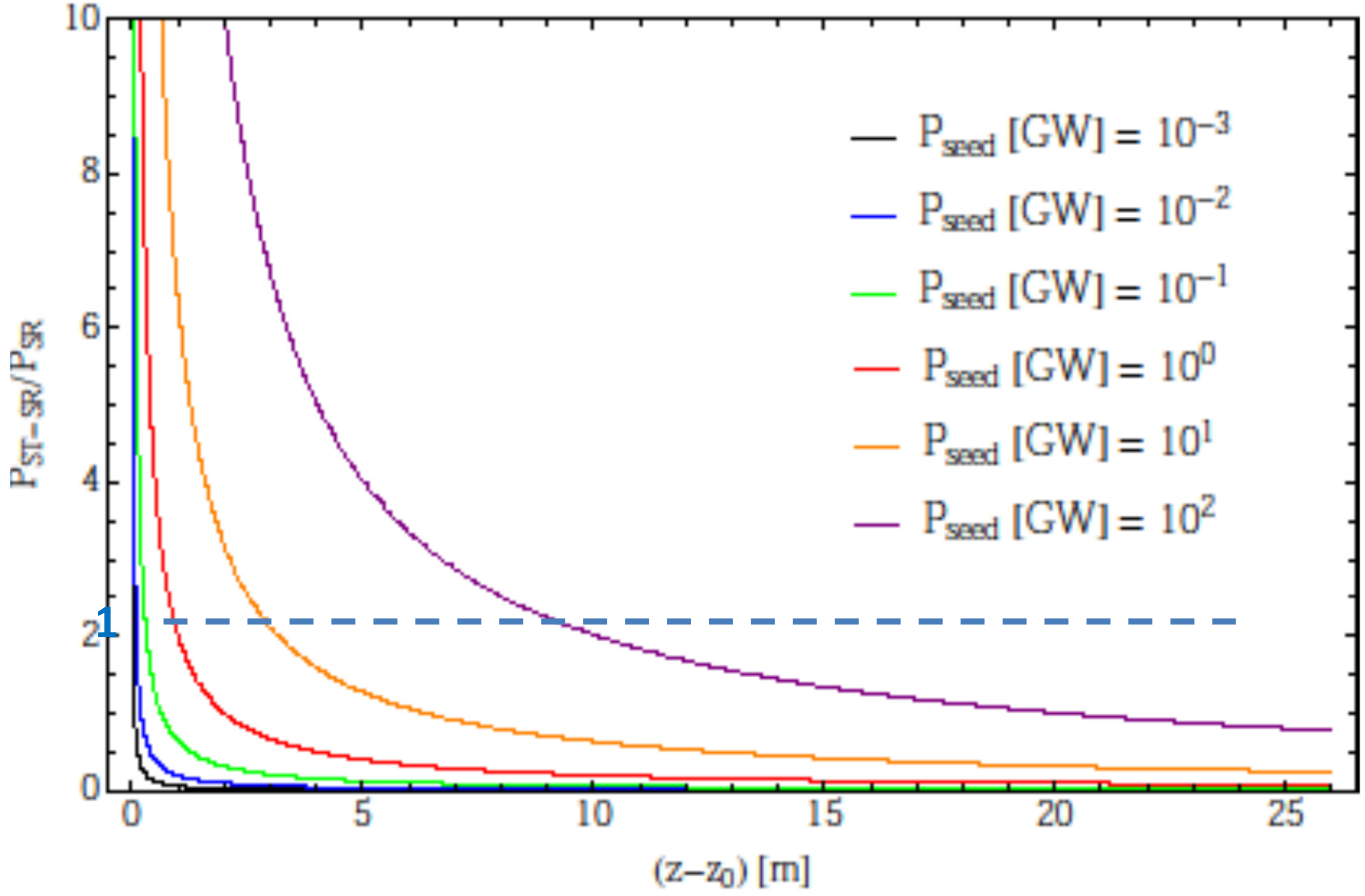
$$P_{ST-SR} = \frac{1}{4}\left|\bar{I}_m\right|\left|E_\perp(0)\right| Fz \cos\left(\varphi_0^r - \varphi_0^b - \theta_0 z/2\right)\operatorname{sin}c\left(\frac{\theta_0 z}{2}\right)$$

Relative weight of ST-SR and SP-SR Power

$$\frac{P_{ST-SR}}{P_{SP-SR}} = \frac{\frac{1}{4}\bar{I}_m\sqrt{\frac{2Z_q}{A_{em}}}\sqrt{P_{in}}\,Fz}{\frac{1}{32}Z_q\left|\bar{I}_m\right|^2 F^2\frac{z^2}{A_{em}}} = 8\frac{A_{em}}{Z_q I_m F z}\sqrt{\frac{27q}{A_{em}}}\sqrt{P_{in}} = 8\frac{A_{em}}{Z_q I_m F z}E_{in}(0)$$

★ E. A. Schneidmiller, M. V. Yurkov, “Optimization of a high efficiency FEL amplifier” PRST-AB 18, 03070 (2015)

Ratio of <u>0-order</u> ST-SR to SP-SR
(calculated for the parameters of the tapered section of LCLS X-FEL)



SR and ST-SR IN THE NONLINEAR REGIME

PERIODIC TIGHT BUNCHING MODEL

$$\mathbf{J}(\mathbf{r},t) = Q_b \mathbf{v}_e(t) f(\mathbf{r}_\perp) \sum_{n=-\infty}^{\infty} \delta[z - z_e(t - nT_b - t_0)]$$

$$\Rightarrow \quad \mathbf{J}(z,t) = \mathbf{J}_0 + \sum_{n=1}^{\infty} 2Re\left[\tilde{\mathbf{J}}_n e^{-in\omega_b t}\right]$$

$$\frac{d\tilde{C}_q(z)}{dz} = \frac{1}{4P_q} \int \tilde{\boldsymbol{J}}(\boldsymbol{r}) \cdot \boldsymbol{E}_q^*(\boldsymbol{r}) d^2\boldsymbol{r}_\perp \qquad \mathrm{P}_{\mathrm{q,n}} = P_{\mathrm{q}} \left|\tilde{\mathrm{C}}_{\mathrm{q,n}}\right|^2$$

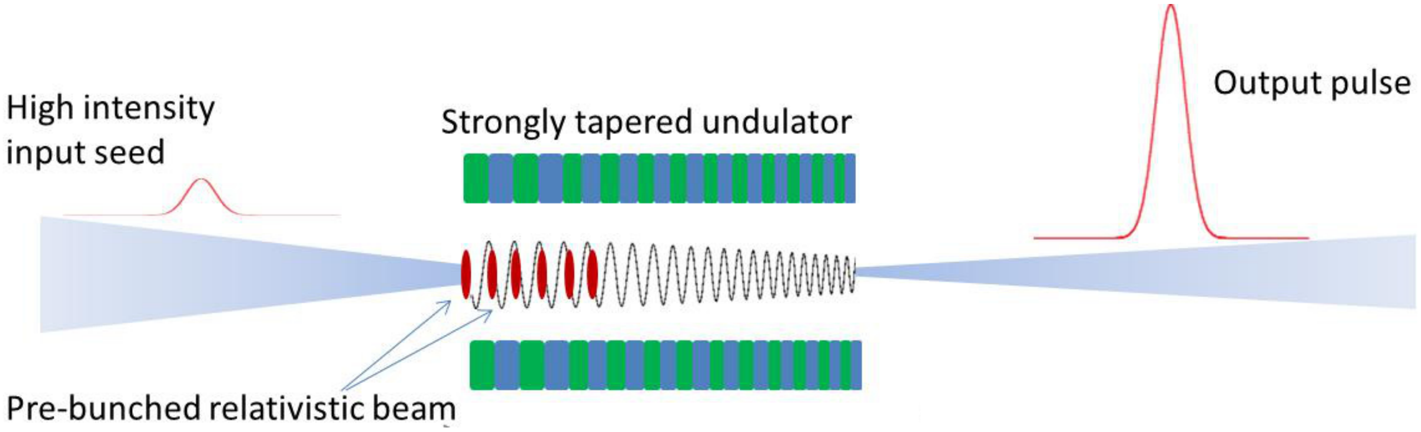
$$\mathrm{N_b mc^2} \frac{\mathrm{d}\gamma}{\mathrm{dz}} = \frac{\mathrm{Q_b}}{\beta_z} \boldsymbol{\beta} \cdot \mathbf{E}\left(\mathbf{r}, \mathrm{t}_e(z)\right) \qquad \mathrm{P_e} = \mathrm{N_b mc^2}\left(\gamma - 1\right) / \mathrm{T_b}$$

Self-consistent nonlinear model formulation for an infinite pulse (or finite pulse with zero-slippage):

$$\frac{\mathrm{dP_{q,n}}}{\mathrm{dz}} + \frac{\mathrm{dP_e}}{\mathrm{dz}} = 0$$

NONLINEAR SR AND ST-SR INTERACTION OF A PREBUNCHED BEAM IN AA WIGGLER

TAPERING-ENHANCED STIMULATED SUPERRADIANT AMPLIFICATION - TESSA



N.M. Kroll, P.L. Morton, M.N. Rosenbluth, IEEE J. Quant. Electron., QE-17, 1981

**N. Sudar, P. Musumeci et al "High Efficiency Energy Extraction …Tapered Undulator"
PRL 117, 174801 (2016)**

**A. Gover, R. Ianconescu, A. Friedman, C. Emma, N. Sudar, P. Musumeci, C. Pellegrini,
"Superradiant and stimulated-superradiant emission of bunchedelectron beams"
Rev. Mod. Phys. 91, 035003 – Published 19 August 2019**

SYNCHRONIZM CONDITION OF A TRAPPED BUNCH

$$\theta(z) = \frac{\omega_0}{v_z(z)} - \overbrace{k_w(z) - k_{zq}(z)}^{pm-wave} \qquad (\omega_0 = n\omega_b)$$

$$\theta(z) = -2k_w(z)\frac{\delta\gamma}{\gamma_r(z)}$$

Controlled

trap dynamics

$$\gamma = \gamma_r(z) + \delta\gamma$$

<u>Resonant energy of a trapped bunch:</u>

$$\theta(z) = 0 \Rightarrow \gamma_r^2(z) = \frac{1+\bar{a}_w^2(z)}{2}\frac{k_0}{k_w(z)} \qquad \left(\bar{a}_w(z) = \frac{e\bar{B}_w(z)}{k_w(z)mc}\right)$$

<u>Phase of ponderomotive (pm) wave relative to bunches:</u>

$$\Psi(z) = -\int_0^z \theta(z')dz' + \Psi(0) + \underbrace{\left[\varphi_q(z) - \varphi_q(0)\right]}_{\approx 0} \qquad \left(\tilde{C}_q(z) = \left|\tilde{C}_q(z)\right| e^{i\varphi_q(z)}\right)$$

RADIATION EMISSION AND BUNCH DYNAMICS – UNIFORM WIGGLER

<u>Single mode:</u> $\tilde{E}(z) = \tilde{C}_q(z)\left|\tilde{\varepsilon}_{q\perp}(0)\right|$ $\qquad \gamma_r = const$

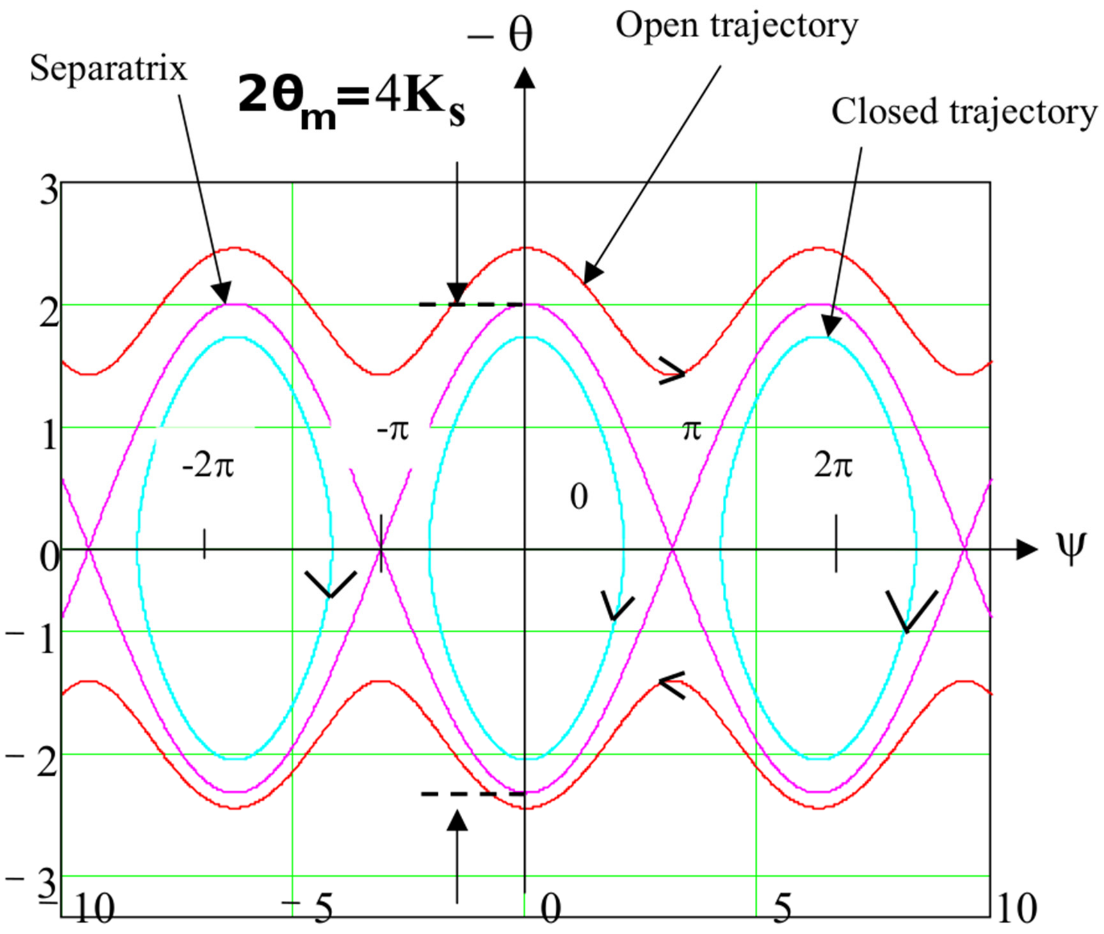
$$\frac{d\left|\tilde{E}\right|}{dz} = b\sin\Psi$$

$$\frac{d\theta}{dz} = K_s^2(z)\sin\Psi \qquad\qquad \frac{d\delta\gamma}{dz} = \frac{\gamma_{zr}^2\gamma_r}{k_0}K_s^2(z)\sin\Psi$$

$$\frac{d\Psi}{dz} = -\theta + \frac{b}{\left|\tilde{E}\right|}\cos\Psi \qquad\qquad \frac{d\Psi}{dz} = \frac{k_0}{\gamma_{zr}^2\gamma_r}\delta\gamma + \frac{b}{\left|\tilde{E}\right|}\cos\Psi$$

$$K_s^2(z) = \frac{k_0 e}{2\gamma_{zr}^2\gamma_r{}^2 mc^2}\bar{a}_w(z)\left|\tilde{E}(z)\right| \qquad\qquad b = \frac{\bar{I}_b\bar{a}_w(z)Z_q}{2A_{em,q}\gamma_r}$$

The Θ-Ψ phase-space trajectories of the pendulum equation



RADIATION EMISSION AND BUNCH DYNAMICS – TAPERED WIGGLER

$$\gamma = \gamma_r(z) + \delta\gamma(z) \qquad \frac{d\theta}{dz} = \frac{k_0}{\gamma_{zr}^2\gamma_r}\left(\frac{d\gamma}{dz} - \frac{d\gamma_r}{dz}\right)$$

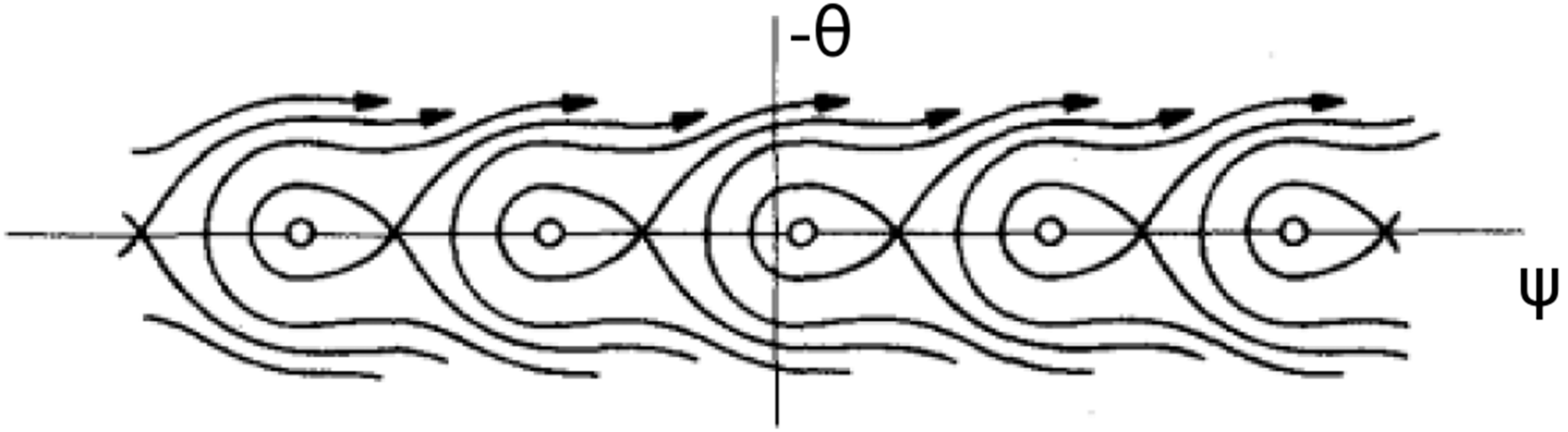
$$\frac{d\left|\tilde{E}\right|}{dz} = b\sin\Psi$$

$$\frac{d\theta}{dz} = K_s^2(z)\left[\sin\Psi - \sin\Psi_r\right] \qquad \frac{d\delta\gamma}{dz} = \frac{\gamma_{zr}^2\gamma_r}{k_0}K_s^2K_s^2(z)\left[\sin\Psi - \sin\Psi_r\right]$$

$$\frac{d\Psi}{dz} = -\theta \quad + \frac{b}{\left|\tilde{E}\right|}\cos\Psi \qquad \frac{d\Psi}{dz} = -\theta \quad + \frac{b}{\left|\tilde{E}\right|}\cos\Psi$$

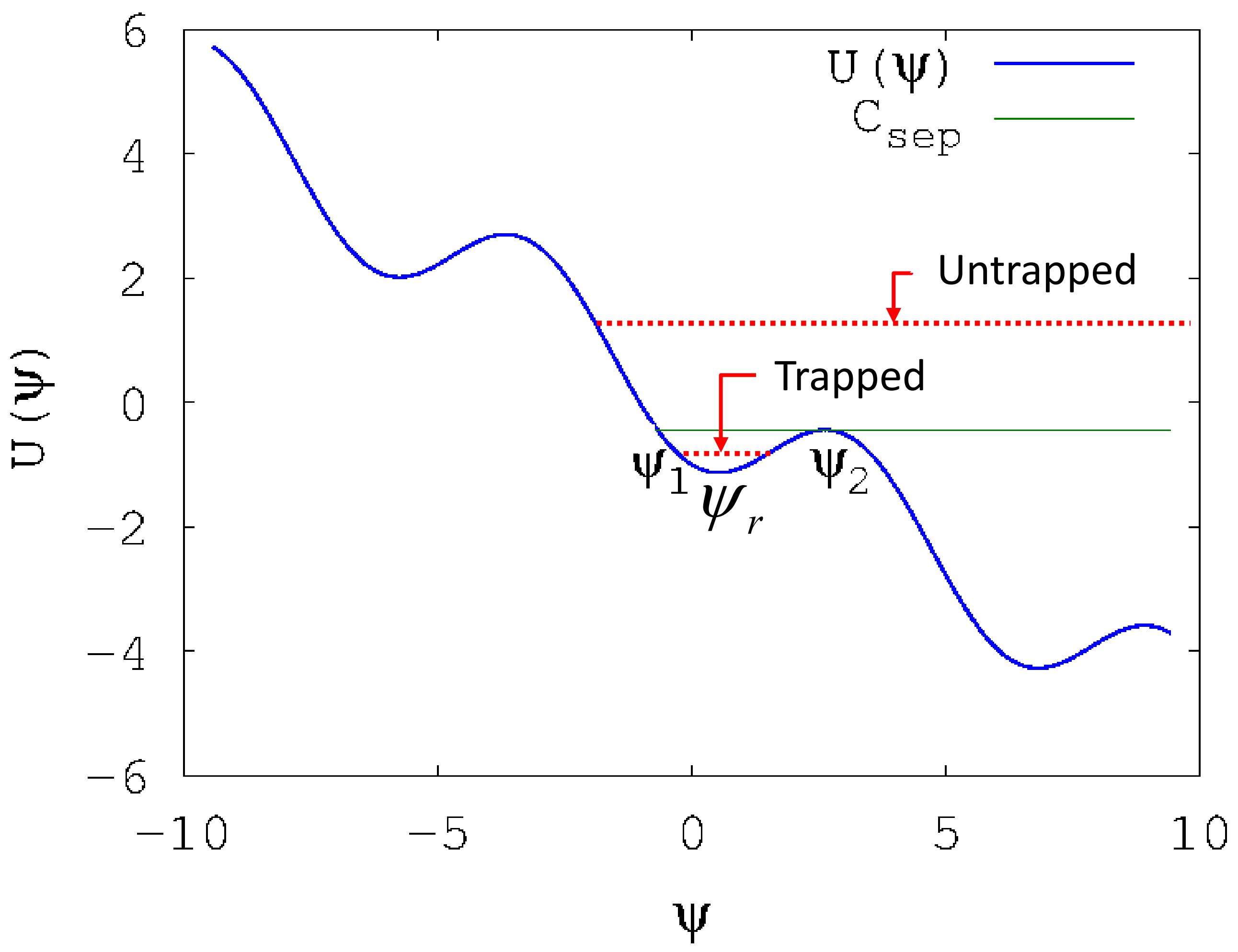
$$\sin\Psi_r = \frac{k_0}{\gamma_{zr}^2\gamma_r K_s^2}\frac{d\gamma_r}{dz}$$

PHASE-SPACE TRAJECTORIES IN A TILTED PENDULUM

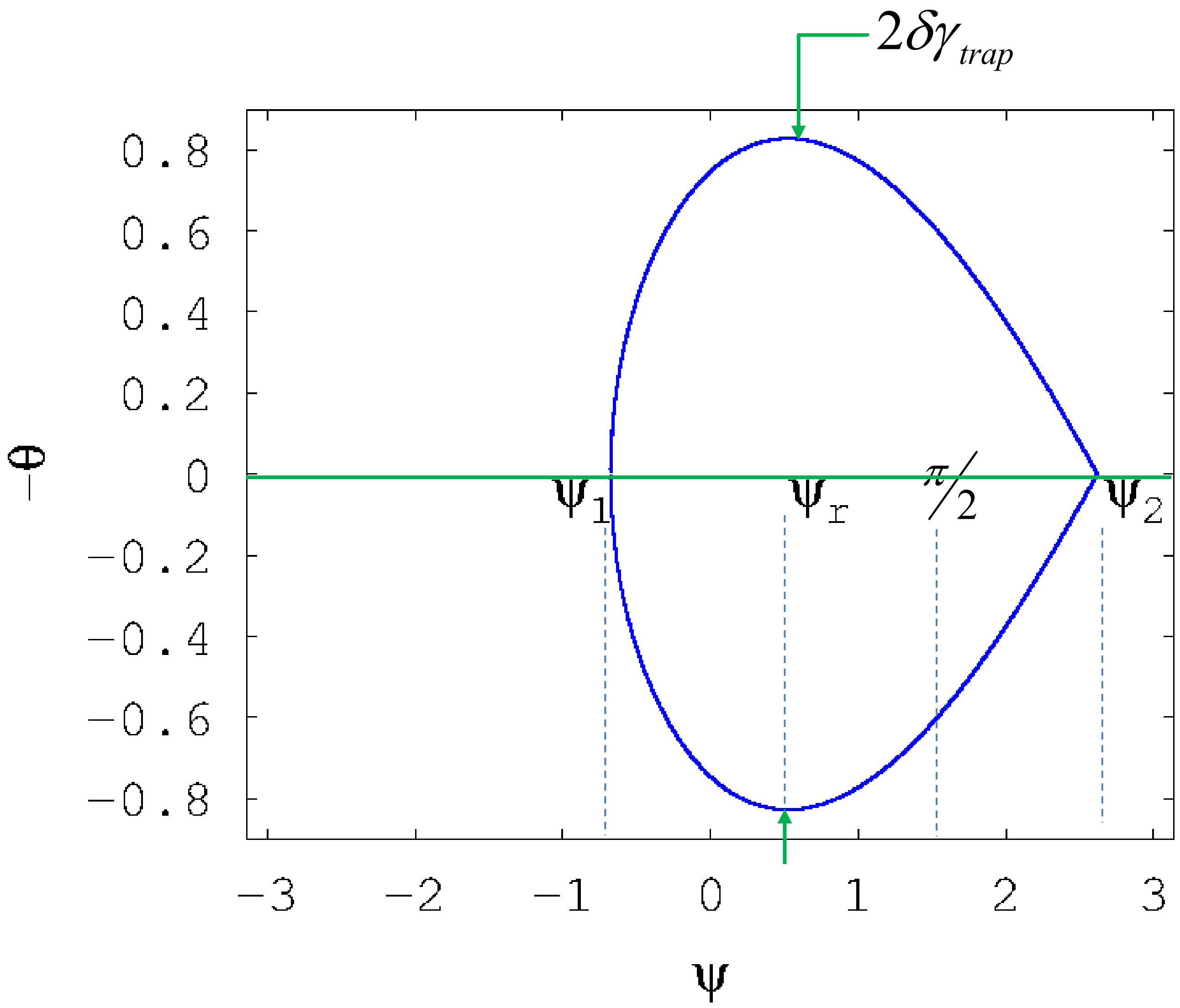


N.M. Kroll, P.L. Morton, M.N. Rosenbluth, "Free-Electron Lasers with Variable Parameter Wigglers", IEEE J. Quant. Electron., VOL. QE-17, NO. 8, AUGUST 1981

The potential energy for a tapered wiggler



SEPARATRIX OF TRAP IN A TAPERED WIGGLER



ST-SR IN THE NONLINEAR REGIME
2nd ORDER PERTURBATIVE SOLUTION

For short interaction length in normalized parameters $(\mathrm{u} = \mathrm{z} / \mathrm{L}_{\mathrm{w}})$:

$$\Delta \mathrm{P}_{\mathrm{q}} / \mathrm{P}_{\mathrm{REF}} = \overline{\mathrm{E}}^{2}(\mathrm{u}) - \overline{\mathrm{E}}^{2}(0)$$

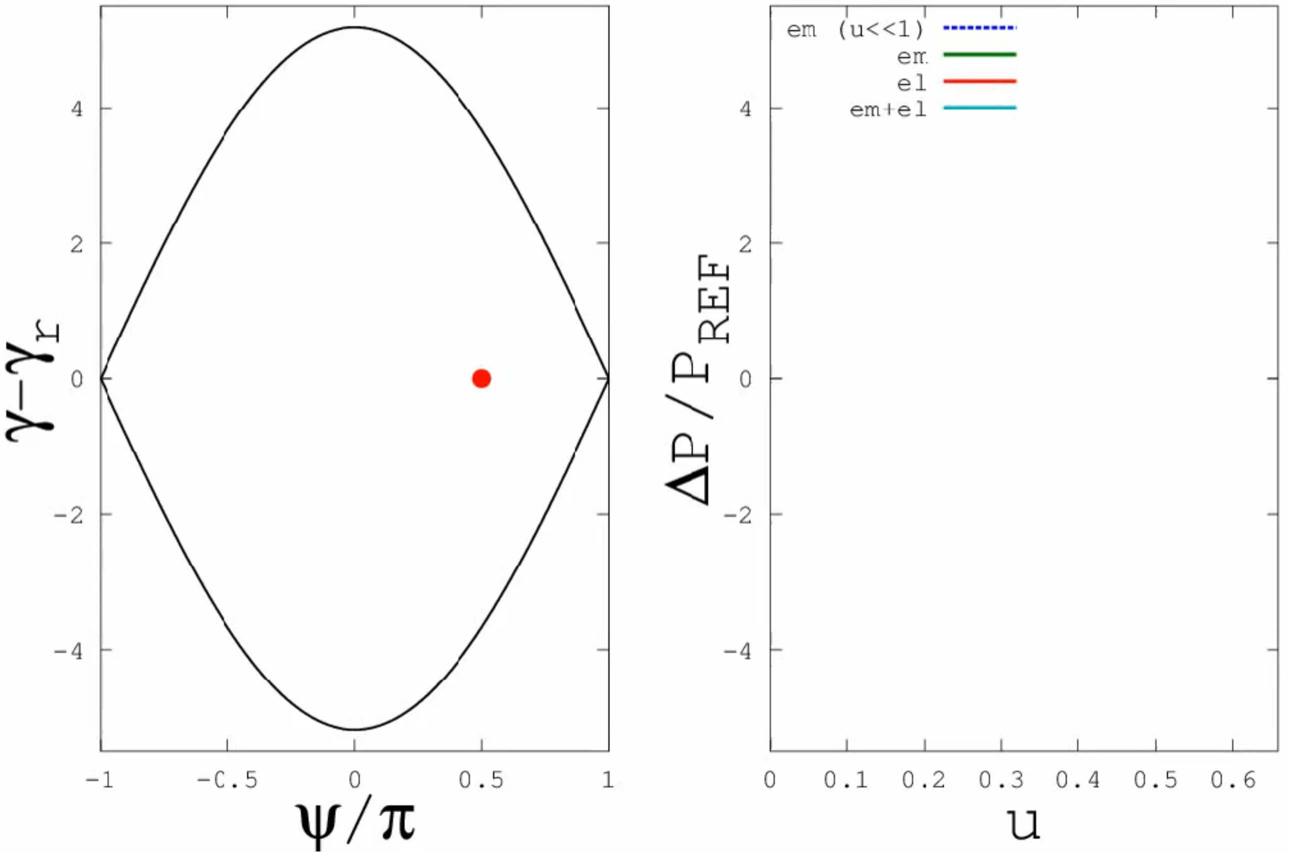
$$= \left[2u\bar{E}(0)\underbrace{\left(\sin\psi(0) - \sin\psi_r\right)}_{\text{ST-SR}} + \underbrace{u^2 \sin^2\psi(0)}_{\text{SR}} \right] + \underbrace{2u\bar{E}(0)\sin\psi_r}_{\text{TAPERING}}$$

For $\psi(0) = \psi_r$

$$\boxed{\Delta P_{em} / P_{REF} = 2u\bar{E}(0)\sin\psi_r + u^2 \sin^2\psi_r}$$

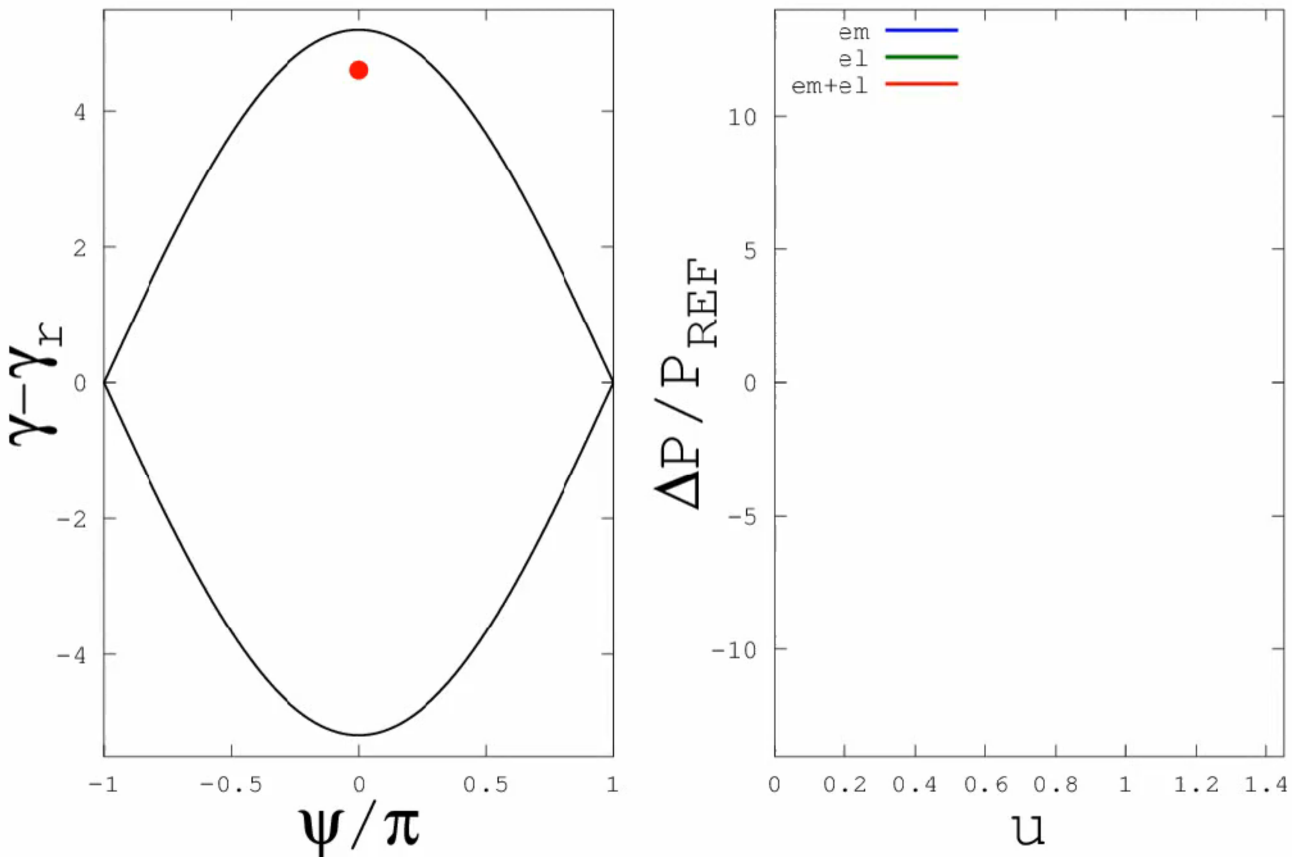
UNIFORM WIGGLER: HIGH GAIN ST-SR

Ψ(0)= 0 π/2



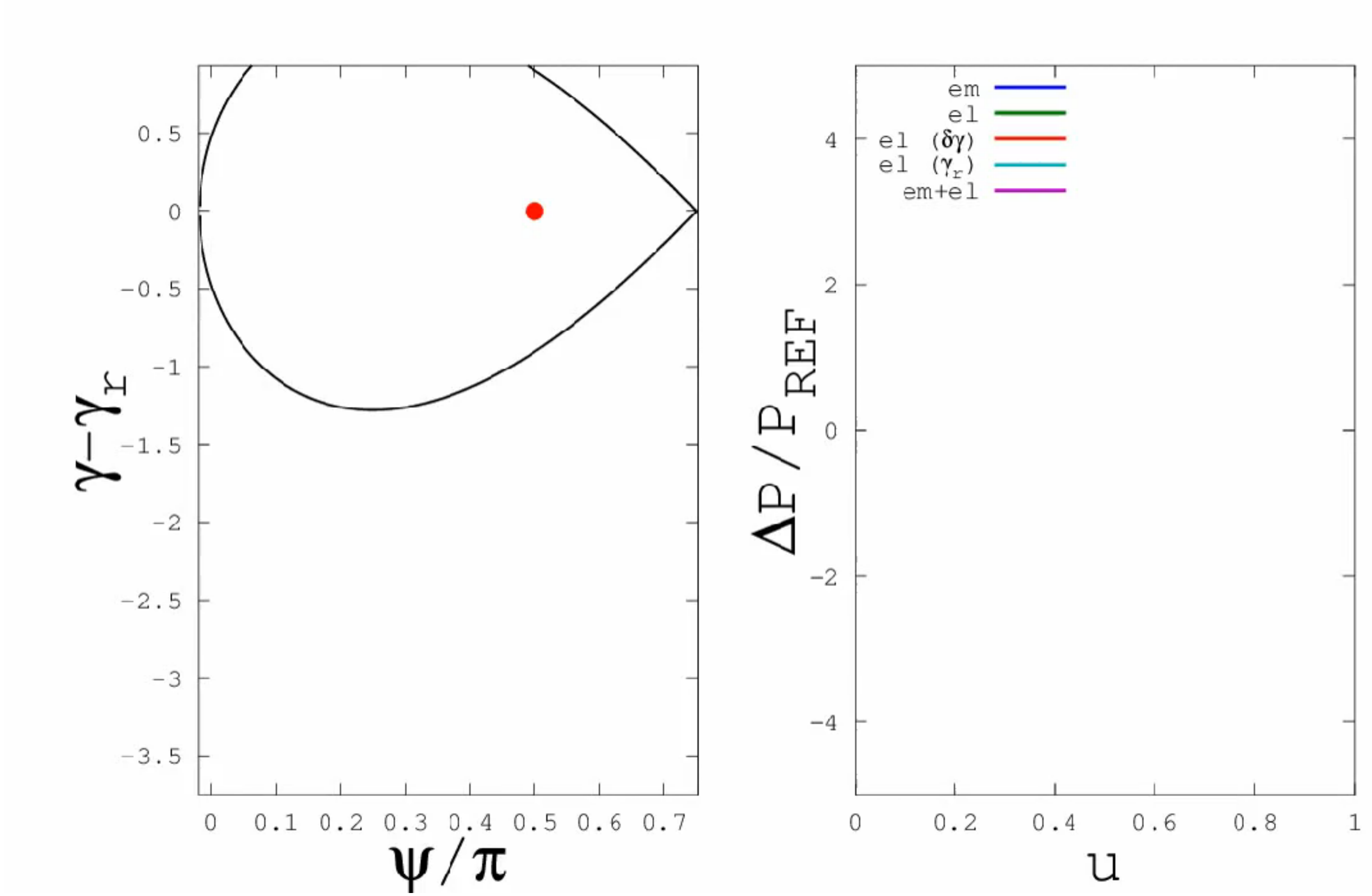
Uniform wiggler: maximal extraction

Ψ(0)= 0 π/2



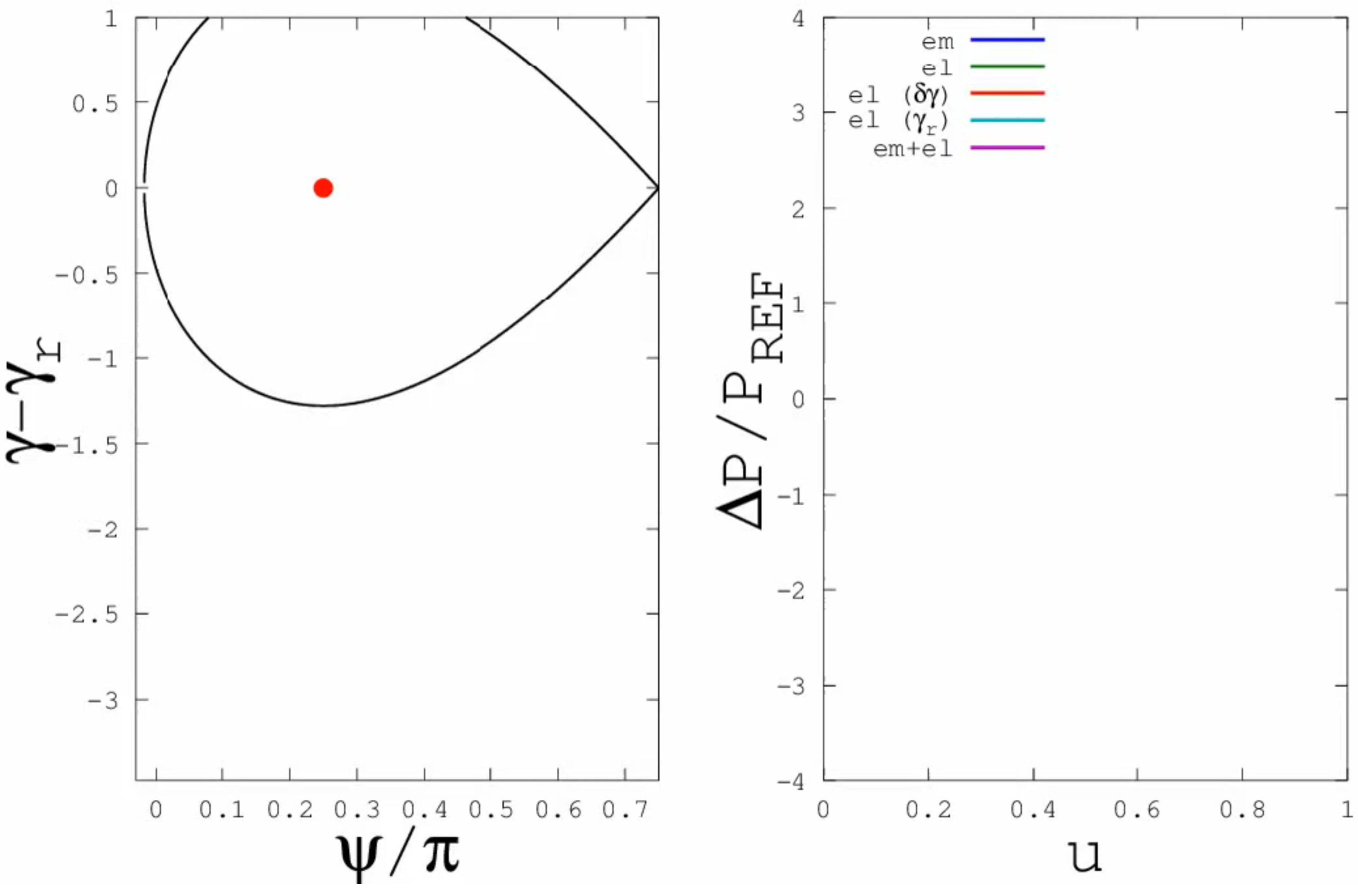
Tapered wiggler: maximal gain ST-SR

$\Psi(0)=$ 0 Ψ_r $\pi/2$



Tapered wiggler: best trapped bunch

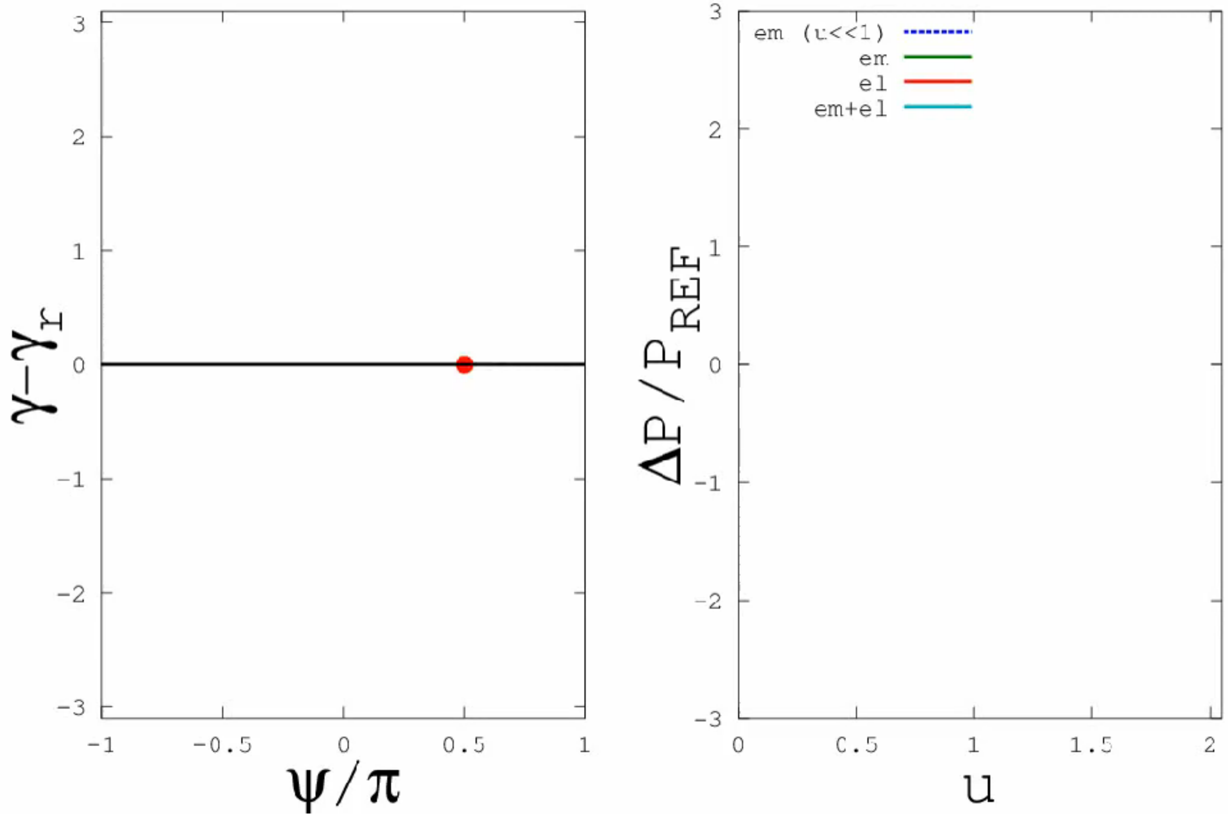
Ψ(0)=   0 Ψ_r π/2



Uniform wiggler: self-interaction

Ψ(0)= 0 π/2

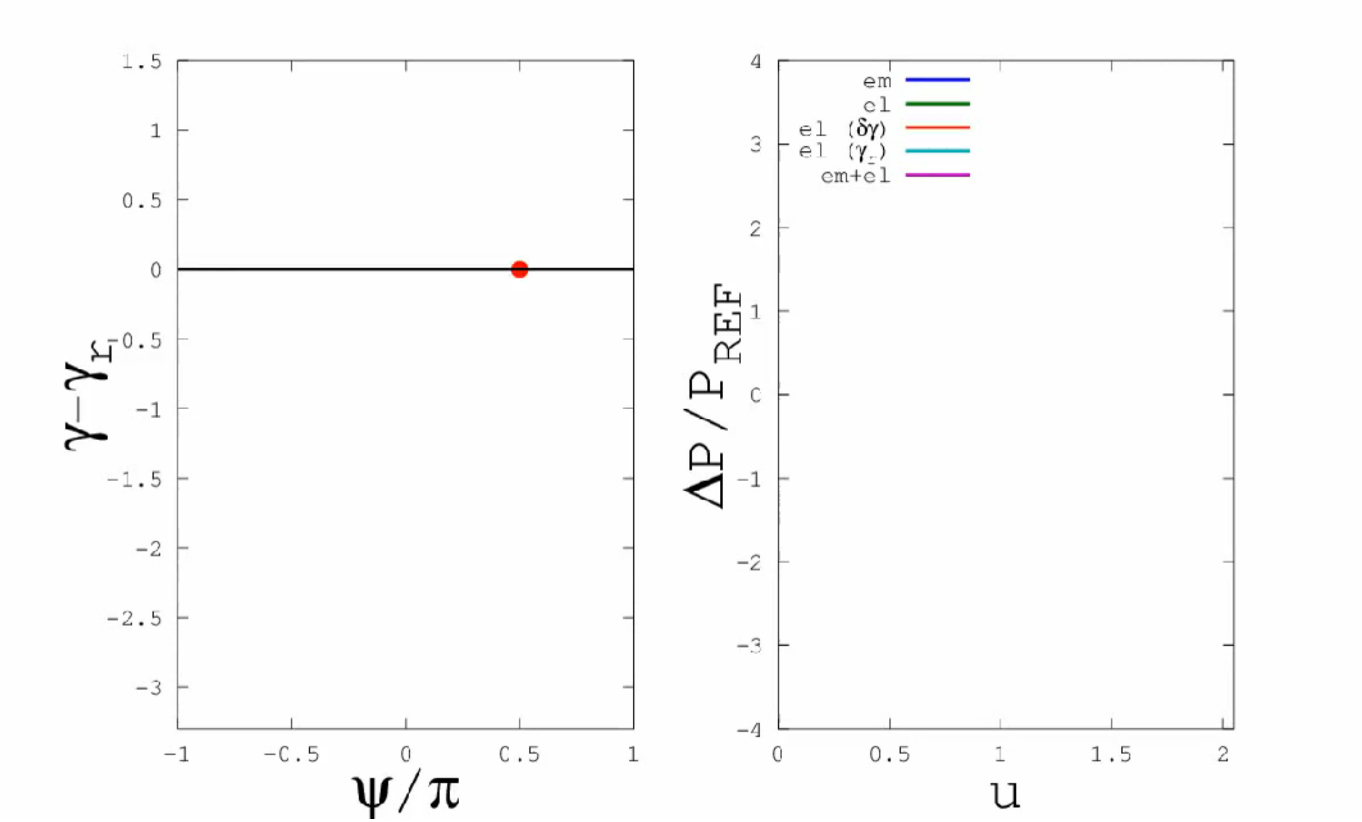
(NO RADIATION REACTION FORCE IS INVOKED)



Abraham-Lorentz Radiation Reaction, Dirac, P. A., 1938, Proc. R. Soc. A 167, 148.

SELF INJECTION: SR TO TESSA

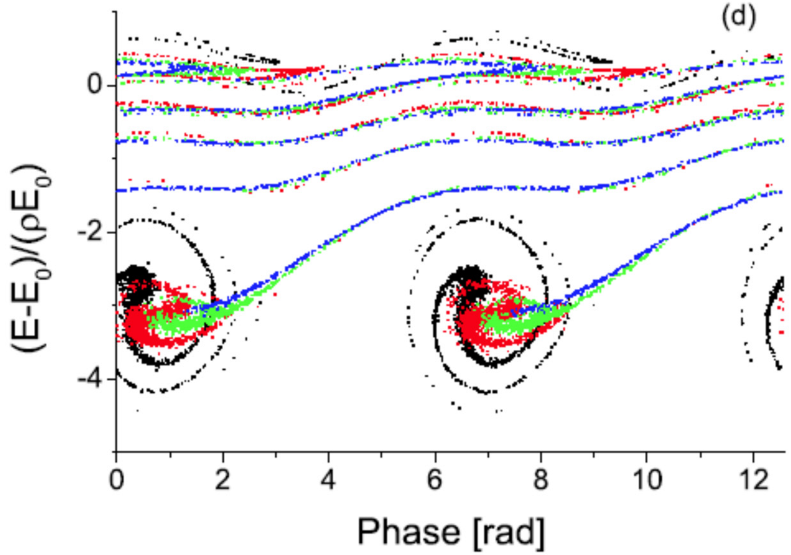
Ψ(0)= 0 π/2

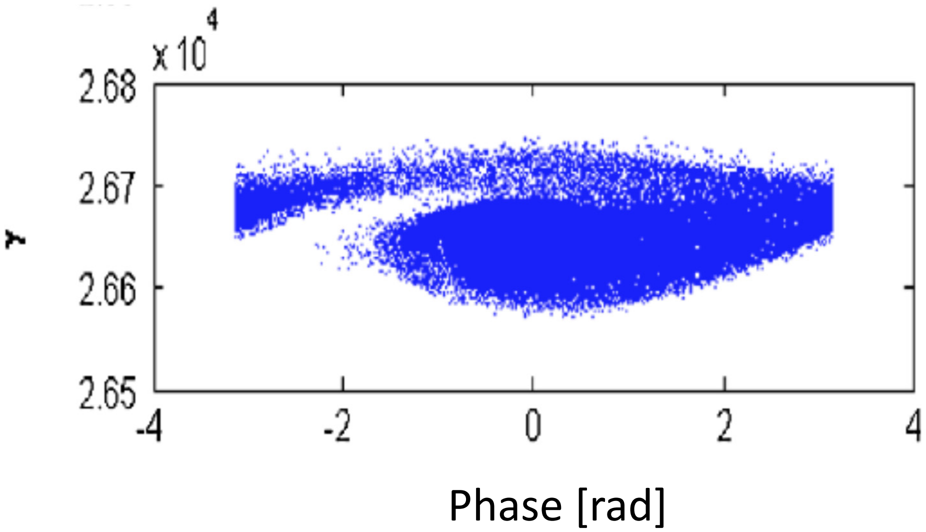


Snively, E. C., Xiong, J., Musumeci, P., & Gover, A. *Optics Express*, *27*(15), 20221(2019). Broadband THz amplification and superradiant spontaneous emission in a waveguide FEL.

EXTENTION TO DISTRIBUTED BUNCH

Phase – space trajectories of a realistic bunch in a tapered wiggler FEL





N. M. Kroll, P. L. Morton, M. N. Rosenbluth, IEEE J. Quant. Electron. 17, 1436 (1981)
Y. Jiao et al, "Modeling and multidimensional optimization of a tapered free electron laser" PRST-AB 15, (050704) (2012)
E. A. Schneidmiller, M. V. Yurkov, "Optimization of a high efficiency FEL amplifier" PRST-AB 18, 03070 (2015)

TESSA: EXTENTION TO DISTRIBUTED BUNCH

Bunches get trapped in the tapered undulator in ponderomotive buckets.

f_t : fraction trapped.

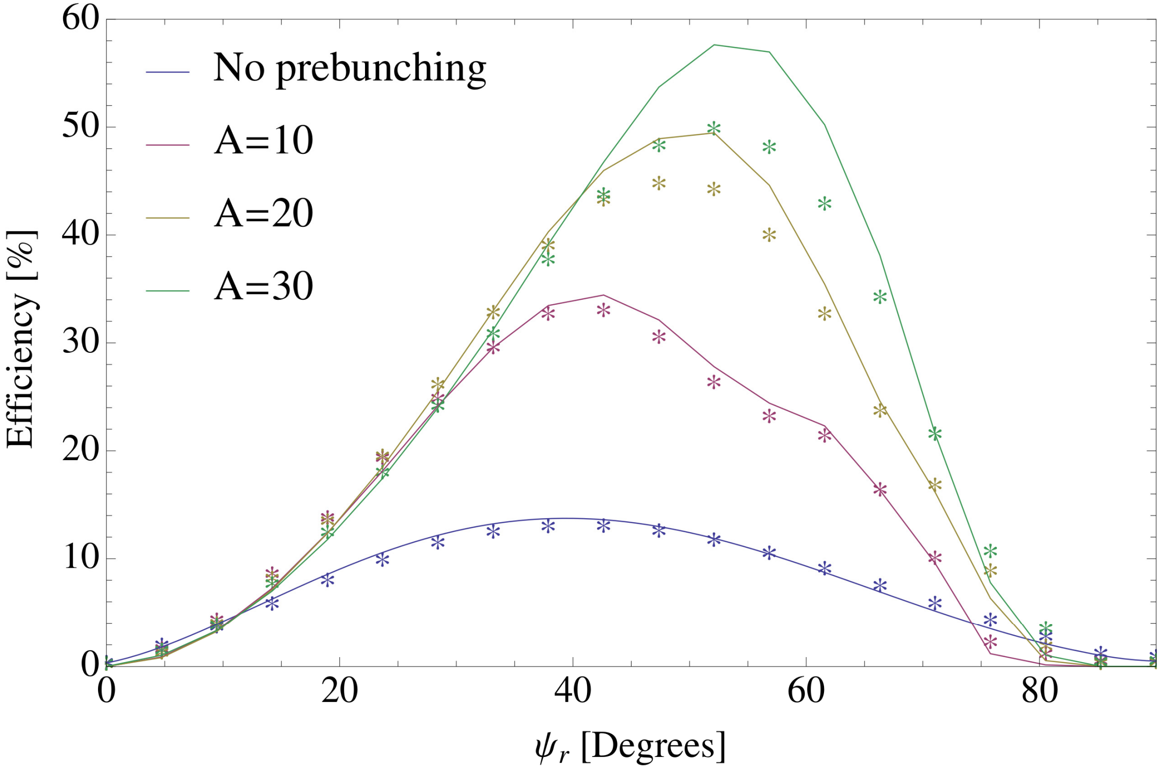
$$P_{rad}(z) = P_0 + E_0 \underset{\underset{\text{TAPERING}}{\uparrow}}{\frac{\bar{a}_w(0)}{\gamma_0}} f_t I z \sin\psi_r + \frac{Z_0}{4A_{emq}} \left(\frac{\bar{a}_w(0)}{\gamma_0}\right)^2 \underset{\underset{\text{SR}}{\uparrow}}{\left(f_t I z \sin\psi_r\right)^2}$$

→ Need to optimize $f_t(\psi_r, \Delta\gamma_{mod}) \sin\psi_r$

N. Sudar, P. Musumeci et al "High Efficiency Energy Extraction …Tapered Undulator"
PRL 117, 174801 (2016)

TRAPPING EFFICIENCY

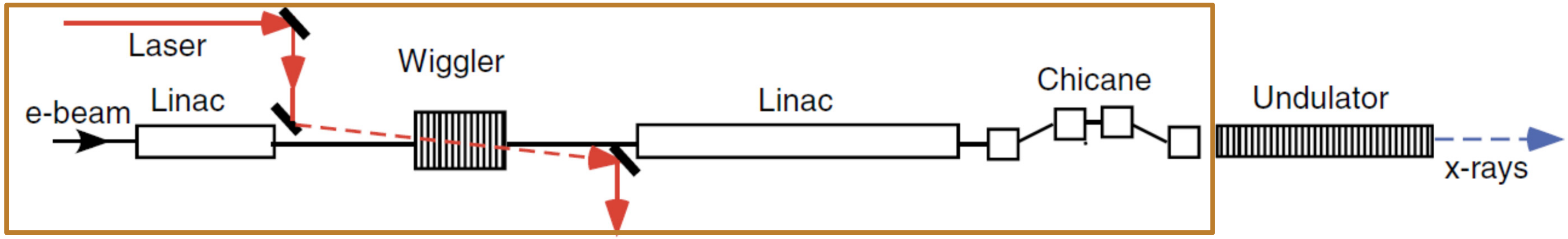
$$f_t = \int_{\Psi_1}^{\Psi_2} d\psi \int_{-\delta\gamma_{trap}/\sigma_{\gamma 0}}^{\delta\gamma_{trap}/\sigma_{\gamma 0}} dp f_0\left(p,\psi\right) \qquad p = \frac{\gamma - \gamma_0}{\sigma_{\gamma 0}} \qquad A = \frac{\Delta\gamma_{\mathrm{mod}}}{\sigma_{\gamma 0}}$$

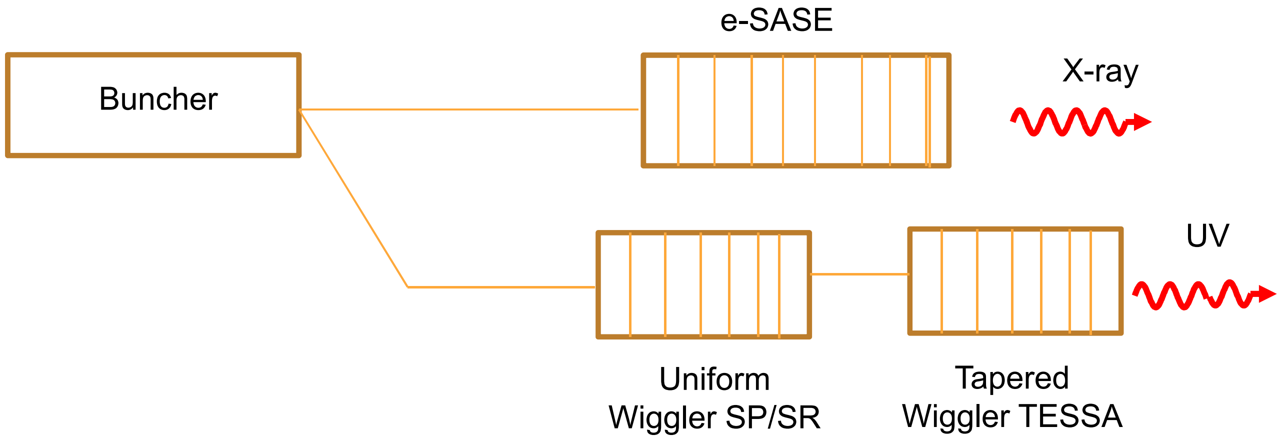


C. Emma *et al* Phys. Rev. A-B 110701 (2017)

EXERCISE # 2 - HIGH HARMONIC UV RADIATION TESSA SOURCE

Buncher




Buncher

e-SASE

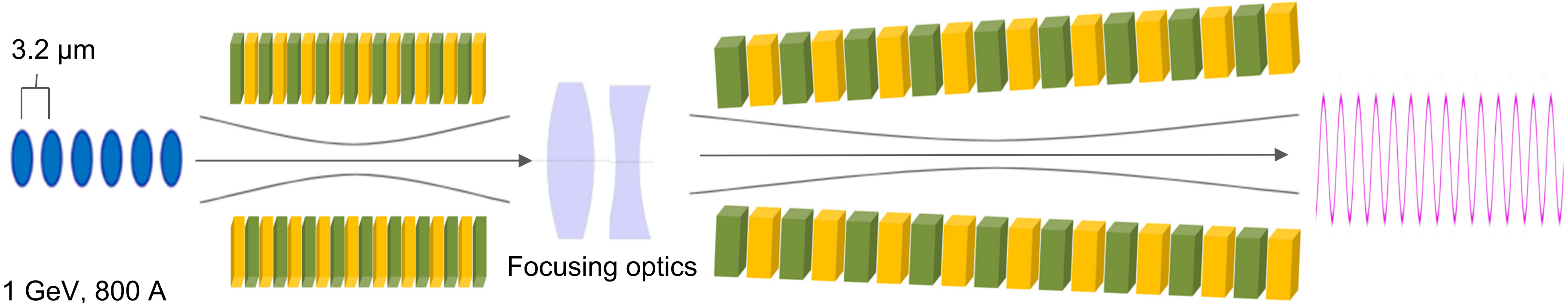
X-ray

UV

Uniform
Wiggler SP/SR

Tapered
Wiggler TESSA

A Model Problem with a SAMURAI beam: 10th harmonic SR and TES*



3.2 μm

1 GeV, 800 A microbunched beam

Short wiggler (10 periods, 24 cm)

$\bar{a}_w = 10$

Focusing optics

Tapered wiggler (125 periods, 3 m)

$\bar{a}(0) = 10$

320 nm radiation

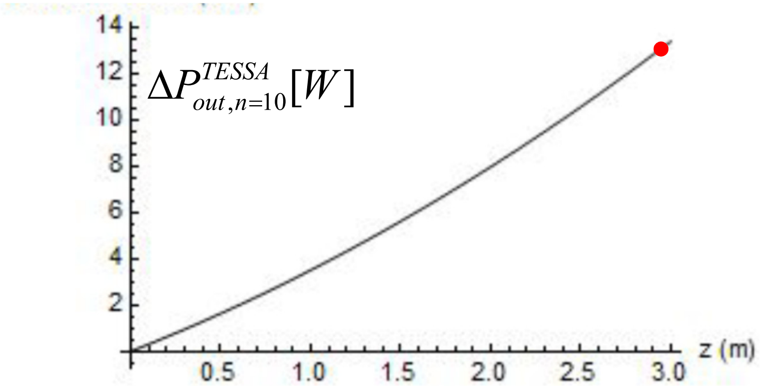
$$\Delta P_{rad}^{TESSA} = E_0 \frac{\bar{a}_w(0)}{\gamma_0} f_t \bar{I} L_w \sin\psi_r + \frac{Z_0}{4A_{emq}} \left(\frac{\bar{a}_w(0)}{\gamma_0} \right)^2 \left(f_t \bar{I} L_w \sin\psi_r \right)^2$$

Summary:

$$P_{in}^{TESSA} = P_{out}^{SR} = 2.4GW$$

$$\Delta P_{out}^{TESSA} = 13.4GW$$

$$P_{out} = P_{in}^{TESSA} + \Delta P_{out}^{TESSA} = 16GW$$



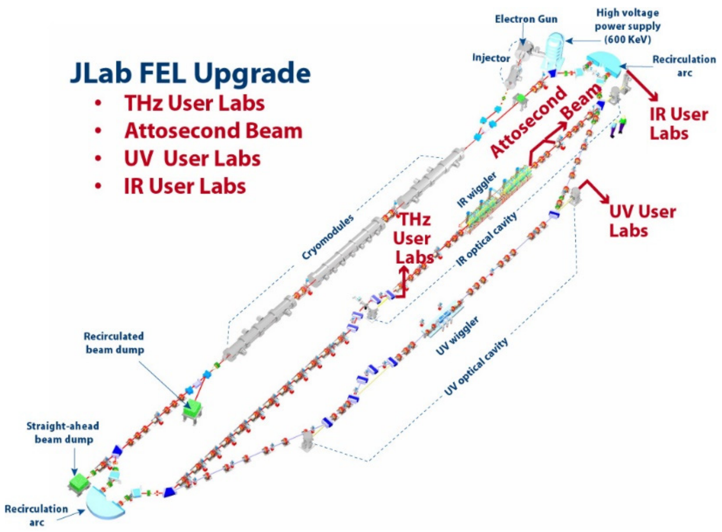
***RIVER ROBLES - STUDENT UCLA PHYSICS DEPT.**

APPLICATION OF SUPERRADIANCE:

Extensive reference list: Gover et al Rev. of Mod. Phys. Vol/EID: 91/035003 (August 2019)

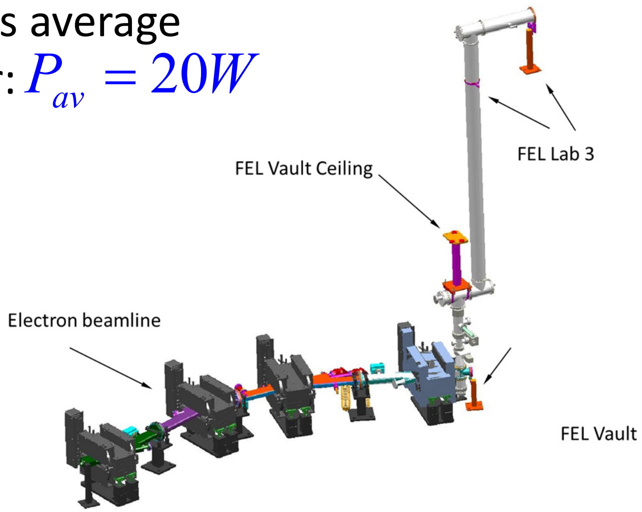
JEFERSON LAB FEL
COHERENT SYNCHROTRON RADIATION

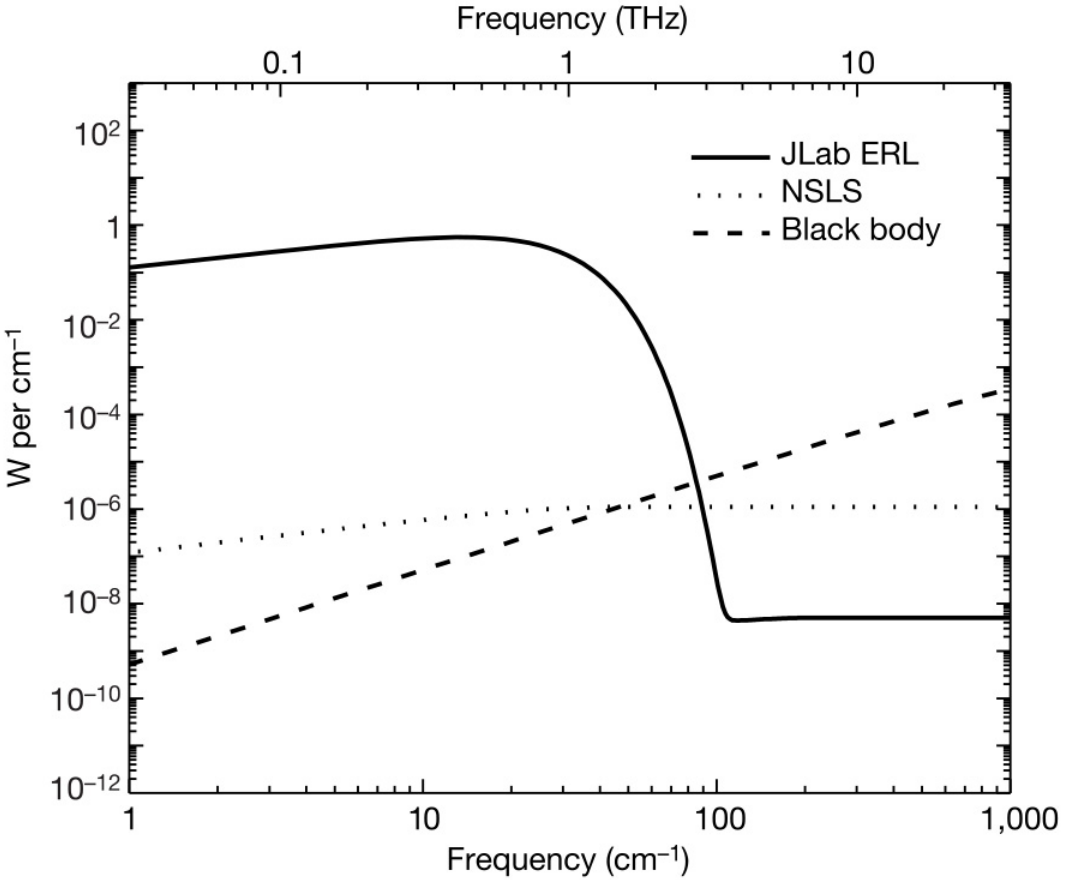
Energy Retrieval LINAC (ERL)



Terahertz beamline transports

Continuous average THz power: $P_{av} = 20W$

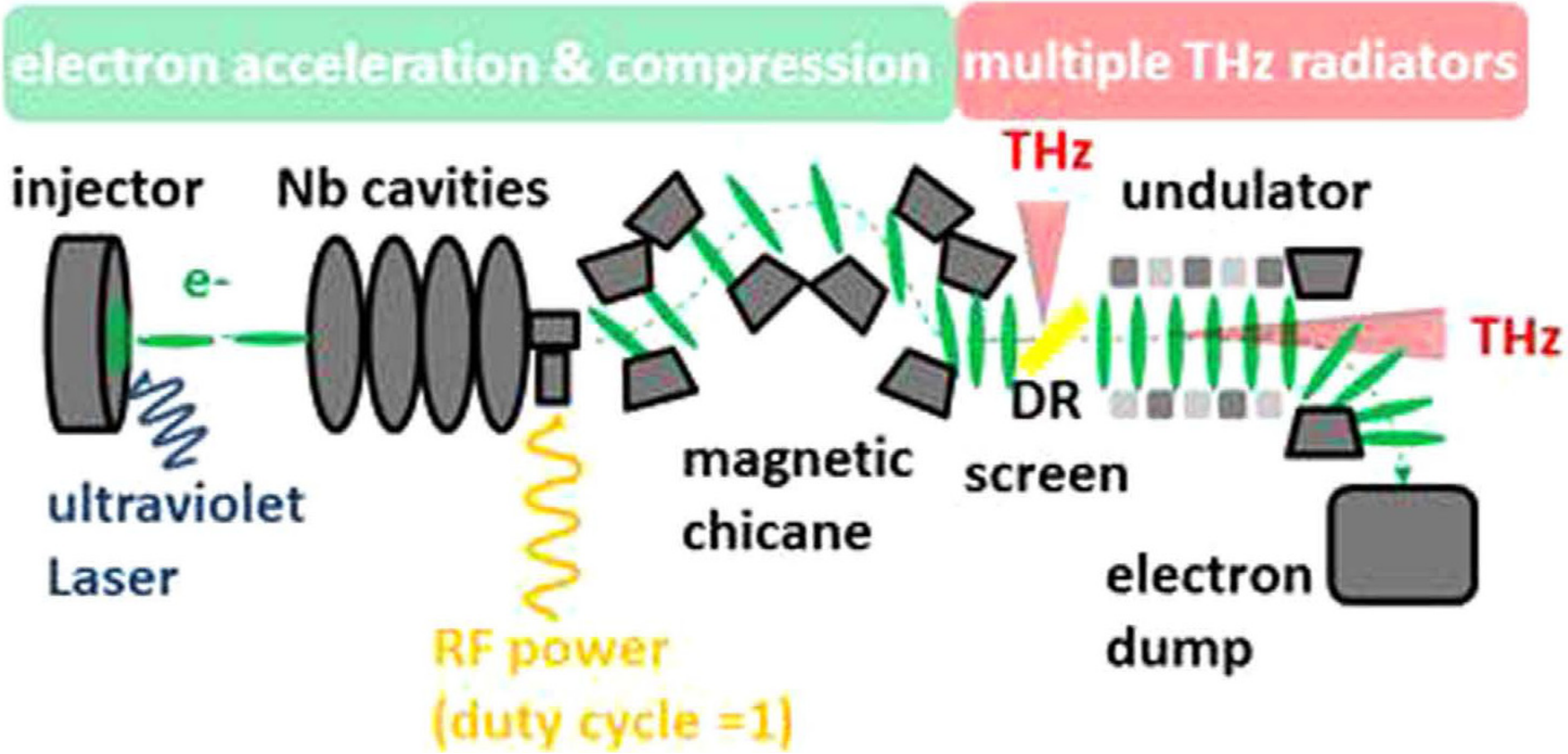




Carr, G. L. *et al*, 2002, Nature 420, 153.

TERA-HERZ ELBE (TELBE) HELMHOLTZ ZENTRUM DREZDEN ROSENDORF

CONTINUOUS TRAIN OF SINGLE PULSES FROM SRF ACCELERATOR THZ SOURCES:
SR UNDULATOR RADIATION,
SR TRANSITION RADIATION



Beam energy: 24 MeV

Charge/pulse: 100 pC

Number of wiggles: 8

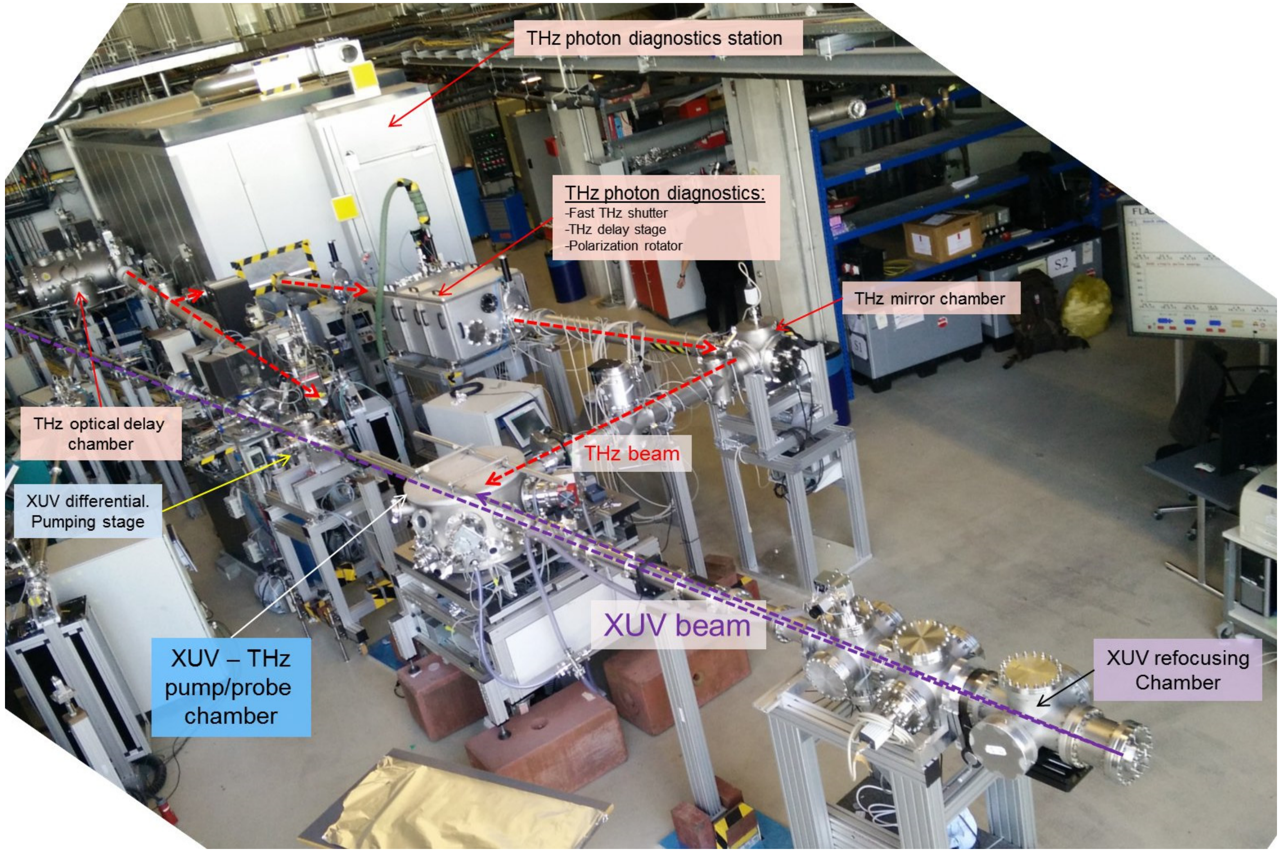
THz frequency: Up to 1.4THz

THz energy/pulse: 1μJ

Green, B., et al., 2016, Sci. Rep. 6, 22256.

g19

FLASH – DESY THz Beamline



tunable: 10 - 300 µm;
up to 100 µJ/pulse;
~10% bandwidth,

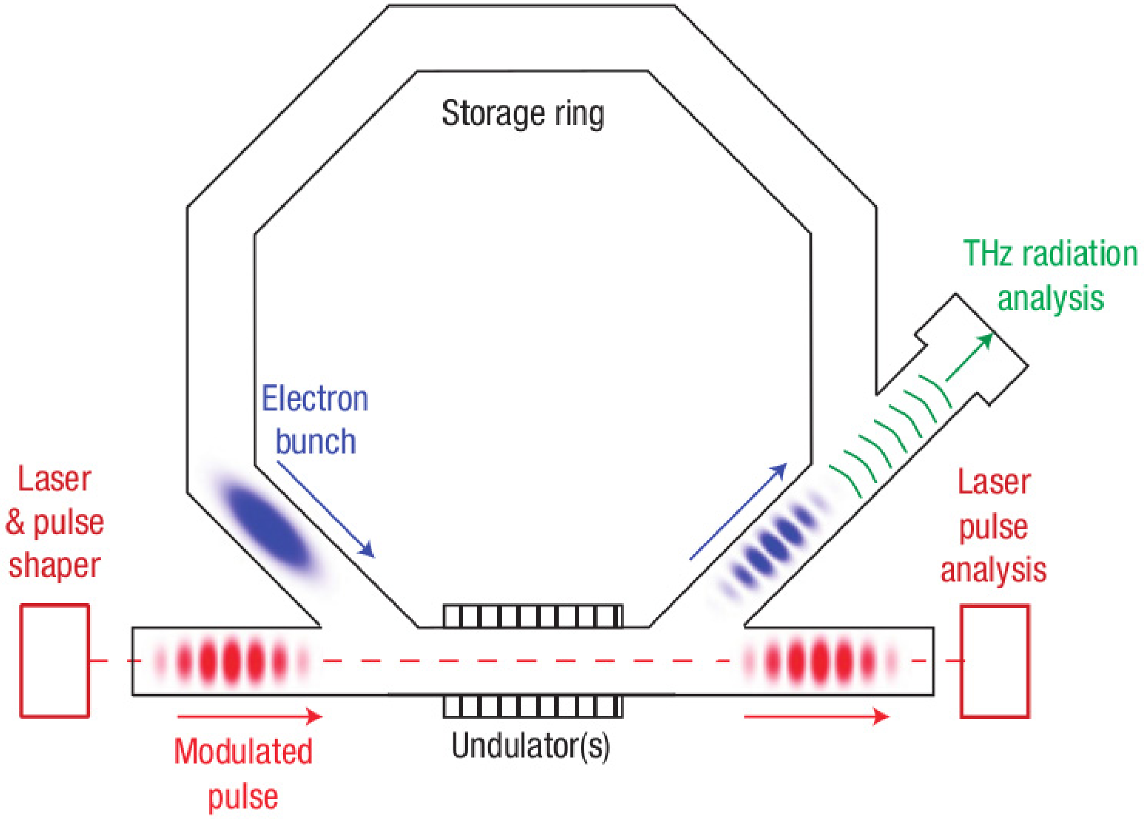
broadband at 200 µm;
up to 10µJ/pulse;
~100% bandwidth

B. Faatz et al NIM A 475 (2001) 363

M. Gensch et al Infrared Physics and Technology 51, 423-425 (2008)

UVSOR OKAZAKI JAPAN

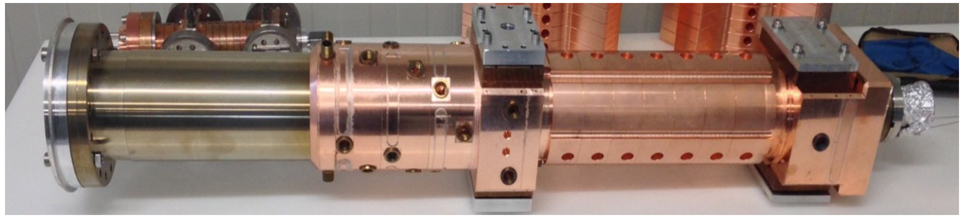
THz SR EMISSION IN A STORAGE RING



- E=750MeV
- The energy of storage beam pulses is modulated in an undulator by interaction with a THz modulated laser beam. They emit Synchrotron SR at the bend.

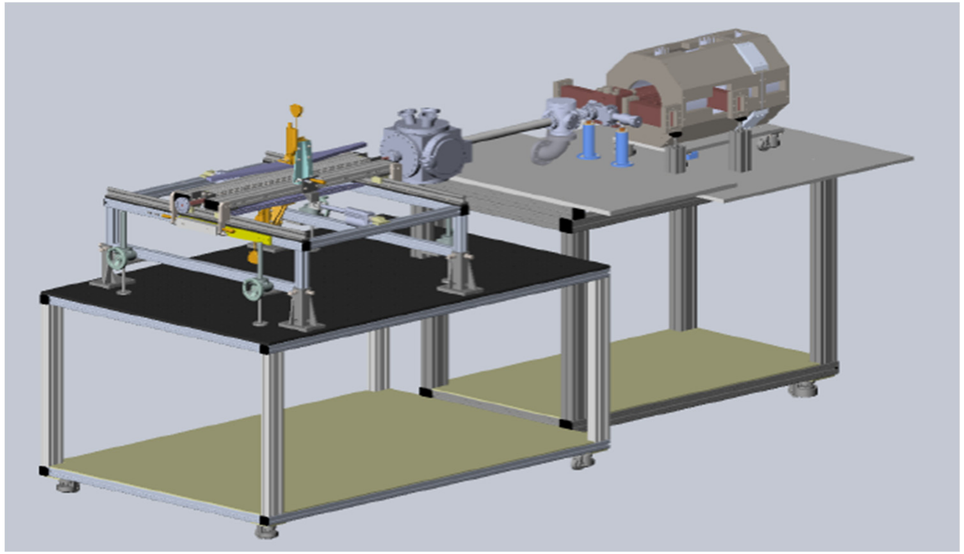
Bielawski, S., et al., 2008, Nat. Phys. 4, 390.

TeraHertz Superradiant FEL – ARIEL/TAU (ISRAEL)



Hybrid photocathode gun PBPL UCLA

Alesini et al, EPAC, Edinburgh 2006

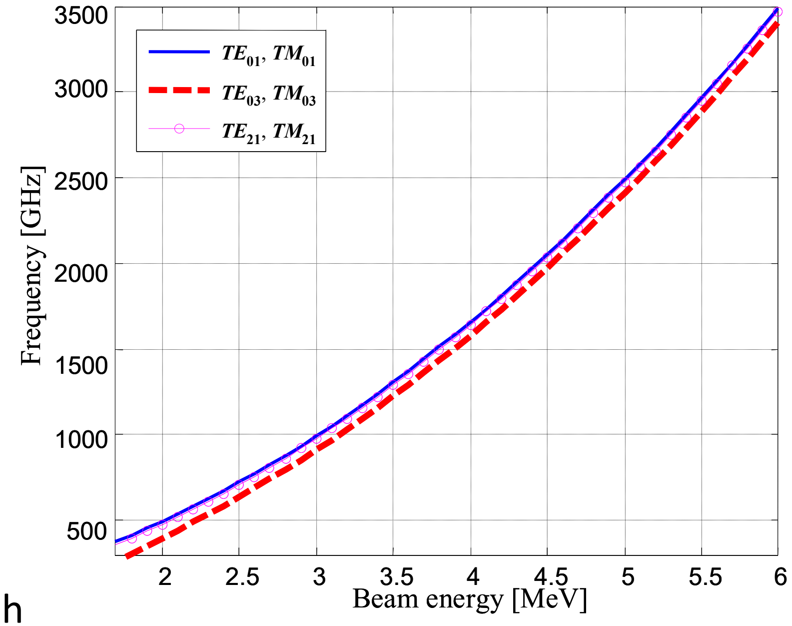


Status of the FEL Facility in Israel

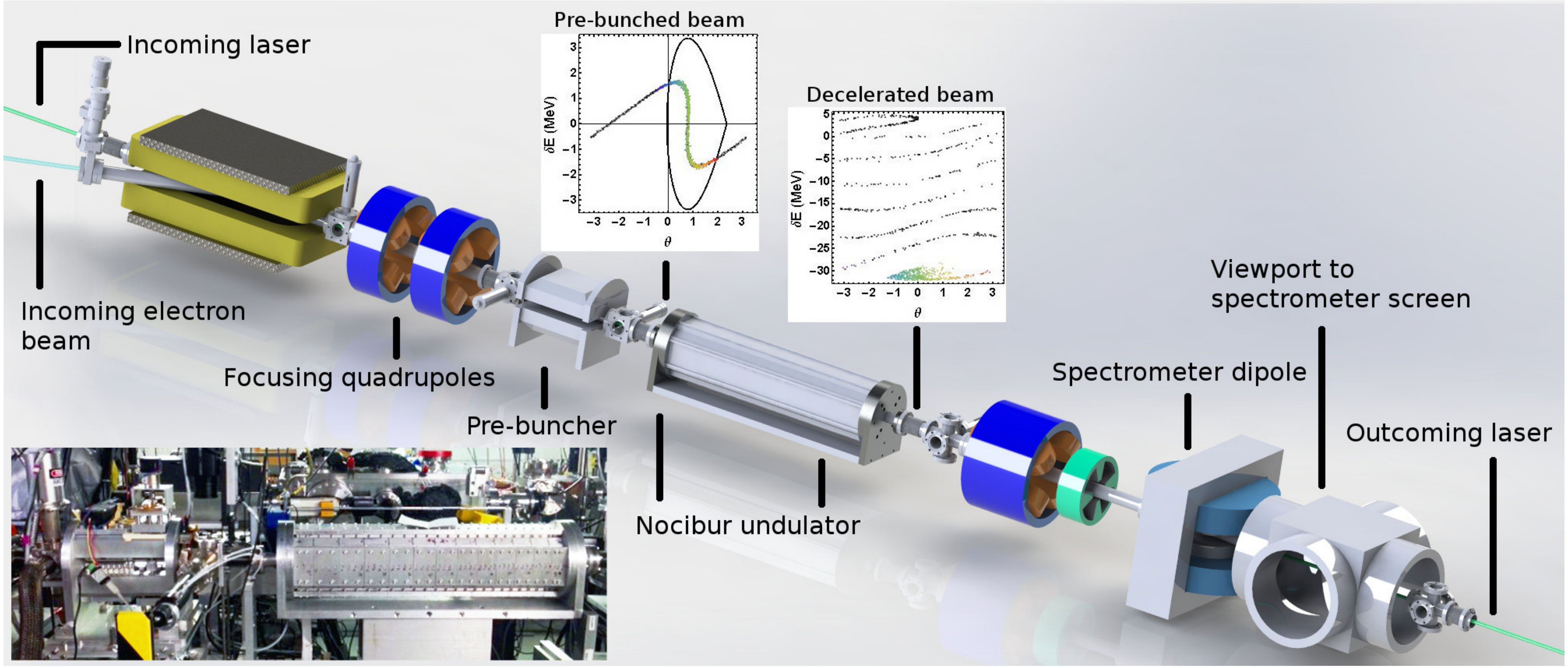
A. Friedman et al, ID: 2151 - THP077

RF-Linac cavity can work for:

Electron Energy at the exit:	3-6.5 MeV
Macrobunch charge:	1 nC
Microbunch duration	sub-pSec
Micrbunch Rep.Rate:	single/3.5 THz
Macrobunch Duration:	12 ps
Macropulse Rep.Rate:	100 Hz

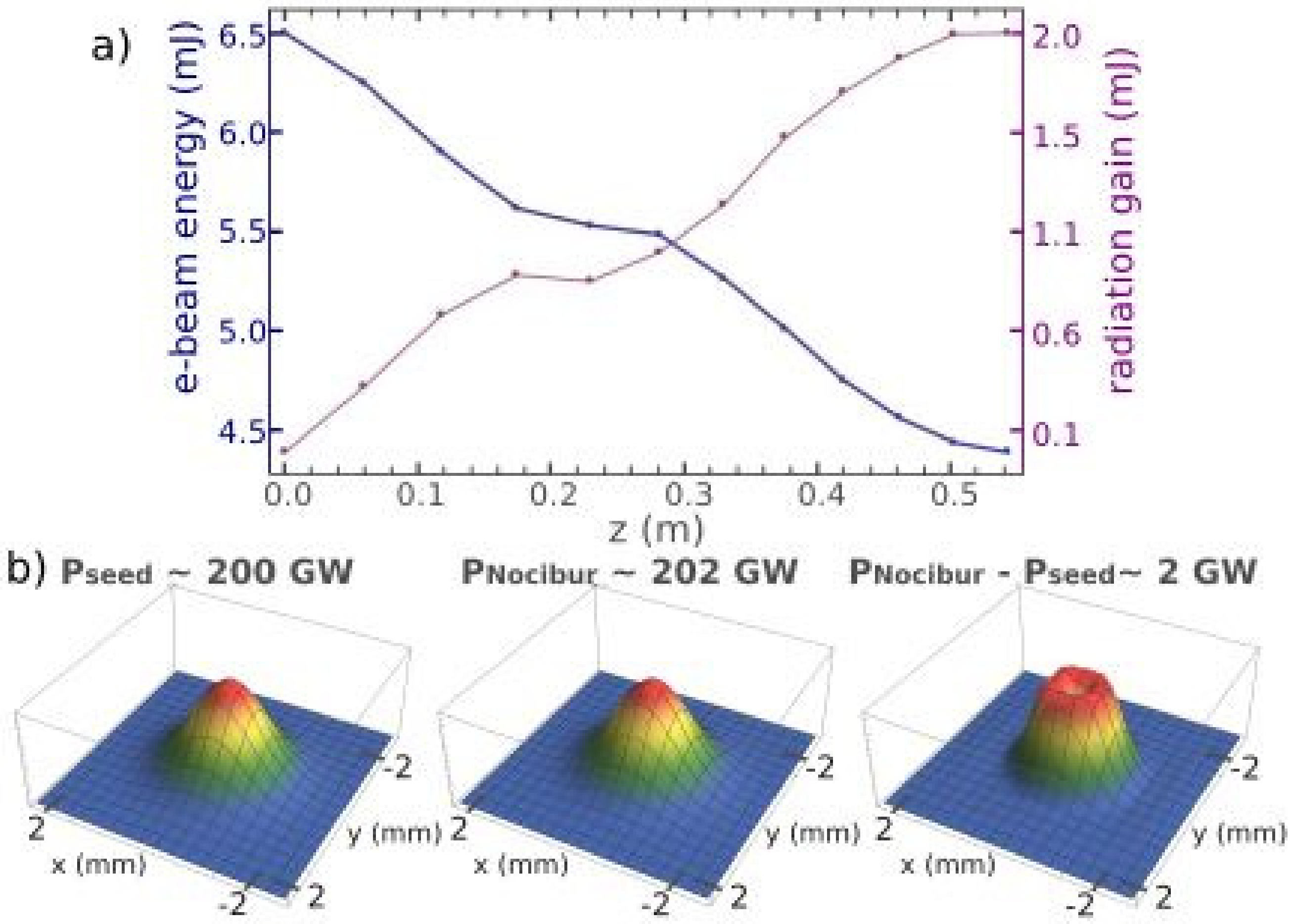


NOCIBUR TESSA EXPERIMET UCLA-ATF

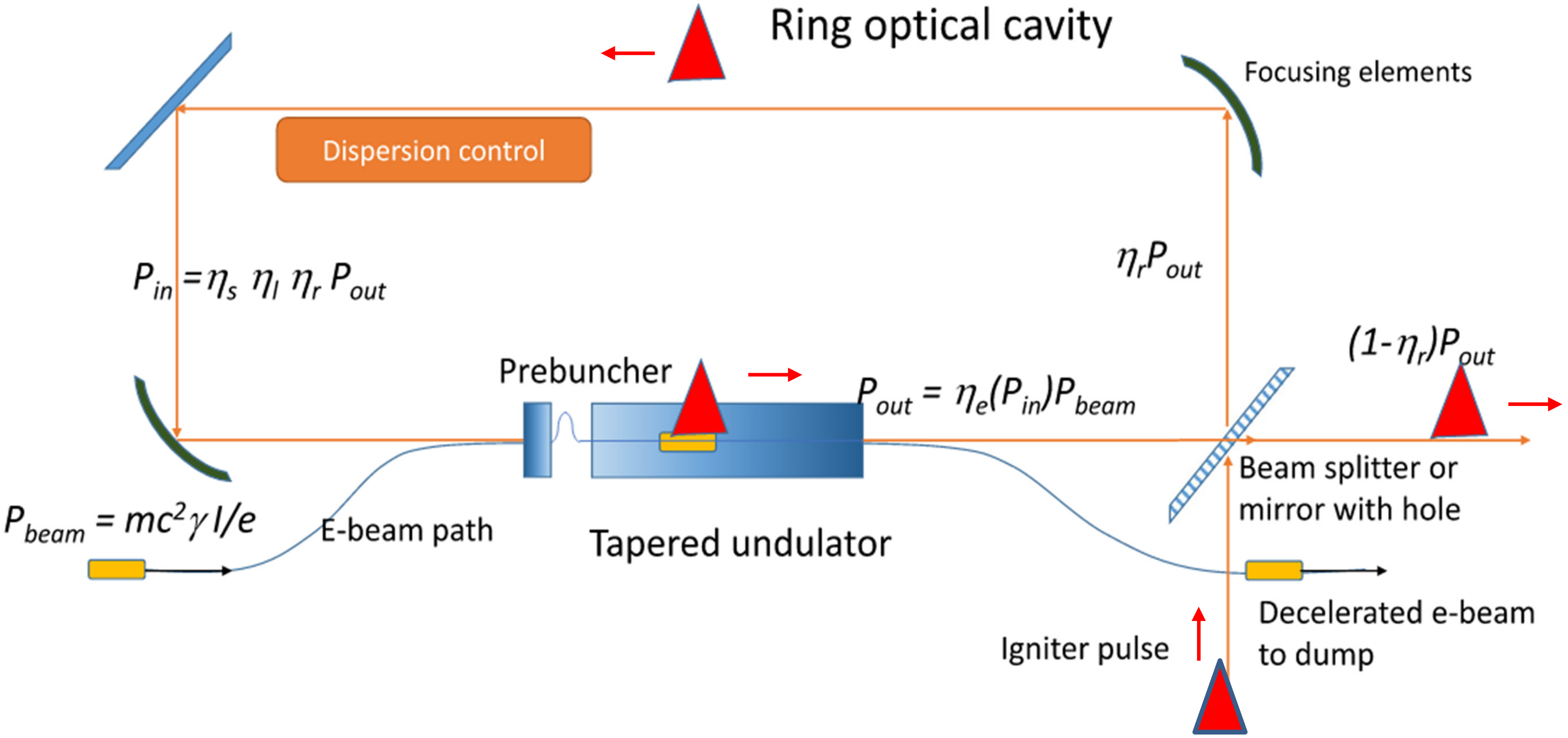


Sudar, et al *Physical review letters* 117.17 (2016): 174801.

30% RADIATIVE ENERGY EXTRACTION EFFICIENCY IN THE NUCIBUR EXPERIMENT @ $\lambda = 10.5\mu$



TAPERING ENHANCED STIMULATED SUPERRADIANCE OSCILLATOR (TESSO)

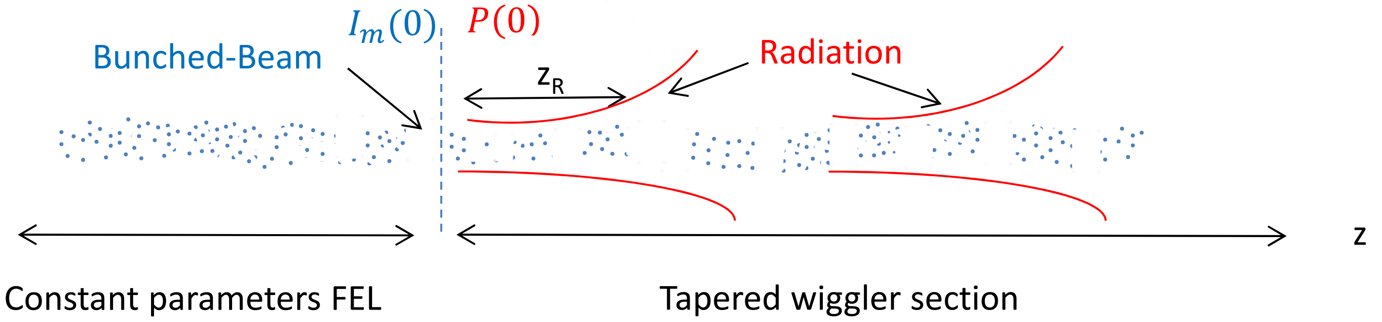


J. Duris, P. Musumeci, N. Sudar, A. Murokh, and A. Gover. Phys. Rev. Accel. Beams **21**, 080705 (2018)

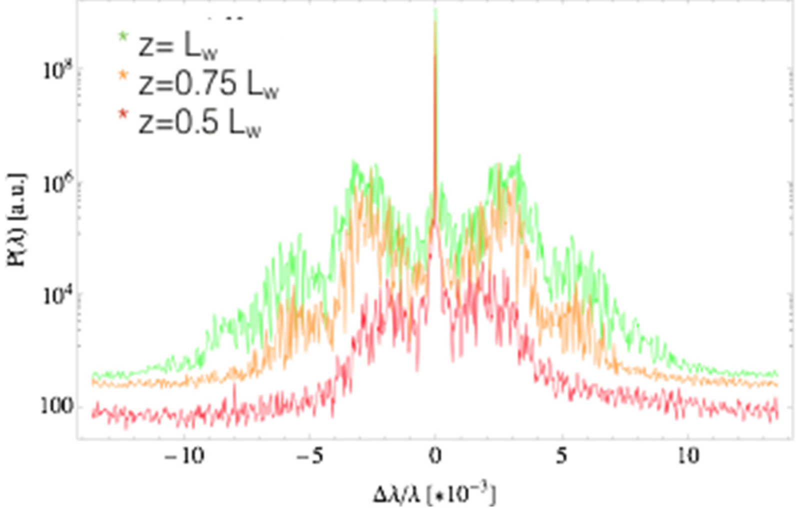
P. Musumeci, “Efficiency and High Gain Amplification at 266 nm” THP073 Thursday Poster Session, 29.8.19.

Tapered wiggler optimization in seed injected FEL

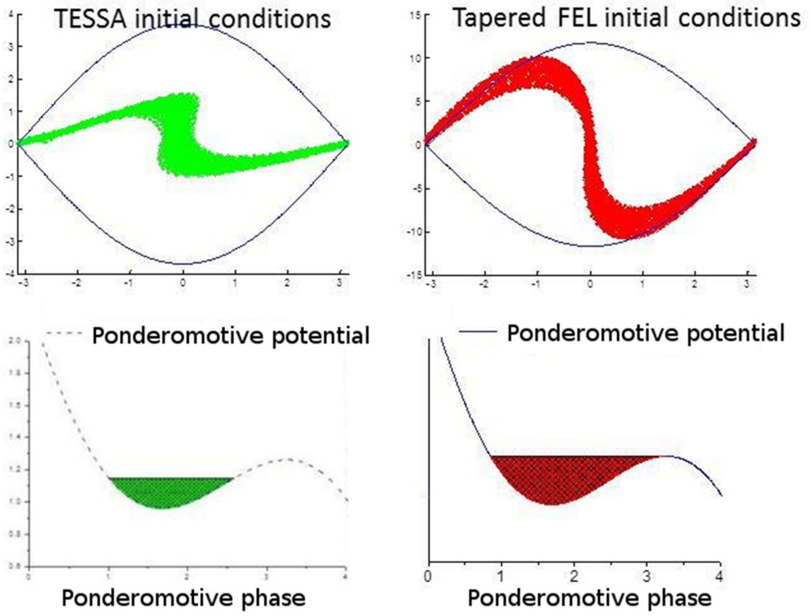
D. Prosnitz et al PRA (1981); Scharlemann T. et al., PRL (1981)



Krol, Morton, Tosenbluth(1981)



Duris, Murokh, and Musumeci (2015)



<u>Effects degrading the fundamental processes:</u>

- 1d theory [1]
-Diffraction[2-7]
-Sideband instability [1,3,4,7,…]
-Spectral effects; Shot-Noise [9,10]
-Phase space spread of injected beam [3,7,8]

<u>Evolving solutions:</u>

-Fresh bunch scheme [11,12]
-Phase shifter [13,14]
-Gain modulation [8]
-Shot-noise suppression [9,10]
-Experimental tests [15]

[1] KMR (1981); Bonifacio, Casagrande (1988) [2] Fawley (1996); [3] Jiao et al (2012); [4] Emma, Pellegrini (2014); [5] Schneidmiller, Yurkov (2015); [6] Tsai, Wu et al (2018); [7] N.Sudar (2019) [8] Emma, Sudar (2017) [9] Gover and Dyunin (2009); [10] Ratner, Huang, Stupakov (2011) [11] Ben-Zvi et al (1992) [12] Emma C. et al (2017) [13] Ratner, D., et al., 2010 [14] Duris, Murokh, and Musumeci (2015) [15] Wu (2017); N. Sudar et al., (2018)

Extensive reference list: Gover et al Rev. of Mod. Phys. Vol/EID: 91/035003 (August 2019)

Conclusions

1. Fundamental radiation emission processes of bunched beam at zero order:
   -Spontaneous Superradiance (SP-SR) $\propto N^2, z^2$
   -Stimulated Superradiance (ST-SR) $\propto N, z, E_0$
2. Model of periodical tightly bunched e-beam interaction with a single radiation mode in the nonlinear regime:
   -SR and ST-SR in a uniform wiggler
   -Tapering Enhanced Superradiance (TES), Tapering Enhance Stimulated Superradiance Amplification (TESSA) and Oscillator (TESSO).
3. Self interaction of a bunched beam in a uniform wiggler and seedless TESSA.
4. Application in THz superradiant sources based on SR emission of sub-picoSec bunches.
5. Applications of TESSA, TESSO in the THz to UV frequencies range.
6. Optimization of tapering strategy in the tapered wiggler section of X-Ray FELs.