# Realistic Modeling of the Muon $g-2$ Experiment Beamlines at Fermilab

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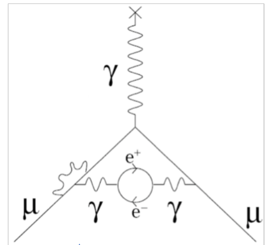
# Outline

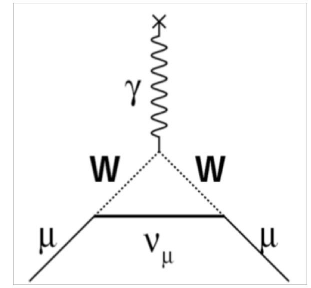
1. Introduction
2. E989 Beam Delivery System
3. Simulation Features
4. Beam Performance
5. Conclusions

# 1. Introduction: Muon $g-2$ Experiment

$$\boldsymbol{\mu} = g\frac{Qe}{2mc}\boldsymbol{S}$$

12672 QED diagrams
(up to 10th-order, Kinoshita)



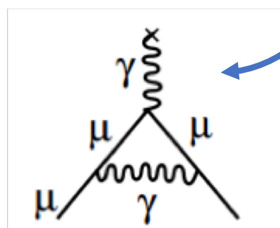




Electroweak contributions

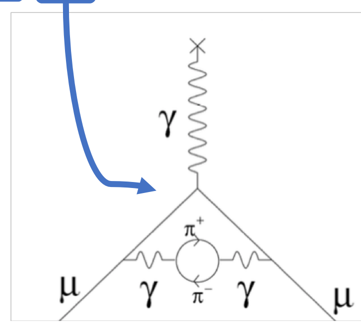
Dirac's equation

Standard Model

$$g_\mu = 2.0023318417 8(126)$$




1st-order QED (Schwinger)

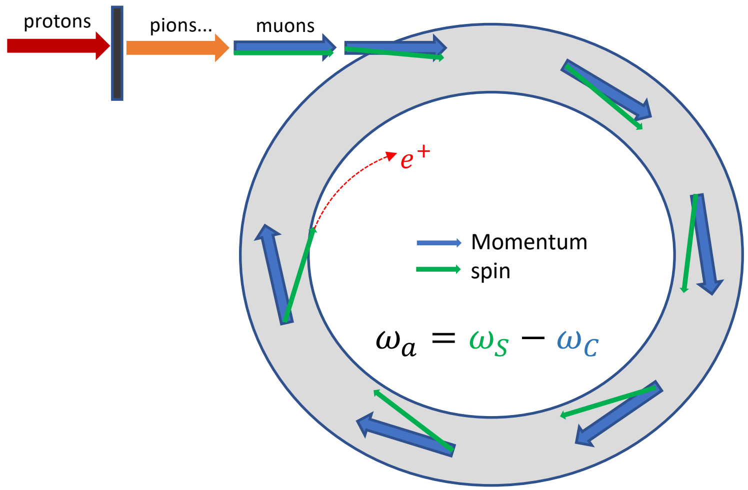



Hadronic contributions

$$a_\mu^{thy} \equiv \frac{g_\mu - 2}{2}$$

$$a_\mu = a_\mu^{QED} + a_\mu^{Had} + a_\mu^{Weak} + a_\mu^{?}$$

# 1. Introduction: Muon $g-2$ Experiment at FNAL (E989)

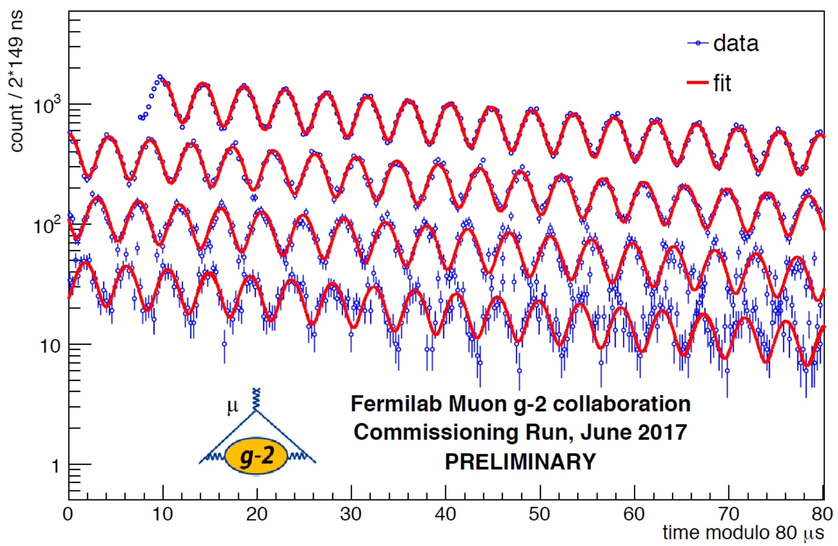



$$a_\mu \approx \frac{m_\mu \omega_a}{eB}$$

- Goal of E989 is to measure $a_\mu$ to 0.14ppm precision or less

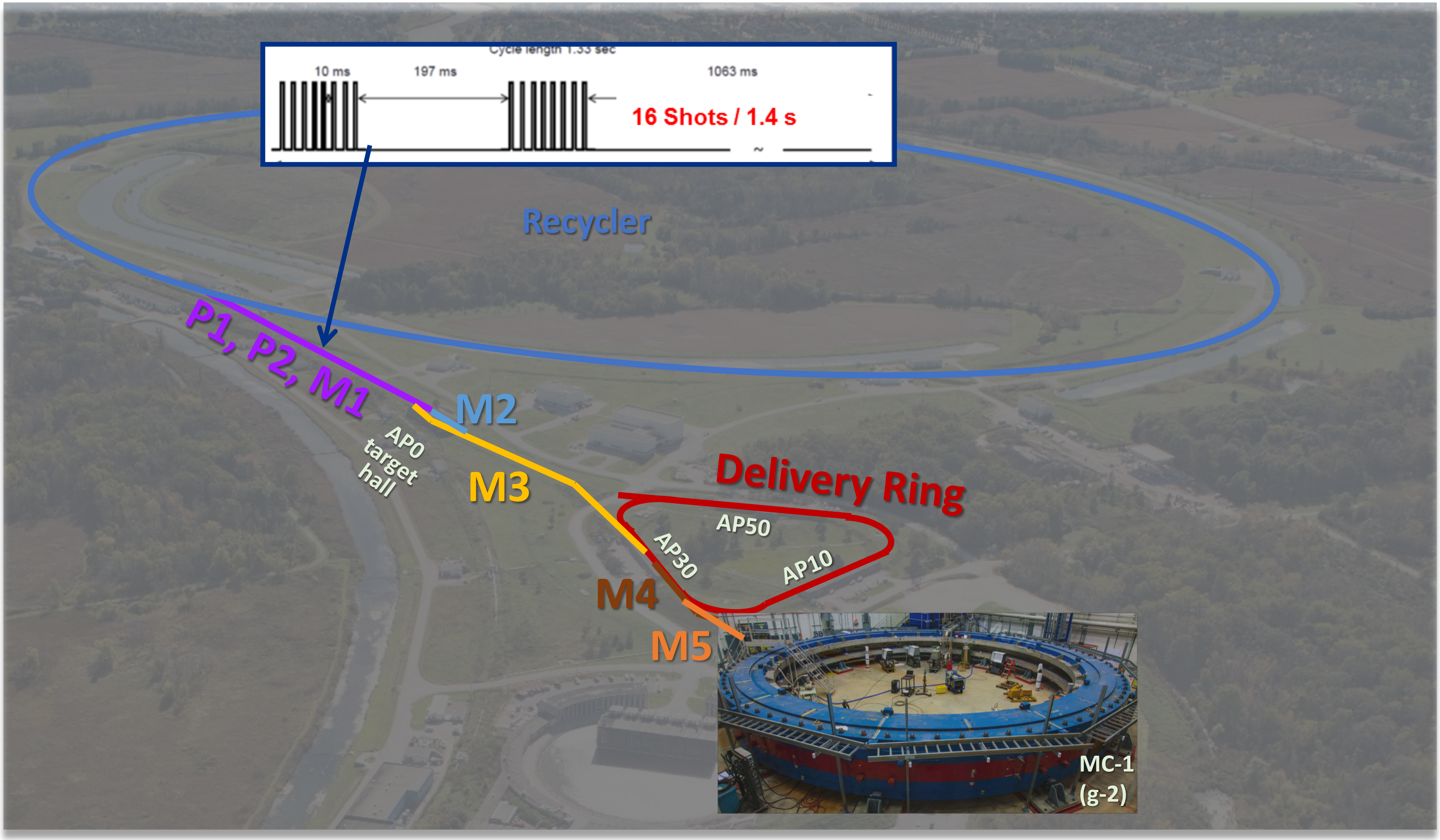
- $\omega_a$ obtained from fit to the wiggle plot

- Reduction of statistical and systematic uncertainties essential.

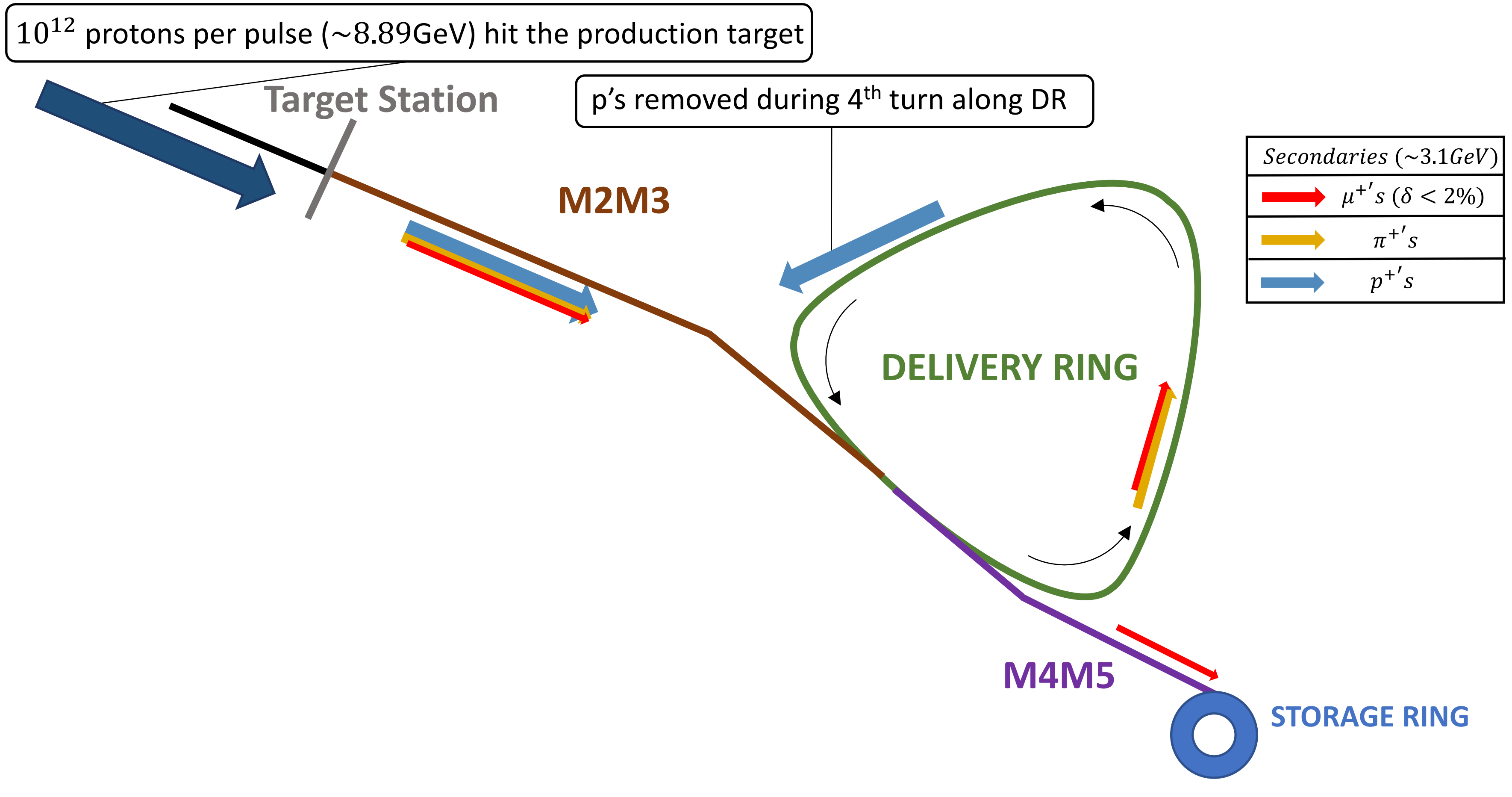



$$N(t, E_{th}) = N_0(E_{th}) \exp^{-t/\gamma\tau_\mu} \left[1 + A(E_{th}) \cos\left(\omega_a t + \varphi_a(E_{th})\right)\right]$$

# 2. E989 Beam Delivery System



# 2. E989 Beam Delivery System



# 3. Simulation Features: COSY INFINITY

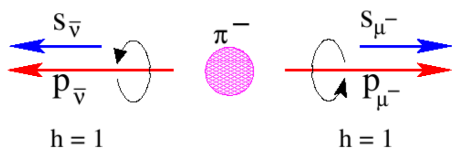
- Beam collimation:

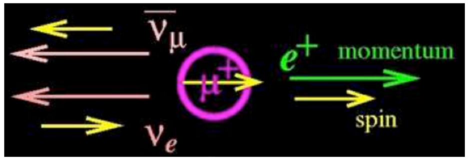
  - Simultaneous tracking of up to 10k particles with DA algorithms
  - Collimation every ~20 cm

- Horizontal and vertical misalignments:

  - Maps transformed based on random misalignments
  - RMS $\sigma = 0.25\text{mm}$

- Decay modes and spin dynamics:




$$P_L = \frac{x(1+b^2) - 2b^2}{x(1-b^2)} \qquad b = m_\mu / m_\pi, \quad x = p_L^\mu / p_\pi$$

# 3. Simulation Features

- High-order effects:

$$r_i = \sum_{l_1, l_2, \ldots, l_6 = 0}^{\leq 4} (r_i | x^{l_1} a^{l_2} y^{l_3} b^{l_4} l^{l_5} \delta^{l_6}) x_0^{l_1} a_0^{l_2} y_0^{l_3} b_0^{l_4} l_0^{l_5} \delta^{l_6}$$

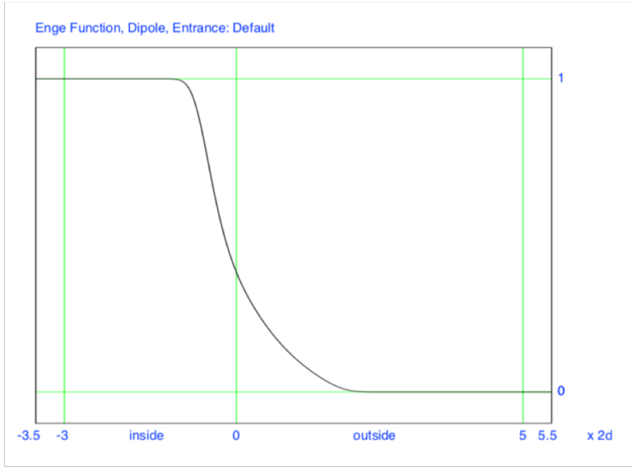
- In addition to linear dynamics, up to 4-th order transport maps considered in simulations
- These effects also included in spin tracking

- Fringe Fields:

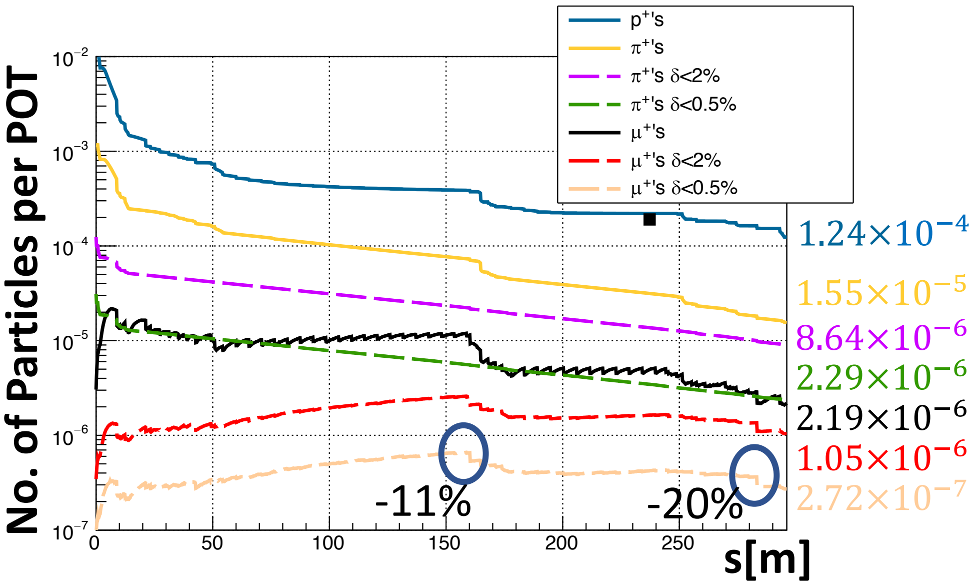
$$F(z) = \frac{1}{1 + \exp\left(a_1 + a_2 \cdot (z/D) + \cdots + a_6 \cdot (z/D)^5\right)},$$

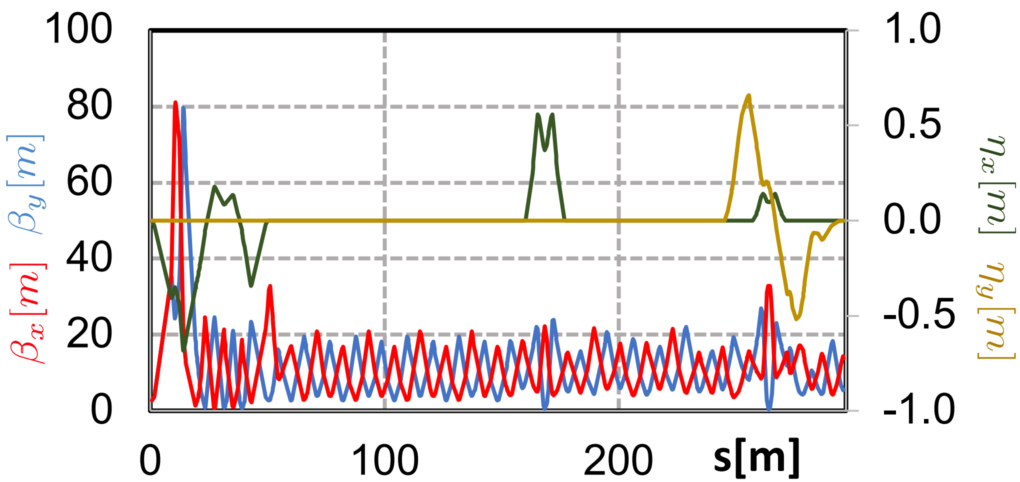
- The $a_i$ coefficients are taken by default based on measured data from PEP at SLAC

- Fringe fields turned on for each beamline element along E989 beam delivery system



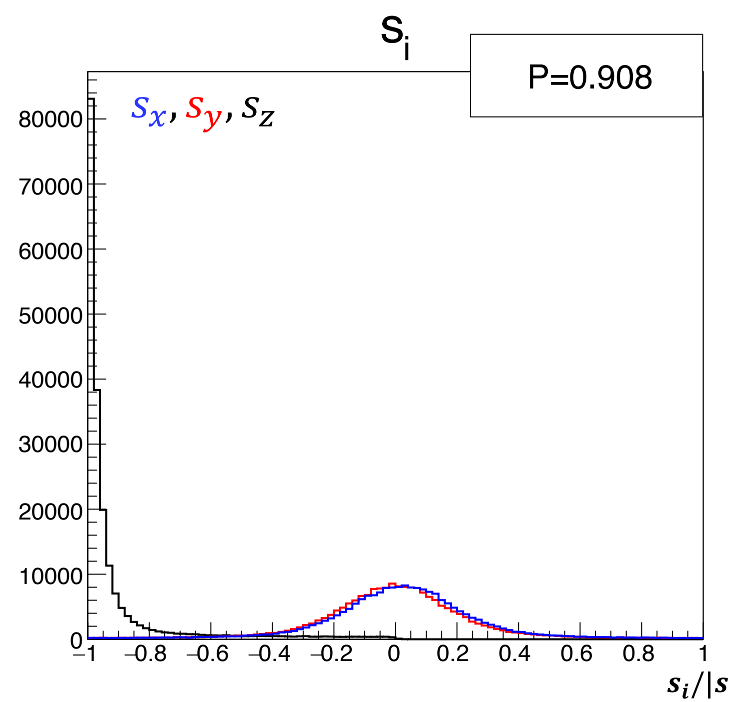
# 4. Beam Performance: **M2M3**




- M2M3 is designed to capture as many magic-momentum muons as possible.

- Main losses occur at s≈163m (H726) and s≈ 285m (ISEP).



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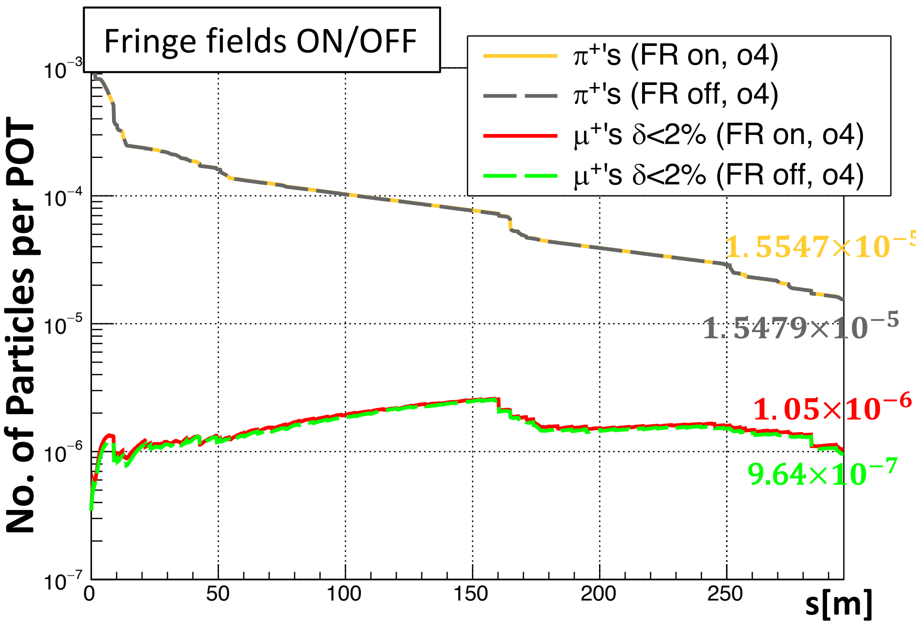
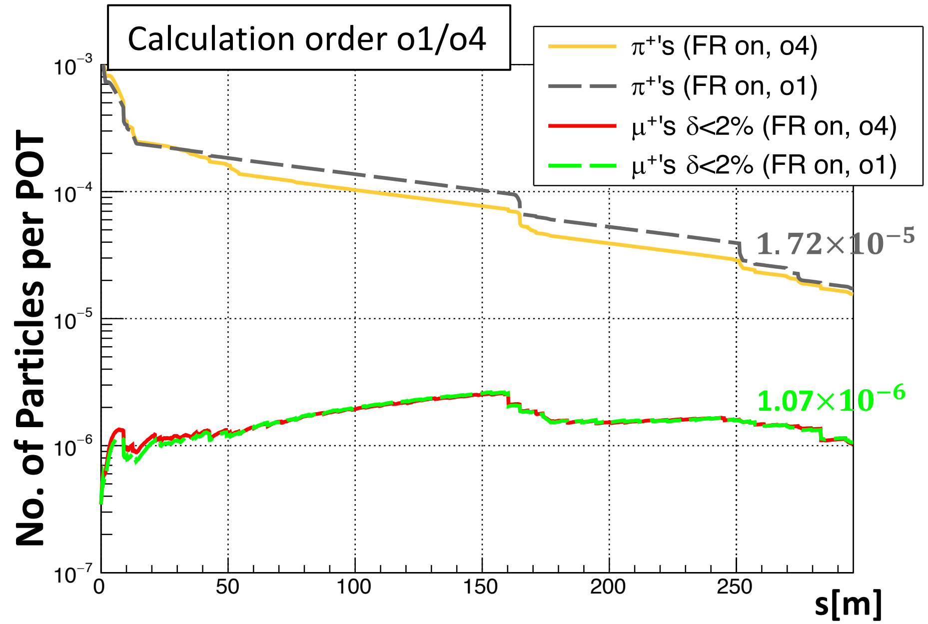
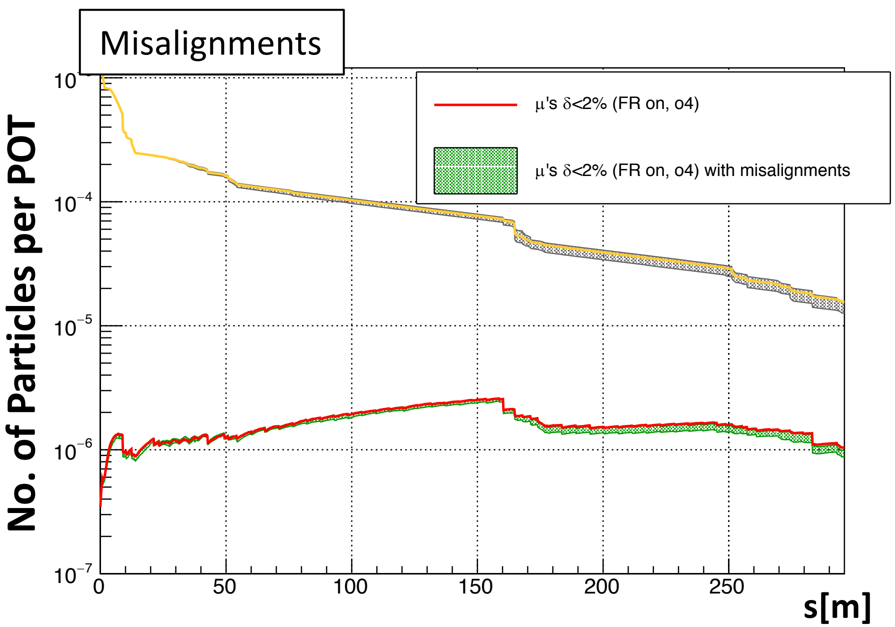


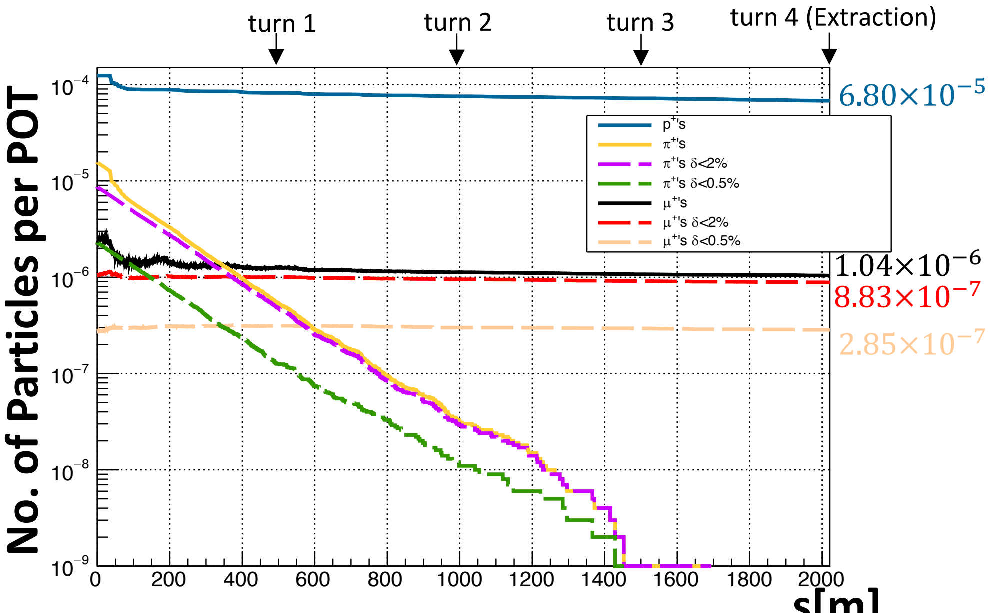



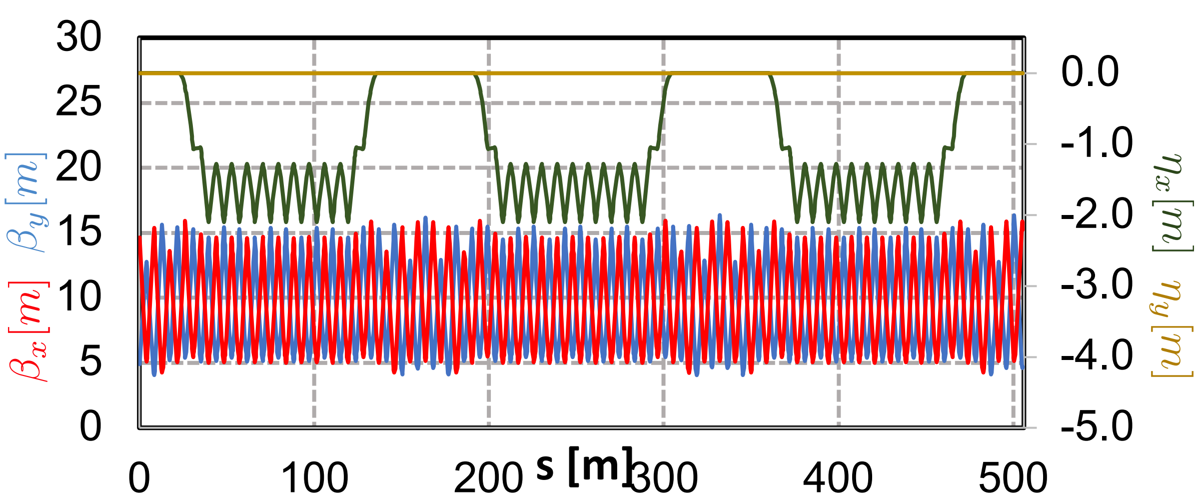

## Effect on particle populations

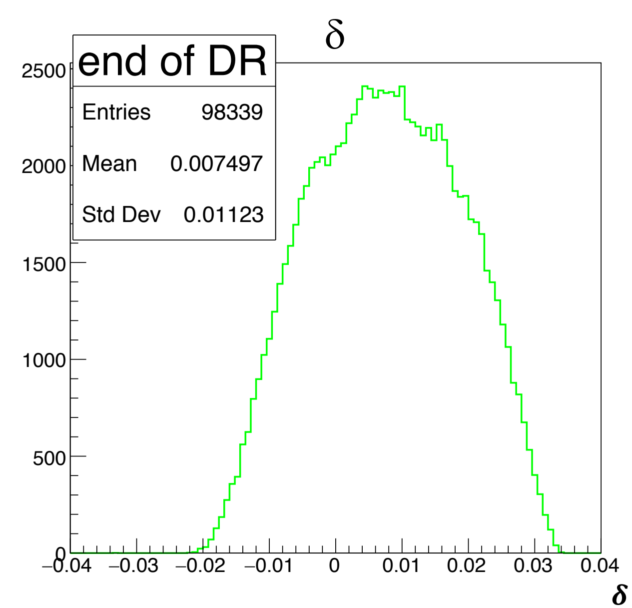
| | $\mu^{+'}s\ (\delta < 2\%)$ | $\pi^{+'}s$ |
|---|---|---|
| Fringe fields | +8.9% | +0.4% |
| Higher-orders | -1.9% | -9.6% |
| Misalignments | -9.2±9.7% | -10.5±5.5% |

# 4. Beam Performance: Delivery Ring




- After the 1st turn, muon losses due to collimation are minimal
- Pions also remain within the acceptance of the DR
- At extraction from the DR, muons with $-2.2\% > \delta > 3.5\%$ survive



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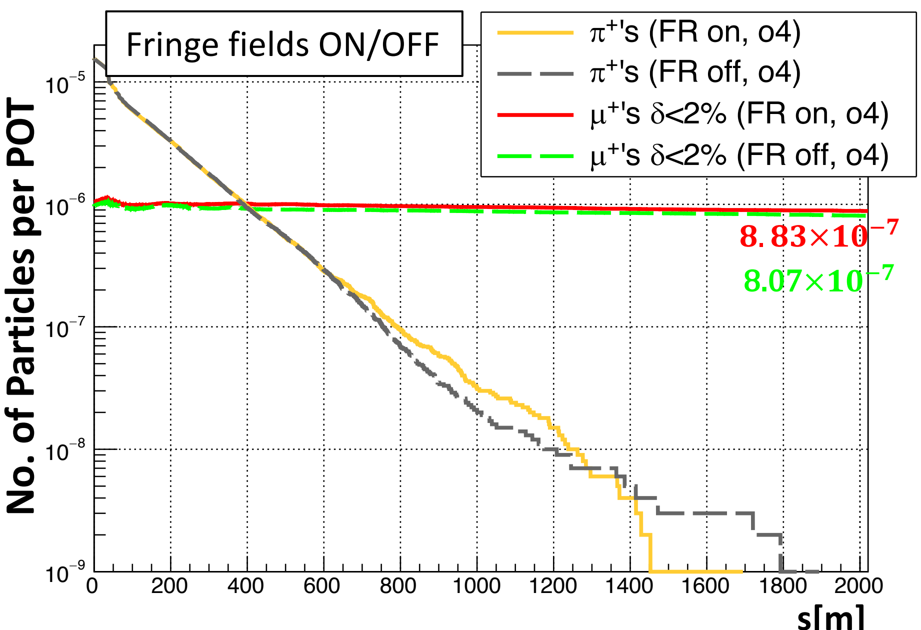
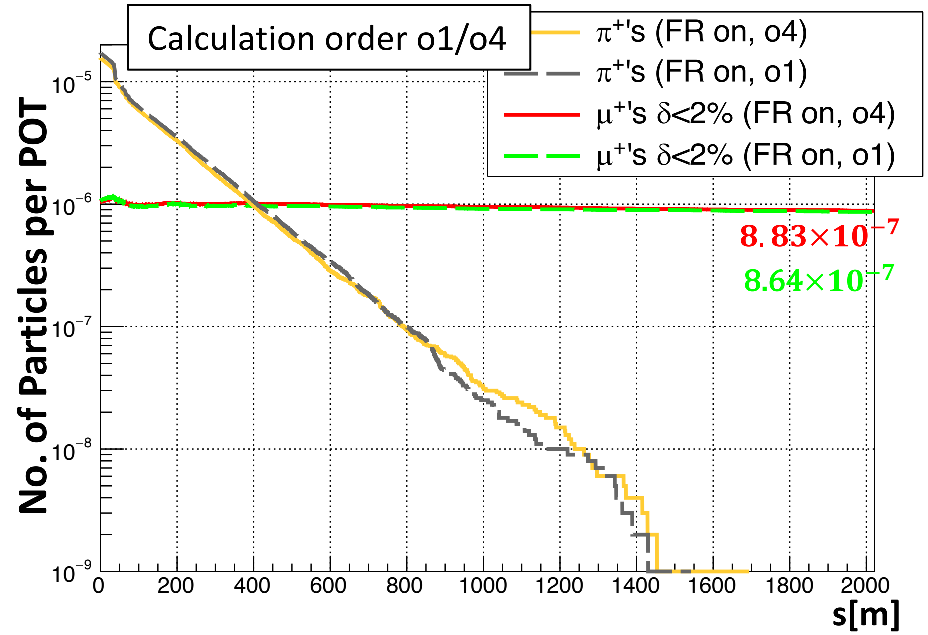
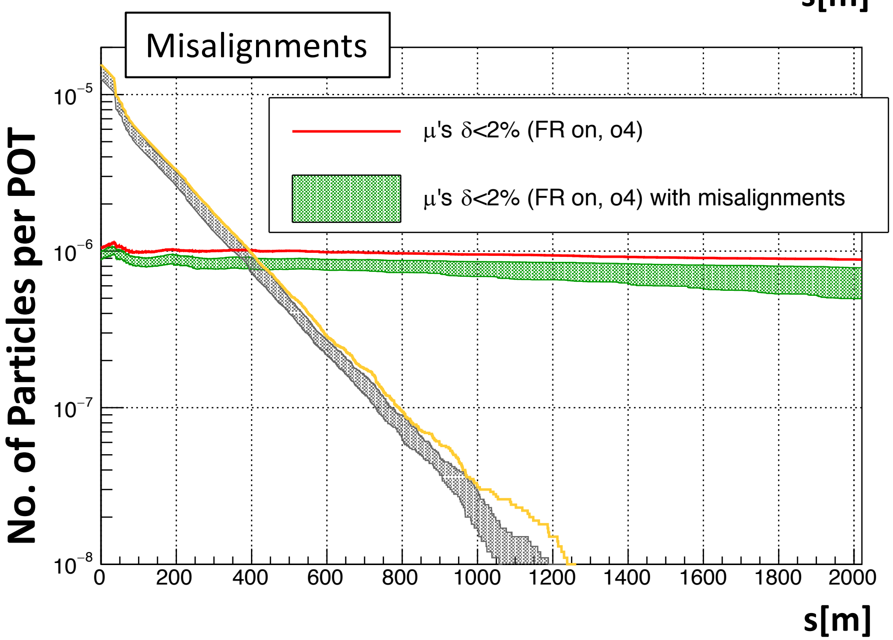


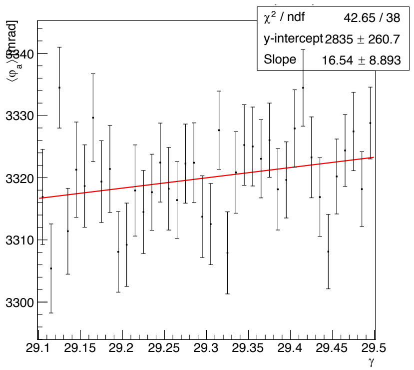




Effect on particle populations at end of DR after 4 turns

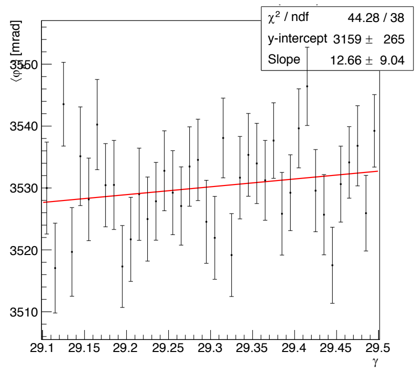
| | $\mu^{+'}s\ (\delta < 2\%)$ |
|---|---|
| Fringe fields | +9.4% |
| Higher-orders | -2.2% |
| Misalignments | -27.5±16.2% |

# 4. Beam Performance: Delivery Ring, spin-orbit correlation studies

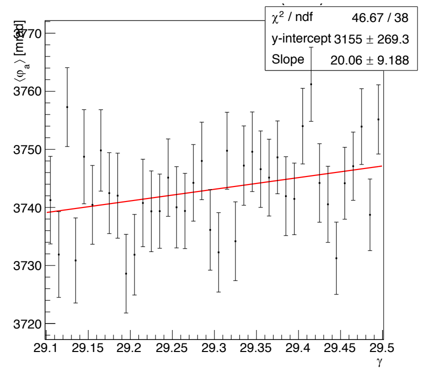
$\langle\varphi_a\rangle - \gamma$ at end of DR (n=1)



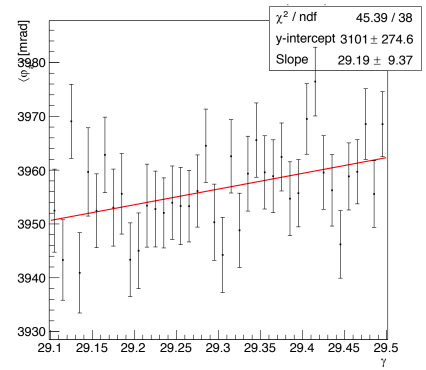

$\langle\varphi_a\rangle - \gamma$ at end of DR (n=2)




$\langle\varphi_a\rangle - \gamma$ at end of DR (n=3)




$\langle\varphi_a\rangle - \gamma$ at end of DR (n=4)




$$\vec{\varphi} = \int_{t_1}^{t_2} \vec{\omega} dt = \int_{s_1}^{s_2} \frac{1}{v} \vec{\omega} \frac{p}{p_s} (1 + hx) ds_0$$

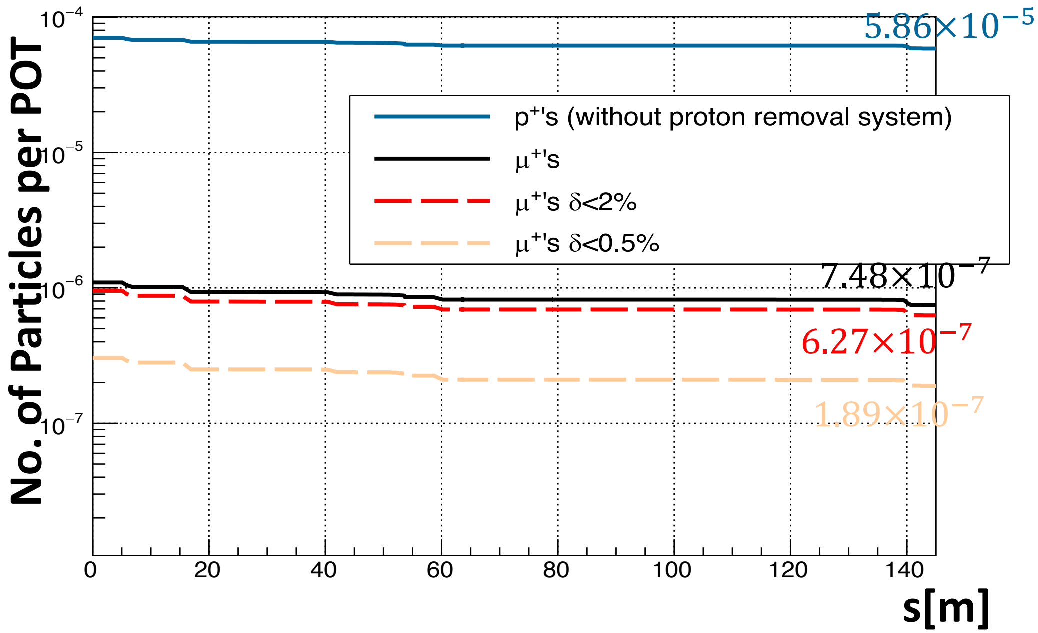
$$\approx \int_{s_1}^{s_2} \frac{1}{v} \vec{\omega} (1 + hx) ds_0$$

$$\frac{d\langle\phi_a\rangle}{d\gamma} = \frac{1}{\gamma_0 \beta_0^2} \frac{\boldsymbol{d\langle\phi_a\rangle}}{\boldsymbol{d\delta}}$$

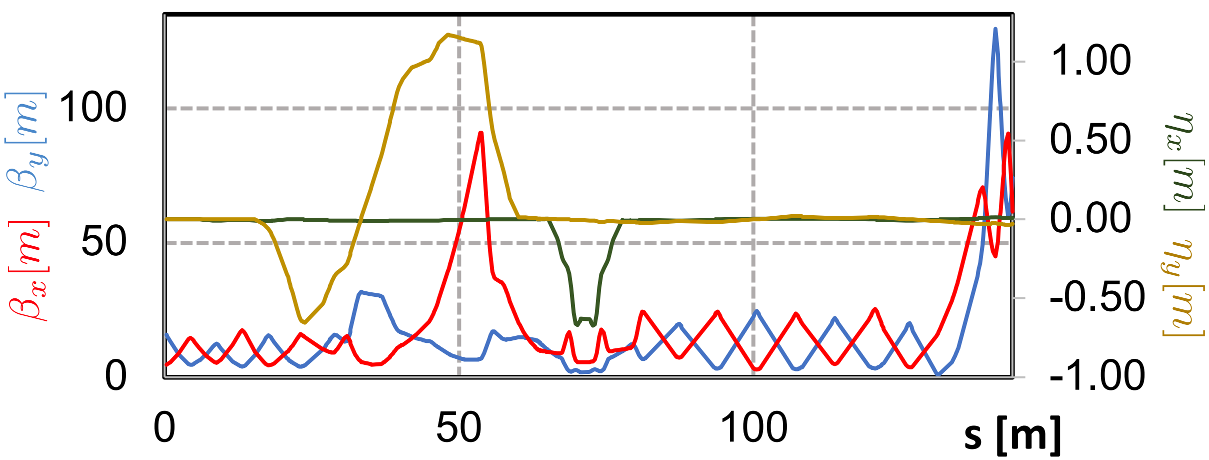
$d\langle\phi_a\rangle/d\gamma$

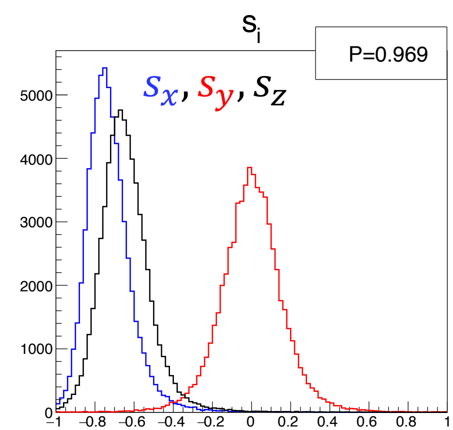
| DR Turn | COSY | |
|---|---|---|
| | Fringe Fields OFF | Fringe Fields ON |
| 1 | 21.1±9.3 | 16.5±8.9 |
| 2 | 37.4±9.5 | 12.7±9.0 |
| 3 | 64.5±9.7 | 20.1±9.2 |
| 4 | 92.1±9.8 | 29.2±9.4 |

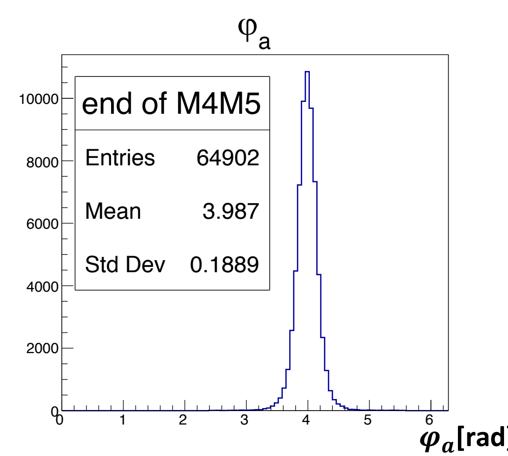
# 4. Beam Performance: **M4M5**



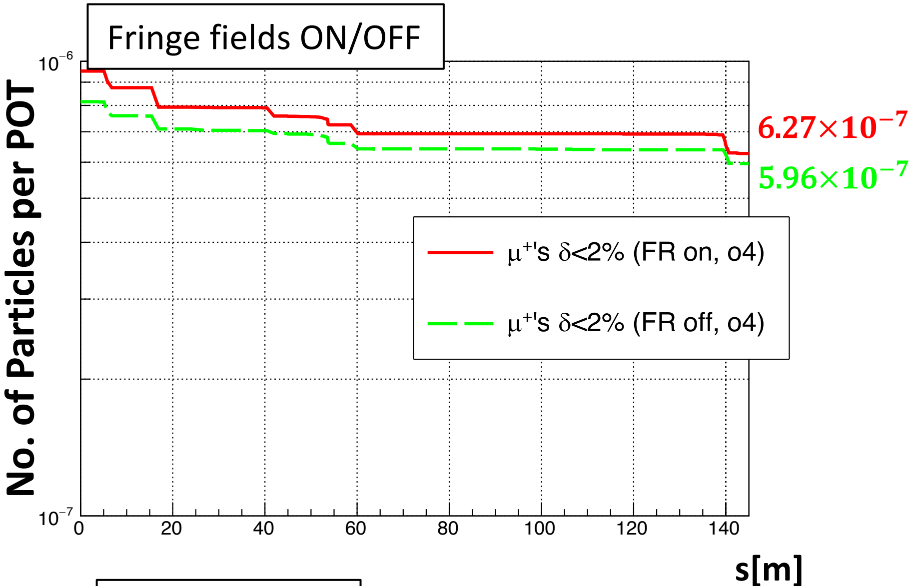

- A fraction of $\sim 1.9\times10^{-7}$ muons with $|dp/p| < 0.5\%$ arrive to the entrance of the inflector

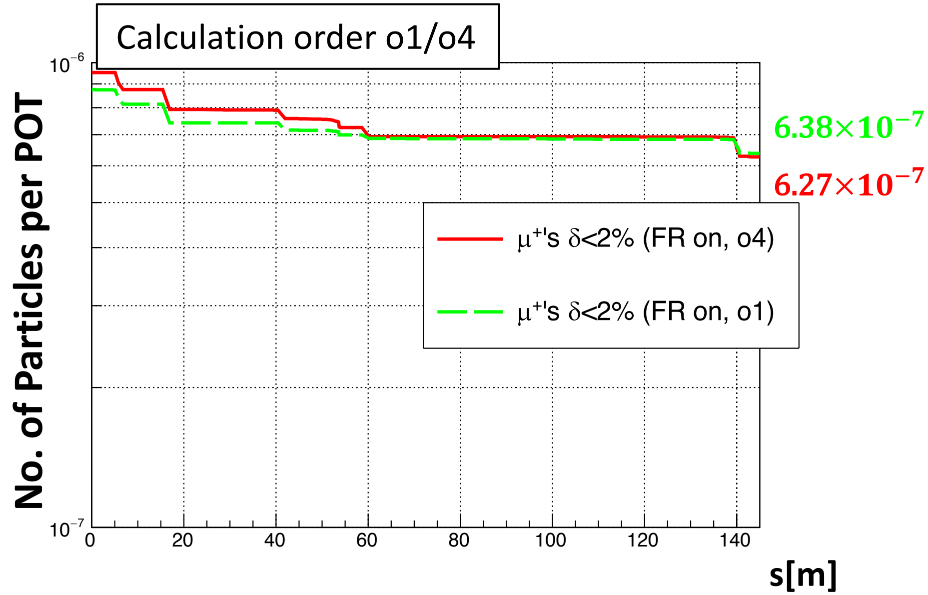


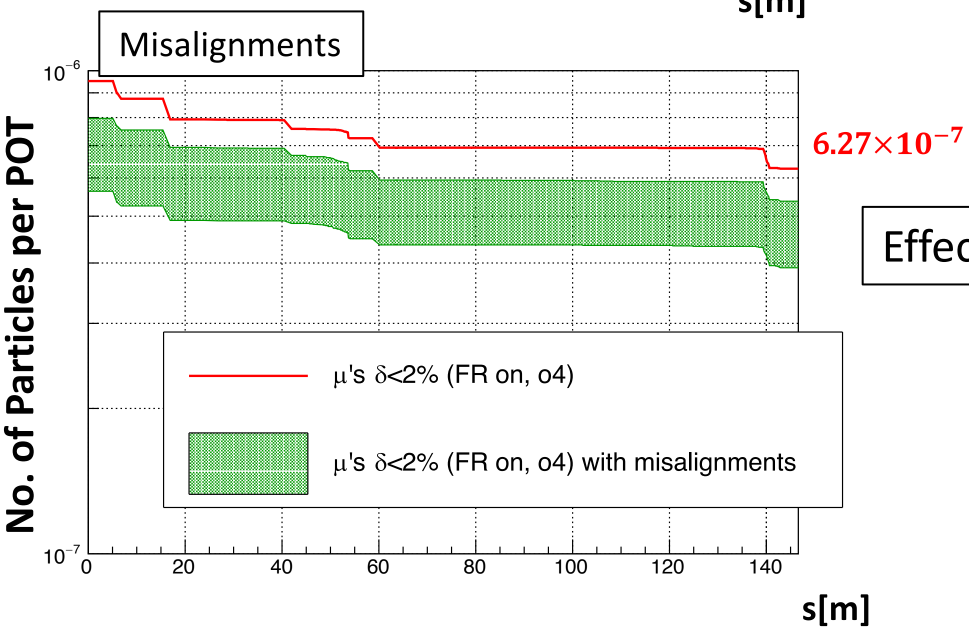



# 4. Beam Performance: M4M5




**Effect on particle populations at the end of M4M5**

| | $\mu^{+'}s\ (\delta < 2\%)$ |
|---|---|
| Fringe fields | +5.2% |
| Higher-orders | -1.7% |
| Misalignments | -26.0±11.6% |

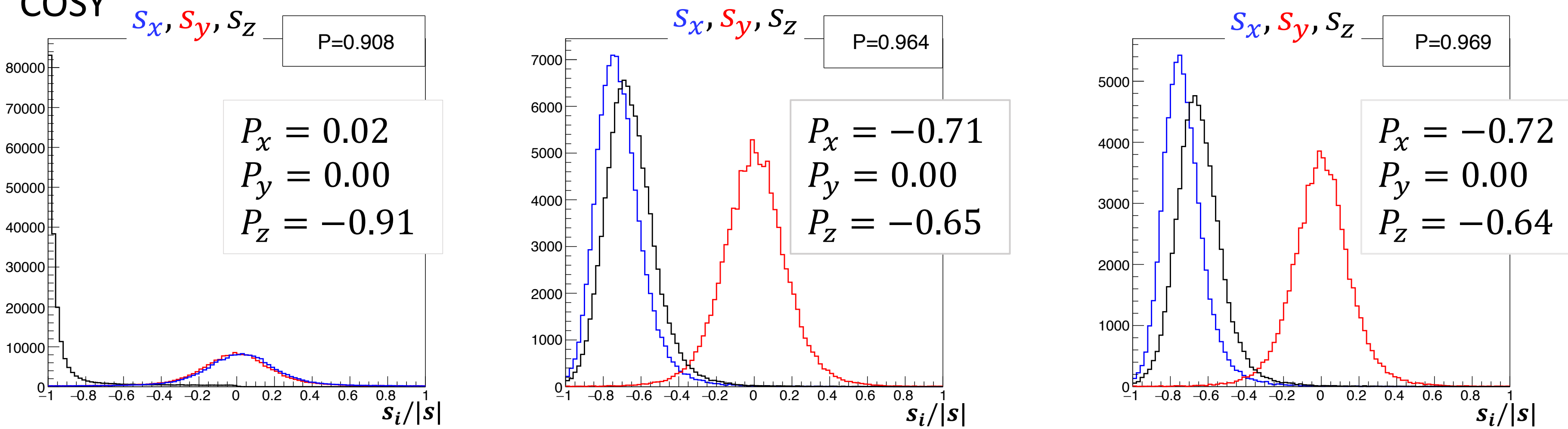
# 4. Conclusions

- Simulations include several aspects of the beamlines to consider nonlinearities on beam
- Beam performance from simulations agree with other numerical descriptions
- Fringe fields and high-order effects do not contribute significantly to statistical nor spin performance
- Though there may be consequences on spin-momentum correlations
- >748k muons/fill expected at entrance of SR
- From simulations, vertical and horizontal random misalignments ($\sigma_x = \sigma_y = 0.25\text{mm}$) reduce the population of muons (with $\delta < 2\%$) by ~$26.0\%$

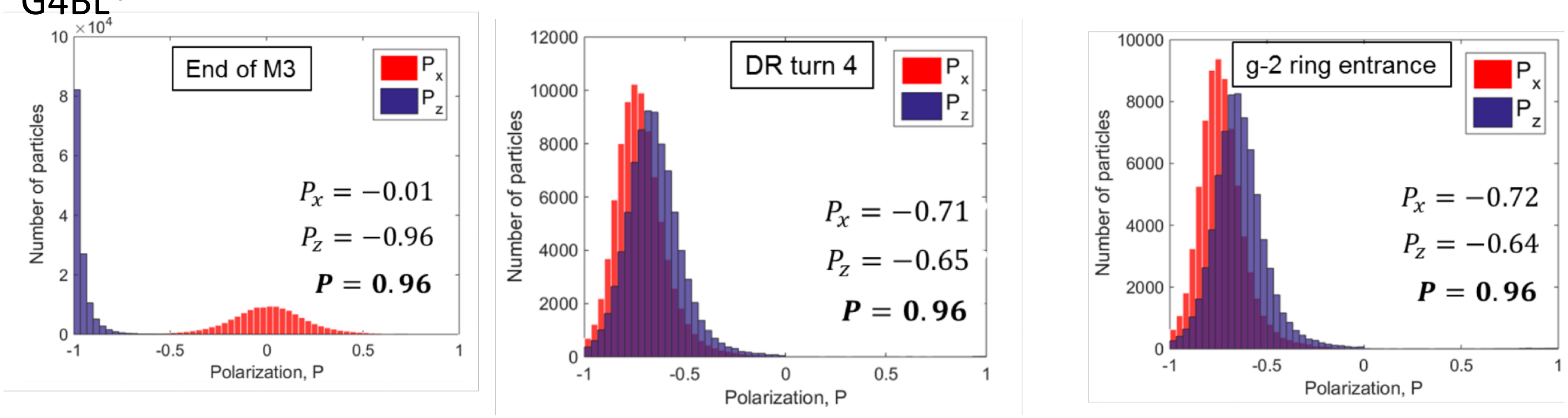
THANK YOU

# Spin dynamics along E989 Beamlines

COSY




G4BL*




*From g2Doc-4504 (Diktys S.)