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PARAMETER STUDIES AND GEOMETRY OPTIMIZATION ON SUPERCONDUCTING MULTICELL RF-CAVITY-RESONATORS[*]

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Abstract

Modern accelerator concepts for high intensity electron beams often require superconducting multicell RF-cavity-resonators in circular accelerators (e.g. in storage rings). Caused by strong beam-cavity interaction and due to high quality factors of superconducting RF-structures, special care of lower order (LOM) and higher order (HOM) modes must be taken. Various numerical studies were performed to compute the dependence of typical figures of merit (e.g. external quality factors Q_{ext}) with respect to the geometry parameters of the RF-structure, focused on the propagation of low and higher order modes. To ease the numerical effort an optimization routine has been developed which automatically optimizes the geometry based on goal functions. In addition the paper summarizes numerical 2D-calculations of monopole-modes of periodic structures with respect to longitudinal power flow which gives a rough estimate on values concerning Q_{ext} when applying HOM-dampening techniques.

INTRODUCTION

Upcoming projects like bERLinPro or BESSY$^{\mathrm{VSR}}$ heavily rely on the use of superconducting multicell RF-cavity-resonators [1, 2]. The resonators discussed consist of a periodic series of base cells (5-7) which are complemented by two endgroups to allow for capture and dumping of beam induced LOM- and/or HOM-power. The main figures of merit are of course covered by the properties of the base cell. Figures of merit such as shuntimpedance, intercell coupling and the ratio of peak and accelerating electric field provide the basis for the fundamental properties of the resonator. The capability to damp the LOM- and HOM-power induced by the beam in first order depends on the longitudinal power flow inside the resonator. This power flow from cell to cell depends on the intercell coupling and the width of the passband of the adjacent modes. It is characterized by the group velocity and thus by the phase advance between neighboring cells. The other important figure of merit regarding external quality factors is the stored electrical respectively magnetic energy, which is roughly proportional to the number of cells. The HOM-damping capability therefore depends mainly on the intercell coupling and the number of cells. Of course this simple examination is a rough estimate only, but provides a lower limit concerning the external quality factors of HOMS.

NUMERICAL CALCULATION

In consequence of field limiting effects like multipacting or field emission, the shapes of superconducting cavities are curved and most often parameterized by two ellipses, hence the name elliptical cavities. Due to the complex geometry of these cavities an analytical solution can not be found and the eigenfrequencies and the corresponding electric and magnetic fields must be calculated numerically. The intrinsic quality factors of superconducting cavities are very high, due to the low surface resistance. We decided to neglect these high intrinsic quality factors of the cavity by using perfect electric boundary (PEC) conditions, taking advantage of the reduced computational effort.

Two-dimensional Calculation

By focusing on the cavity structure and without including power couplers or HOM dampeners which would break the axisymmetry, one is able to solve for eigenfrequencies in a reduced two-dimensional problem. This greatly reduces the computational effort and enables the computation of HOMs with frequencies beyond the usual calculation done with 3D models e.g. [3].

Geometry Parameter Optimization

In order to optimize the various figures of merit of a cavity, a program has been developed which is able to optimize the cavity geometry (see Fig. 1) of the base cells of elliptical cavities based on goal functions. The optimization routine uses a random search algorithm in order to optimize non differentiable goal functions and hence easily put constraints on

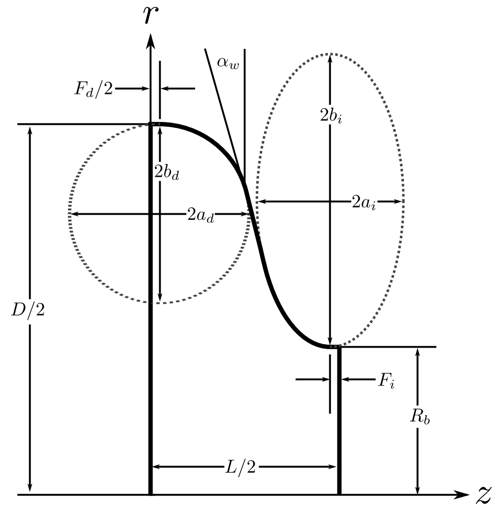

Figure 1: Geometry parameters of an elliptical cavity.

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certain figures of merit. Some general goal functions were tested in order to verify convergence of the used optimization algorithm (see Fig. 2). The program heavily relies on the use of the solver Superfish [5] and the supplementary program Ellfish. Superfish is able to solve monopole modes in two dimensions utilizing the axisymmetry. Ellfish simplifies the geometry input and is able to automatically tune certain geometry parameters in order to maintain the design frequency. All figures of merit calculated by Superfish are available for the goal function. The geometry parameters which should be used for tuning are also selectable.

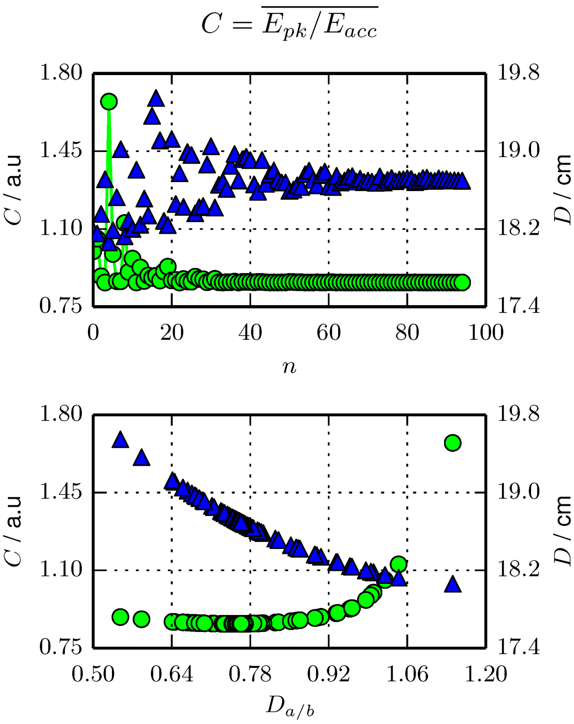

Figure 2: Optimization of the ratio $\overline{E_{pk}/E_{acc}}$ of a scaled 1.5 GHz TESLA cavity [4] which is normalized to the value of the starting geometry. The dome half axis ratio $D_{a/b} = a_d/b_d$ (see Fig. 1) was used to optimize the goal function. The green dots show the value of the goal function, the blue triangles represent the diameter D.

Passband Studies

For the ease of modeling we decided to use spline parametrization for the passband and external quality factor studies. It was shown earlier that this parametrization yields similar figures of merit than the elliptical parametrization [6] while additionally significantly reducing the geometry parameter space (see Fig. 3). Since we wanted to make the external quality factor studies as general as possible we decided to calculate the power flow through the iris in an infinite periodic multicell structure (see Fig. 5) based on a prototype design for the bERLinPro main linac (geometry parameters are listed in Table 1). Because Superfish only supports electric and magnetic boundary conditions we used COMSOL Multiphysics [7] for this task. We used Floquet-periodic boundary conditions with $k_z \in [0, \pi/l]$. Calculating the external quality factor can be done with [8]

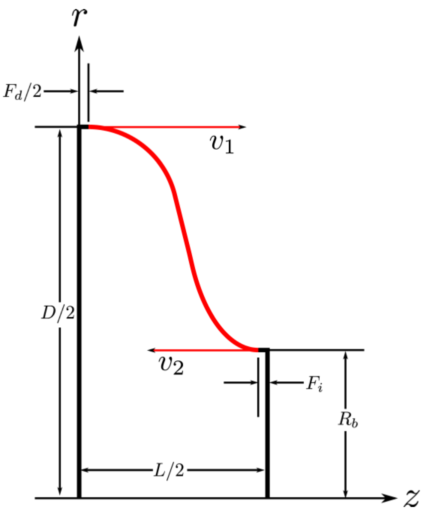

Figure 3: Geometry parameters of a cubic spline cavity.

Table 1: Geometry Parameter of the Cubic Spline Cavity

D / cm	**L / cm**	**R_b / cm**	**v_1 / cm**	**v_2 / cm**
20.320	11.530	3.600	6.128	2.882

$$Q_{\text{ext}} = \frac{\omega_0 U}{P_z} = \frac{\omega_0 \iiint_V \frac{1}{2}\epsilon_0 |\vec{E}|^2 dV}{\iint_A \rho_z dA},$$

where ω_0 represents the angular frequency, $\vec{E}$ the electrical field and ρ_z the power flow through the iris. The power flow of the modes with $k_z = 0$ and $k_z = \pi/l$ is $\rho = 0$ W/m^2 since the group velocity becomes zero, leading to infinite external quality factors. These modes were left out of consideration since we are most interested in the lowest external quality factors respectively the maximum power flow through the iris of each passband. All studies were performed with a maximum mesh size of $h = \lambda_{010}/96$ where λ_{010} represents the wave length of the fundamental TM$_{010}$-mode. Further more 31 phase advances of each passband have been calculated ranging from $k_z = 0$ to $k_z = \pi/l$. The first fifteen monopole modes excluding modes with a phase advance of 0 and π are shown in Fig. 4. It can be seen that passbands with a greater frequency spread correlate to a smaller external quality factor respectively greater power flow through the iris due to stronger cell coupling. The third mode, which happens to be the first TE-mode, has significantly higher external quality factors than its surrounding TM-modes. This has to be taken into consideration when designing multicell cavities, since these are basically the minimal external quality factors one can achieve when neglecting intrinsic quality factors with perfect coupling.

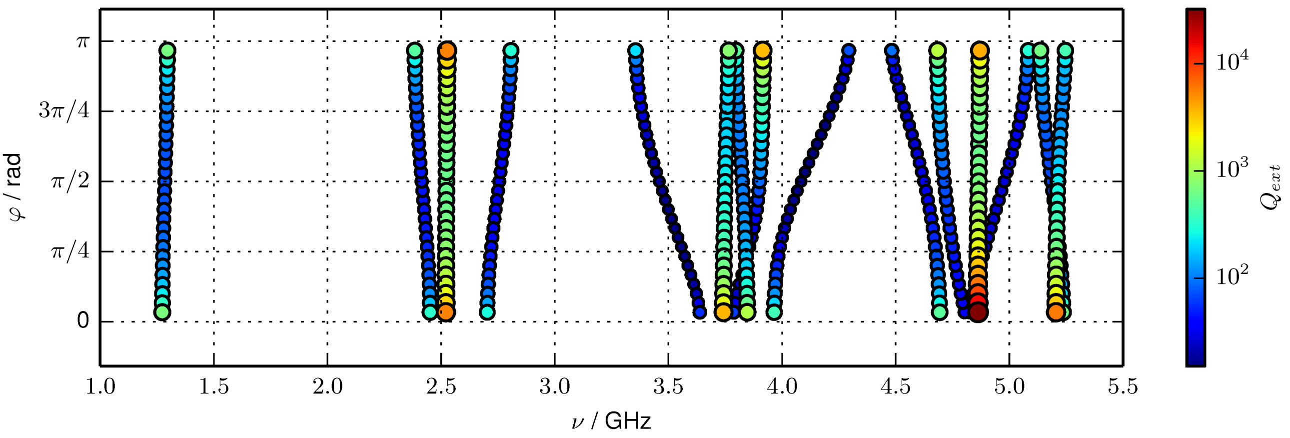

Figure 4: External quality factors for the first 16 mode passbands of the cubic spline cavity.

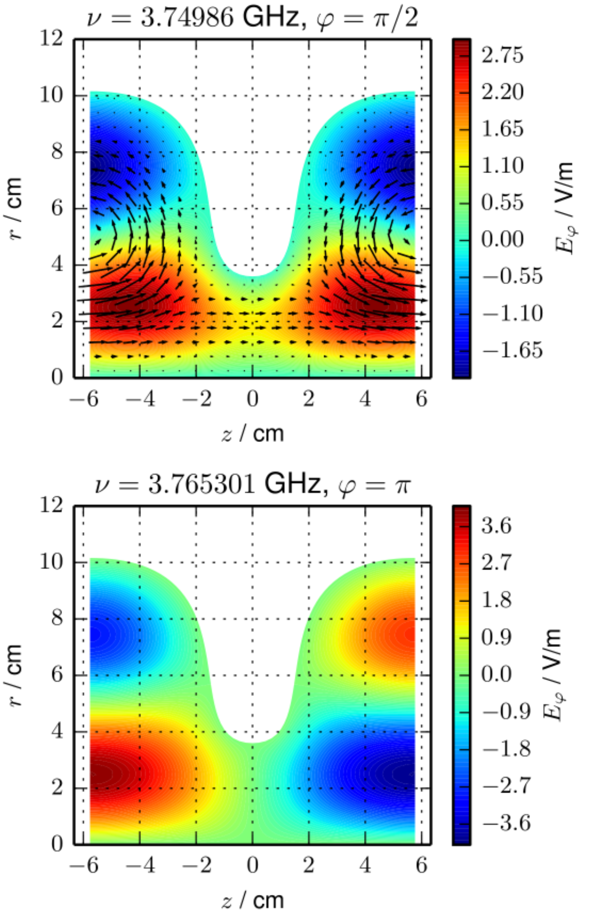

Figure 5: Field distribution and power flow of the periodic structure with $k_z = \pi/2l$ and $k_z = \pi/l$ respectively a phase advance of $\varphi = \pi/2$ and $\varphi = \pi$.

CONCLUSION

We developed a program to automatically optimize the base cell geometry of elliptical cavities based on goal function. The used random search algorithm showed good convergence. Making use of the axisymmetry significantly reduced the computational effort. General studies on external quality factors involving the mode propagation in infinitely periodic multicell structures based on cubic spline parametrization additionally showed the benefit of reducing the problem complexity to two dimensions. While using the axisymmetry we were able to easily solve up to frequencies of about 5.5 GHz while maintaing a reasonable mesh resolution of about 96th of the wavelength of the fundamental mode. Further studies are going to be made using azimuthal wave number $m \neq 0$ to study the propagation of multipole modes involving 2.5 dimensional computations. Future studies might also involve calculations with lossy material in order to take the finite surface resistance of superconducting RF-resonators into account.

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