# QUASI-FROZEN SPIN METHOD FOR EDM DEUTERON SEARCH*

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## Abstract

To search for proton Electric Dipole Moments (EDM) using proton storage ring with purely electrostatic elements, the concept of frozen spin method has been proposed [1]. This method is based on two facts: in the equation of spin precession, the magnetic field dependence is entirely eliminated, and at the "magic" energy, the spin precession frequency coincides with the precession frequency of the particle momentum. In case of deuteron we have to use the electrical and magnetic field simultaneously as will be explained later, keeping the frozen spin direction along the momentum as in the pure electrostatic ring. In this article, we suggest the concept of the quasi-frozen spin, in which the spin oscillates around the momentum direction within the half value of the advanced spin phase in magnetic arcs, each time returning back in electrostatic arcs. Due to the low value of the anomalous magnetic moment of deuteron, an effective contribution to the expected EDM effect is reduced only by a few percent compared with frozen spin method.

## INTRODUCTION

The frozen spin method [1] is based on the fact that at a certain so-called "magic" energy, the particle spin begins to rotate with the frequency of the momentum and is always directed along the momentum. Under this condition, the signal growth of presumably existing electric dipole moment is maximized. This is clearly evident from the T-BMT equations:

$$\frac{d\vec{S}}{dt} = \vec{S} \times \left( \vec{\omega}_G + \vec{\omega}_{edm} \right)$$

$$\vec{\omega}_G = -\frac{e}{m} \left\{ G\vec{B} + \left( \frac{1}{\gamma^2 - 1} - G \right) \frac{\vec{\beta} \times \vec{E}}{c} \right\}$$

$$\vec{\omega}_{edm} = \frac{e\eta}{m} \left( \vec{\beta} \times \vec{B} + \frac{\vec{E}}{c} \right)$$   (1)

$$G = \frac{g-2}{2},$$

where $G$ is the anomalous magnetic moment, $g$ is the gyromagnetic ratio, $\omega_G$ is the spin precession frequency due to the magnetic dipole moment (hereinafter called MDM precession) relative to the momentum, $\omega_{edm}$ is the

spin precession frequency due to the electrical dipole moment (hereinafter called EDM precession), and $\eta$ is the dimensionless coefficient defined by the relation $d = \eta e\hbar / 4mc$ in (1).

It is reasonable to implement the frozen spin method in a purely electrostatic machine with electrical deflectors holding a beam on orbit. The advantages of purely electrostatic machines are especially evident at the "magic" energy, where

$$G - \frac{1}{\gamma_{mag}^2 - 1} = 0 \,,$$   (2)

and the spin oriented in the longitudinal direction rotates in horizontal plane with the same frequency as the momentum, which is $\omega_G = 0$ [2].

However, this method cannot be used for deuterons having the negative anomalous magnetic moment $G = -0.142$, which follows from condition (2). Therefore, the only possible method in this case is a storage ring with both electric and magnetic fields [3]. It was proposed to store a longitudinally polarized deuteron beam of 1 GeV/c total momentum in an electro-magnetic storage ring of 0.5 T, where they minimize $\omega_G$. It should be small, but not zero. This can be done by applying a radial electric field of magnitude to cancel the $G \cdot B$ contribution to $\omega_G$ in Eq. (1):

$$E_r = \frac{GBc\beta\gamma^2}{1 - G\beta^2\gamma^2} \approx GBc\beta\gamma^2$$   (3)

However, studying the proton spin-orbital dynamics in a purely electrostatic ring [4], we realized that the condition (2) is feasible only for the reference particle. For all other particles the spin is not frozen and changes in time. In addition, condition (3) forcibly couples the values of the electric and magnetic fields, assuming both of them exist in each element. Firstly, the latter determines the rate of EDM signal increase, and secondly, it complicates the design of the element with incorporated E and B field.

## QUASI-FROZEN SPIN CONCEPT

Thus, the only requirement of the "frozen" spin condition is to maximize the EDM signal growth. However, if the spin oscillates in the horizontal plane with respect to frozen spin direction with amplitude $\Phi_s$, then the EDM growth decreases proportionally to the

factor $J_0(\Phi_s) \approx 1 - \dfrac{(\Phi_s)^2}{4}$ . We take into account that the anomalous magnetic moment for deuteron $G = -0.142$ has a small value and assume that the spin oscillates around the momentum direction within the half value of the advanced spin phase $\pi \cdot \gamma G / 2n$, each time returning back by special optics with $n$-periodicity. Due to the low value of the anomalous magnetic moment of deuteron, the effective contribution to the expected EDM effect is reduced only by a few percent.

This allows us to proceed to the concept of quasi-frozen spin, where the spin is not frozen with respect to the momentum vector, but continually oscillates around some average fixed direction coinciding with the momentum direction. After all we have to answer the question how to implement a variable MDM spin precession in the storage ring and to provide a sufficient EDM signal growth.

Now let us consider the spin equation in an electric deflector and a bending magnet separately. From the T-BMT equation in the laboratory coordinate system, it follows that MDM spin precession in the horizontal driving electrical field is

$$\omega_G^E = -\frac{e}{m}\left\{ \left(G + \frac{1}{\gamma+1}\right)\frac{\vec{\beta}\times\vec{E}}{c} \right\},\qquad(4)$$

At the same time, the frequency of particle momentum precession in the laboratory coordinate system with E field is

$$\Omega_p^E = \frac{e}{mc}\frac{\vec{E}\times\vec{\beta}}{\gamma\beta^2}.\qquad(5)$$

Subtracting (5) from (4) and normalizing the difference by (5), we obtain the spin tune in the electrical deflector relative to the momentum

$$\nu_s^E = \left(\frac{1}{\gamma^2-1} - G\right)\cdot\gamma\beta^2.\qquad(6)$$

We can do the same for the magnetic field. In the bend magnet, the frequency of MDM spin precession in the laboratory coordinate system is

$$\omega_G^B = \frac{e}{mc}\left(G + \frac{1}{\gamma}\right)\cdot cB,\qquad(7)$$

and the frequency of momentum precession in real space with B field is

$$\Omega_p^B = \frac{e}{mc}\frac{Bc}{\gamma}.\qquad(8)$$

Similarly, we find the spin tune in a magnetic field relative to the momentum:

$$\nu_s^B = \gamma G.\qquad(9)$$

Now let us to define the ratio of (6) to (9). Figure 1 shows the ratio

$$K = \nu_s^E / \nu_s^B$$

between spin tune in electrical and magnetic fields relative to particle momentum versus energy.

Thus, we can see that there is an energy region, where the MDM spin oscillation in the electric field is several times faster than in the magnetic field. Due to this fact, the idea of quasi-frozen structure can be implemented on the basis of two types of arcs: magnetostatic and electrostatic with inverse curvature of the later.
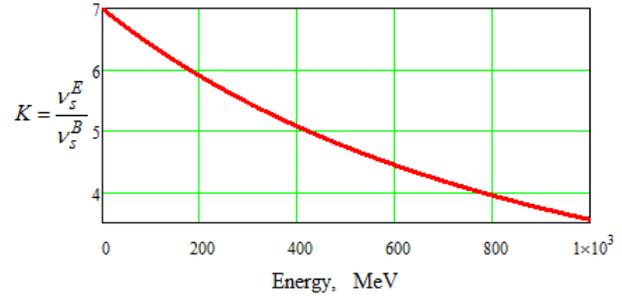


Figure 1: The ratio $\nu_s^E / \nu_s^B$ vs beam energy.

That is, the lattice is created by two parts: two magnetic arcs with bend magnets, rotating the particle by angle $\Phi^B = (\pi + 2\alpha)$ per arc and providing the MDM spin rotation in horizontal plane relative to the momentum by an angle $\Phi_s^B = \nu_s^B \cdot \Phi^B$, and two electrostatic arcs with electrical deflectors of negative curvature, rotating the beam by an angle $\Phi^E = -2\alpha$ per arc and providing the MDM spin rotation in the horizontal plane relative to the momentum in opposite direction by an angle $\Phi_s^E = \nu_s^E \cdot \Phi^E$. To realize the quasi-frozen spin concept, we have to fulfil this condition and ensure $\Phi_s^E = -\Phi_s^B$. Since in the electrostatic deflector the spin is rotated relative to the momentum with frequency, which is by the factor of $K = \nu_s^E / \nu_s^B$ faster than in magnetostatic structure, we have the basic relation for two different arcs:

$$\nu_s^B \cdot (\pi + 2\alpha) = \nu_s^E \cdot 2\alpha \ \text{ and } \ \alpha = \frac{0.5\cdot\pi}{\nu_s^E / \nu_s^B - 1}.$$

Figure 2 shows the ring for the deuteron energy 75 MeV as an example of its visual appearance.
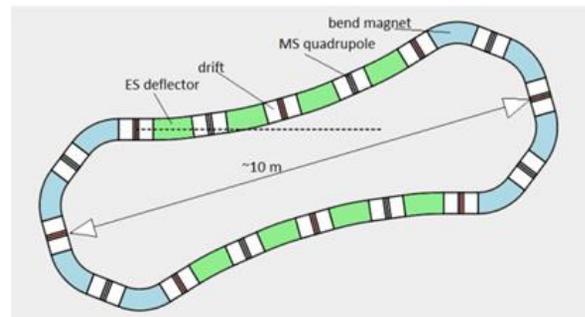


Figure 2: Ring lattice based on QFS concept.

Following the principles of this idea, it is obvious that the electrostatic and magnetostatic parts have an arbitrary geometry with the single condition

$$\sum_i \Phi_i^E \cdot v_s^E = \sum_j - \Phi_j^B \cdot v_s^B , \qquad (10)$$

where $\Phi_i^E, \Phi_j^B$ are the momentum angle rotation in *i-th* electrostatic and *j-th* magnetostatic element of structure respectively. The sequence of magnetic and electrostatic elements in the ring is also arbitrary and determined by the beam dynamics. So, turn by turn, the MDM spin rotation in magnetostatic part is compensated by MDM spin rotation in electrostatic part.

Obviously, this oscillation should lead to EDM signal reduction. However, due to the small amplitude $\Phi_s^E$ and $\Phi_s^B$, the growth of EDM signal is reduced insignificantly by factor

$$J_0(\Phi_s^{E,B}) \approx 1 - \frac{\left(\Phi_s^{E,B}\right)^2}{4} \sim 0.98$$

in comparison with fixed spin direction.

In addition, there is another lowering factor directly affecting the EDM signal, and it is common for both the "frozen" and quasi-"frozen" spin concepts. As one can see from T-BMT equations, the EDM signal in electrostatic and magnetostatic parts of ring will grow with different signs, partially compensating itself. Let us estimate this lowering factor as well. Since the EDM signal is proportional to the Lorentz force

$$dS_{E,B}^{edm} \propto \frac{e\eta}{mc}\left(c\beta_z B_y - E_x\right)\cdot dt , \qquad (11)$$

it is obviously proportional to the path length $L^E, L^B$ and inversely proportional to the radius of curvature $R^E, R^B$ in electrostatic and magnetostatic parts, that is, to the angle of beam rotation $\Phi^E, \Phi^B$ in the corresponding structure:

$$dS_E^{edm} \propto L^E / R^E = \Phi^E; \quad dS_B^{edm} \propto L^B / R^B = \Phi^B . \quad (12)$$

As a result, we can say that the ratio between values of the EDM signal is determined by the ratio of rotation angles in two structures, which in turn is determined by $K$:

$$\frac{dS_B^{edm}}{dS_E^{edm}} = \frac{\Phi^B}{\Phi^E} = \frac{v_s^E}{v_s^B} \approx 7 \div 5 .$$

Thus, the second lowering factor is about 0.85÷0.80, which is not a substantial reduction of the EDM signal. Besides, this lowering factor for the quasi-frozen spin concept is much larger than in case of the "frozen" spin concept.

To make sure that this is true, we have done a numerical simulation of 3D spin-orbital motion using two
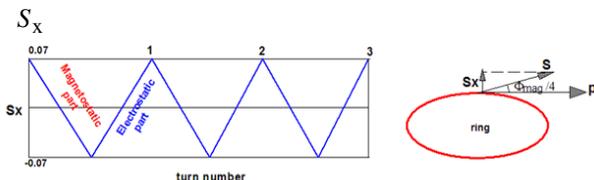
$S_x$



Figure 3: $S_x$ component vs. turn number.

codes COSY-Infinity [5] and MODE [6] and have obtained the following results.

Figure 3 shows the variation of the horizontal component of spin $S_x$ along the ring. One sees that the deviation of the spin in the horizontal plane is periodic and that it remains at the same level ~±0.07. Due to oscillation, which is assumed in the quasi-frozen spin method, the EDM signal decreases by 1%. Besides, in each magnet, the EDM signal grows by $2.14*10^{-16}$, and in each deflector it decreases by $3.2*10^{-17}$.
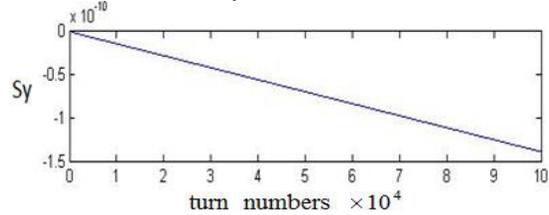


Figure 4: EDM signal growth vs. turn number.

As a result, the total EDM signal grows by $-1.4*10^{-15}$ per turn, and in order to get the total EDM signal to be ~$10^{-6}$, we have to keep the beam in ring for the duration of $N_{turn}$ ~$10^9$ or ~800 sec (see Fig. 4).

## CONCLUSION

The quasi-frozen spin method for deuteron is based on the following fundamental principles. Firstly, in a certain region of energy, the MDM spin precession relative to the momentum in the electric field is faster by factor 6-7 than in the magnetic field. Secondly, in the same region of energy, the EDM spin precession is faster by the same factor 6-7 in the magnetic field than in the electric field. Therefore, in case of different signs of curvature in the magnetic and electrostatic field, the MDM spin rotation relative to the momentum can be compensated and we should observe EDM signal.

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