SYSTEMATIC ERRORS INVESTIGATION IN FROZEN AND QUASI-FROZEN SPIN LATTICES OF DEUTERON EDM RING

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Abstract

The search for electric dipole moment (EDM) in storage rings raises two questions: how to create conditions for maximum growth of the total EDM signal of all particles in bunch, and how to differentiate the EDM signal from the induced magnetic dipole moment (MDM) signal. The T-BMT equation distinctly addresses each issue. Because the EDM signal is proportional to the projection of the spin on the direction of the momentum, it is desirable to freeze the spin direction of all particles in a bunch along the momentum. This can be successfully implemented in Quasi Frozen (QFS) and Frozen (FS) Spin structures. However, in case of magnet misalignments, the induced MDM signal may arise in the same plane as the EDM signal and thereby prevent its registration. In this paper, we analyze the effect of errors together with the spin-tune decoherence of all particles in the bunch for FS and QFS options.

INTRODUCTION

Currently, the JEDI collaboration develops the conceptual design of a storage ring specifically for the search of the deuteron electrical dipole moment (dEDM). For the design of such a ring, we need to address three major challenges:

- the lattice should meet the conditions of stability of motion, minimizing of beam loss, and it has to have incorporated straight sections to accommodate the accelerating station, equipment for injection and extraction of beam, a polarimeter, and sextupoles;
- using an RF cavity and a certain number of sextupole families, the horizontal plane polarization lifetime of the beam must be around ~1000 seconds;
- systematic errors have to be minimized to eliminate the induced fake EDM signal.

In this paper, we will analyze two types of structures: the frozen spin (FS) and quasi-frozen spin (QFS) lattices described in [1]. Our FS lattice is based on the "frozen spin" principle [2], where the spin of the reference particle is always orientated along the momentum. The FS lattice contains two arcs and two straight sections and is based on "E+B" elements with electric and magnetic fields combined in one element. In case of the quasi-frozen spin lattice, we have two options. In the first option, the electrical and magnetic fields are fully drawback of spatially separated [1]. However, this concept inherits the cylindrical electrodes, namely the whole set of high-order nonlinearities. Therefore, in second option of QFS lattice we introduced a magnetic field of small value ~80 mT, compensating the Lorentz force of the electric field. Both QFS lattices consist of two arcs and two straight sections with approximately similar circumference to that of the FS lattice.

SPIN TUNE DECOHERENCE

Initially, the problem of spin tune decoherence arose due to the requirement of having a maximum EDM signal. For horizontally oriented spin, the spread of spin tune leads to a multi-directional EDM signal for different particles and ultimately to a reduction of the total EDM signal. Later on, this problem was amplified by understanding the fact that considering systematic errors, in particular due to the electric and magnetic elements misalignment, spin decoherence can be transferred from the horizontal plane into the vertical plane, where we expect to see the EDM signal—that is, we get a "fake" EDM signal. The latter is a stronger argument than the geometric phase considered in [2], and it puts forward much greater demands on the limitation of the spin tune decoherence.

Now let us briefly mention the main causes of decoherence. Expanding in Taylor series the well-known expression for the deviation of the spin tune in the vicinity $\Delta\gamma$ of an arbitrary point γ_0 in electric and in magnetic fields

$$\begin{aligned}\Delta\nu_s^B &= \Delta\gamma\cdot G\\ \Delta\nu_s^E &= \Delta\gamma\cdot\left[-G-(1+G)/\gamma_0^2\right]+\Delta\gamma^2\cdot(1+G)/\gamma_0^3+..\end{aligned}, \quad (1)$$

we see that the spin tune in an electric field has all orders of non-linearity. Obviously, the linear term gives the maximum contribution to the spin tune decoherence, and a simple estimate shows that the spin coherence time is limited to a few milliseconds. Introduction of an RF cavity allows averaging and practically reducing the linear term contribution to zero. However, it has been shown in [3] that the $\Delta\gamma$ deviation follows the expression:

$$\Delta\gamma=\Delta\gamma_m\cdot\cos\Omega_s t-\frac{\beta^2}{\eta}\left[\frac{\left(\alpha_1-\dfrac{\eta}{\gamma^2}\right)}{\beta^2}\cdot\Delta\gamma_m^2+\gamma^2\left(\frac{\Delta L}{L}\right)_\beta\right], \quad (2)$$

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where $(\Delta L/L)_\beta = \left[\left\langle p_{xm}^2\right\rangle + \left\langle p_{ym}^2\right\rangle\right]/4$ is the orbit lengthening due to the betatron motion, γ_m is amplitude of γ synchrotron oscillation, $\eta = \alpha_0 - 1/\gamma^2$ is the slip factor, $\alpha_0 = 1/\gamma_{tr}^2$ is the first order momentum compaction factor, α_1 is the second order momentum compaction factor, and Ω_s is the synchrotron frequency.

Despite that the linear term is practically reduced to zero with RF, the time independent term in (1) and the term proportional to $\Delta\gamma^2$ in equation (2) restrict the spin coherence time to several hundred seconds. The final step to reduce the spin tune decoherence is based on the sextupoles, which change the orbit length depending on the momentum deviation and the dispersion [3]. Detailed numerical consideration of decoherence effects has been done using COSY INFINITY [4].

SYSTEMATIC ERRORS

In the EDM ring experiment systematic error arises due to the misalignments of electric and magnetic elements of the ring and causes a fake EDM signal. By their nature of origin being random errors, the misalignments create conditions for systematic errors in EDM experiments. The installation errors (misalignments) are associated with limited capabilities of the geodetic instruments. The bend magnet (or the electric deflector) can be rotated in three planes. We consider only the rotation around the longitudinal and transverse axis, because the rotation around the vertical axis does not introduce a systematic error. First, let us consider the case of the magnet rotated relative to the longitudinal axis (see Fig.1).

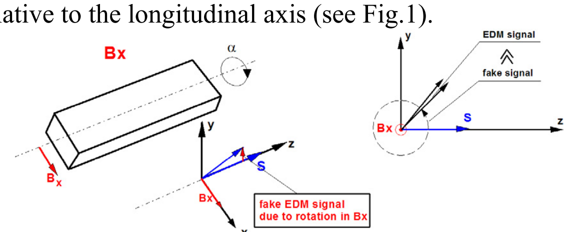

Figure 1: Magnet rotating relative to longitudinal axis.

Due to such a rotation, a horizontal component of the magnetic field arises and causes the spin rotation $\Omega_x = \Omega_{Bx}$ in the same plane where we expect the EDM rotation. To illustrate this, let us write the solution of T-BMT equations with initial condition $S_x = 0,\ S_y = 0,\ S_z = 1, \Omega_z = 0$ in simplest form:

$$S_x(t) = \frac{\Omega_y \sin(\sqrt{\Omega_x^2+\Omega_y^2}\,t)}{\sqrt{\Omega_x^2+\Omega_y^2}};\ S_y(t) = -\frac{\Omega_x \sin(\sqrt{\Omega_x^2+\Omega_y^2}\,t)}{\sqrt{\Omega_x^2+\Omega_y^2}} \quad (3)$$

Taking into account the above, we can present components: $\Omega_x = \Omega_{EDM} + \Omega_{Bx}$ and $\Omega_y = 0 + \langle \delta\Omega_{decoh} \rangle$, where Ω_{EDM} is the frequency of spin rotation due to the presence of an EDM, $\langle \delta\Omega_{decoh} \rangle$ is the spin tune decoherence in the horizontal plane, and it is allowed to reach an rms value of 1 rad for spin coherence time t_{SCT}=1000 sec.

The magnets are supposed to be installed at the technically realized accuracy of 10μm, which corresponds to the rotation angle of the magnet around the axis of about $\alpha_{max} = \pm 10^{-5}$ rad. Using COSY INFINITY [4], we have calculated the MDM spin rotation due to B_x, which is $\Omega_{Bx} \approx 3$ rad/sec. At the same time, at presumable EDM value of 10^{-29} e·cm, the EDM rotation should be $\Omega_{EDM} = 10^{-9}$ rad/sec, that is $\Omega_{EDM}/\Omega_{Bx} \approx 10^{-9}$, and expression (3) can be simplified:

$$\langle S_x(t) \rangle = \frac{\langle \delta\Omega_{decoh} \rangle}{\Omega_{Bx}} \sin \Omega_{Bx} t;\ S_y(t) = -\sin(\Omega_{Bx} + \Omega_{EDM})t \quad (4)$$

We can see from the first equation of (4) that the spin decoherence in the horizontal plane is not growing and is stabilized at the level of $\frac{\langle \delta\Omega_{decoh} \rangle}{\Omega_{Bx}} \approx 10^{-3}$. This is a significant positive feature. But to be fair, we should understand that, since $\Omega_{Bx} = \frac{e}{m\gamma}(\gamma G + 1) B_x$ we will now get spin frequency decoherence in the vertical plane (around radial axis), which we can minimize by the same methods (sextupole, RF) as in horizontal plane. In addition, we are really deprived of ability to measure the EDM signal by growth of the vertical component of spin suggested in [2], since $\Omega_{Bx} >> \Omega_{EDM}$ and S_y reaches a maximum after a very short time.

Therefore, the only solution is to measure the total frequency, but in order to split out the EDM signal from the sum signal, we need an additional condition. Such a condition is to measure the total spin frequency in the experiment with a counter clock-wise (CCW) direction of the beam $\Omega_{CCW} = -\Omega_{Bx}^{CCW} + \Omega_{EDM}$ and compare with clock-wise (CW) measurements $\Omega_{CW} = \Omega_{Bx}^{CW} + \Omega_{EDM}$. If we assume that we can measure the spin frequencies $\Omega_{CW}, \Omega_{CCW}$ with a relative accuracy of 10^{-10} already experimentally demonstrated in [5], we will be able to determine the EDM frequency

$$\Omega_{EDM} = (\Omega_{CW} + \Omega_{CCW})/2 + (\Omega_{Bx}^{CCW} - \Omega_{Bx}^{CW})/2 \quad (5)$$

up to 10^{-4} rad/sec, which corresponds to the EDM measurement on the level of 10^{-24} e·cm. However, we need to be sure that when the sign of the magnetic field B_y for the CW-CCW is changed, the magnetic field component B_x is restored with required precision not lower than 10^{-10}, since the difference $\Delta\Omega_{Bx} = \Omega_{Bx}^{CCW} - \Omega_{Bx}^{CW}$ actually determines the accuracy of the EDM measurement. Therefore, we suggest calibrating the field of the magnets using the relation between the

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beam energy and the spin precession frequency in the horizontal plane, that is determined by the vertical component B_y. Since the magnet orientation remains unchanged, and the magnets are fed from one power supply, the calibration of B_y will restore the component B_x with the same accuracy 10^{-10}, that is the difference $\Omega_{Bx}^{CCW} - \Omega_{Bx}^{CW}$ as well. Besides, we should mention that the calibration in the horizontal plane does not involve the EDM signal. Figure 2 shows the results of a numerical simulation of the EDM measurement procedure. We purposely took the initial EDM value 10^{-21} when $\Omega_{EDM} = 0.1$ rad/sec in order to reduce the duration of the simulation. Then, following the above described procedure, we "have measured" EDM and got EDM= 10^{-21}. Thus, we have proved the method of EDM measurement.

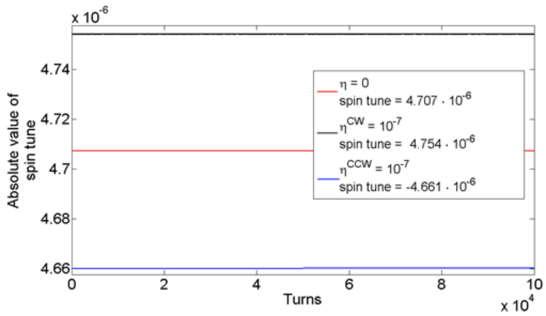

Figure 2: Results of numerical simulation of EDM measurement.

Nevertheless, the fundamental question of how to calibrate the field B_y using the spin tune measurement in a horizontal plane, if due to misalignments the spin rotates in the vertical plane with relatively high frequency Ω_{Bx} ~10 rad/sec, remains. To solve this problem, we plan for the calibration time only to introduce the inhibitory vertical field, for example by means of a horizontal coil. Having inhibited rotation in the vertical plane to the reasonable value of Ω_{Bx}~0.1 rad/sec and calibrated, then we turn off the coil. In this case we do not need to know the value of the field in the coil.

Up to this point, we have discussed only how to calibrate the magnetic field. But our ring consists of magnetic and electrical elements. Here we rely on the fact that calibrating the magnetic field and taking into account that the electric polarity is not changed and the unique connection of the magnetic field with the electric field for each energy value, we calibrate the electric field as well.

We have to mention that the idea of measuring EDM by introducing a horizontal magnetic field and measuring the spin precession in the vertical plane has been proposed in the wheel concept by I. Koop [6], but it differs from the method considered here. The wheel method uses a special horizontal coil, assuming calibration of the field in the coil by splitting of CW and CCW trajectories and measuring the distance between the separated beams. Besides, in the wheel concept, the issue with the change of field direction in presence of misalignments remained to be unresolved.

Finally, let us consider the case where systematic errors arise due to magnet rotation around the transverse axis, and we get the longitudinal component $B_z \neq 0$. The longitudinal component is not mixed with the EDM signal directly, but it can transform by spin decoherence from the horizontal plane into the vertical plane where we expect a signal of EDM. Now, let us suppose we do not have the systematic errors B_x=0 in vertical plane, but B_z≠0. The solution of the T-BMT equations with initial condition $S_x = 0,\ S_y = 0,\ S_z = 1,\ \Omega_x = 0$ at condition $\Omega_z = \Omega_{Bz}$, $\Omega_y = 0 + \langle \delta\Omega_{decoh} \rangle$ and $\Omega_{Bz} << \langle \delta\Omega_{decoh} \rangle$ is:

$$\langle S_x(t) \rangle = \sin\langle \Omega_{decoh} \rangle t; \langle S_y(t) \rangle = \frac{\Omega_{Bz}}{\langle \Omega_{decoh} \rangle}\left[1 - \cos\langle \Omega_{decoh} \rangle t\right] \quad (6)$$

How we can see the fake signal depends on the ratio between $\langle \Omega_{decoh} \rangle$ and Ω_{Bz}. Therefore, the only way is to minimize the longitudinal component of the magnetic field with $\Omega_{Bz} \sim 10^{-9}$ rad/turn, using additional trim coils with the longitudinal magnetic field.

CONCLUSION

In the paper we analyzed the frozen and quasi-frozen spin structures, taking into account the effect of spin decoherence and systematic errors. It has been shown how you can measure the EDM in an imperfect ring. These estimates show that the lower limit of detection of presumably existing EDM can be 10^{-24} e·cm.

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