

CHROMATICITY COMPENSATION SCHEMES FOR THE ARC LATTICE OF THE FCC-EE COLLIDER*

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Abstract

FCC-ee is a 100 km e^+e^- collider that is being designed within the Future Circular Collider Study organized by CERN. It's layout is optimized for precision studies and rare decay observations in the range of 90 to 350 GeV center-of-mass energy with luminosities in the order of $10^{35} \text{ cm}^{-2}\text{s}^{-1}$. Extremely small vertical beta functions of 1 - 2 mm are required at the two interaction points to reach this goal. The strong focusing required in the final doublet quadrupoles drives the chromaticity to more than -2000 units, far beyond the values that had been achieved in previous storage rings. As a consequence a pure linear chromaticity compensation scheme will not be sufficient to obtain the required $\pm 2\%$ energy acceptance. A state of the art multi-family sextupole scheme will have to be combined with a local chromaticity correction. This paper presents the design of the arc lattice, optimised for highest momentum acceptance and the results of systematic studies of the sextupole scheme in the arcs in order to gain highest chromaticity performance.

INTRODUCTION

The Future Circular Collider Study's lepton collider FCC-ee is designed as a 100 km electron-positron collider for operation at four different centre-of-mass energies to allow precision measurements from the Z peak at 90 GeV to the $t\bar{t}$ -threshold at 350 GeV. Additional measurements above the WW threshold (180 GeV) and at the energy of maximum Higgs production (240 GeV) are foreseen [1]. A summary of the baseline parameter set is given in table 1. For each energy the parameters are optimised to provide best luminosity performance with regard to beam-beam effect and beamstrahlung requirements.

Table 1: FCC-ee Parameters at Four Design Energies [1]

	Z	W	H	t
Beam energy (GeV)	45.5	80	120	175
Current (mA)	1450	152	30	6.6
Bunch number	91500	5260	780	81
Hor. Emittance (nm)	0.09	0.26	0.61	1.3
Vert. Emittance (pm)	1	1	1.2	2.5
β_x function at IP (m)	1	1	1	1
β_y function at IP (mm)	2	2	2	2
\mathcal{L}/IP ($10^{34} \text{ cm}^{-2}\text{s}^{-1}$)	90	19.1	5.1	1.3

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Layout and Lattice of FCC-ee

The FCC layout is shown in Fig. 1. The six straight sections of 1.4 km are clustered to groups of three straight sections connected by mini-arcs with the length of 3.2 km. Two extended straight sections of 4.2 km each are placed in the middle of the long arcs and will provide space for RF installations. The two experiments of the lepton collider will be housed in points A and G. The experiments in points F and H will only be used for the hadron collider.

The basic cell follows the FODO design with a phase advance of 90° in the horizontal plane and 60° in the vertical plane with a length of 50 m [2]. A draft of the FODO cell is shown in Fig. 2. The quadrupoles are accompanied by sextupoles on each side to locate their average effect precisely at the centre of the quadrupole. Considerations of synchrotron radiation define a maximum dipole length of approximately 10 m. Inbetween the dipoles absorbers need to be installed to shield the light fan. The design already includes room for bellows, flanges and absorbers.

For the following calculations standard mini-beta insertions were used without any local chromaticity correction.

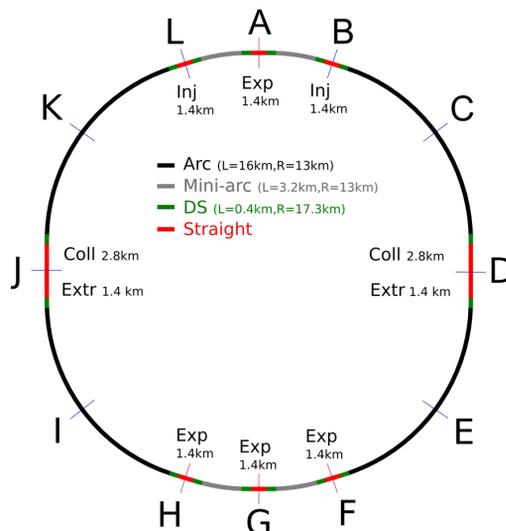


Figure 1: The FCC layout [1].

Natural Chromaticity

The design goal for luminosity is in the order of $10^{35} \text{ cm}^{-2}\text{s}^{-1}$. Therefore very small vertical beta functions of $\beta_y^* = 1-2 \text{ mm}$ at the interaction points (IPs) are required, which is achieved by very strong final doublet quadrupoles. For comparison in the former high-energy lepton collider LEP the design value was $\beta_y^* = 70 \text{ mm}$ [3]. As a consequence the natural chromaticity dominated by the effect of

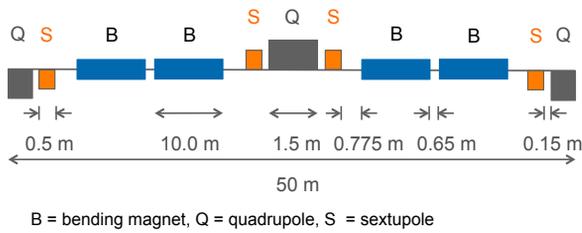


Figure 2: Draft of an FCC-ee FODO cell in the arcs. Sextupoles are placed at both sides of each quadrupole.

the interaction regions (IRs) reaches values of more than -2000 units in the vertical plane (LEP: -144.55 [3]). In order to achieve the momentum acceptance of $\pm 2\%$, which is required due to the severe energy loss by beamstrahlung during the interaction process, it is not sufficient to apply a simple correction of the linear chromaticity. Higher-order terms of the tune function expanded to the Taylor series

$$Q(\delta) = Q_0 + Q'\delta + \frac{1}{2}Q''\delta^2 + \frac{1}{6}Q^{(3)}\delta^3 + \dots \quad (1)$$

need to be compensated as well. In the following section methods are presented to correct the chromaticity including higher-order terms by a global approach with the sextupoles of the arcs. The main goal will be to increase the momentum acceptance as much as possible.

Table 2: First Four Orders of Chromaticity Calculated with a MAD-X Macro [4] the Corresponding Tune Shifts for $\beta_y^* = 2\text{ mm}$ and $\delta p = 0.1\%$

	$\beta_y^* = 1\text{ mm}$	$\beta_y^* = 2\text{ mm}$	ΔQ
Q'_x	-584.26	-587.67	-1.18
Q''_x	-3818.40	-3847.84	-0.01
$Q_x^{(3)}$	-1.43×10^8	-1.52×10^8	-0.20
$Q_x^{(4)}$	1.45×10^{13}	-1.43×10^{13}	-9.40
Q'_y	-2059.23	-860.42	-1.72
Q''_y	-4.18×10^6	-1.04×10^6	-2.09
$Q_y^{(3)}$	-1.19×10^{11}	-0.21×10^{11}	-27.75
$Q_y^{(4)}$	-4.53×10^{15}	-0.53×10^{15}	-351.67

GLOBAL HIGHER-ORDER CHROMATICITY CORRECTION

Correction of the Montague W Functions

As a first approach the sextupoles of the arc FODO cells are arranged into an interleaved sextupole scheme of two families per arc in the horizontal plane and three families per arc in the vertical plane. The two sextupoles next to a certain quadrupole belong to the same family. The phase advance between two sextupole pairs of each family is thereby 180° applying a $-I$ transformation of the beam coordinates. This scheme allows to iteratively correct the MONTAGUE W functions [5], which describe the distortion of the beam's

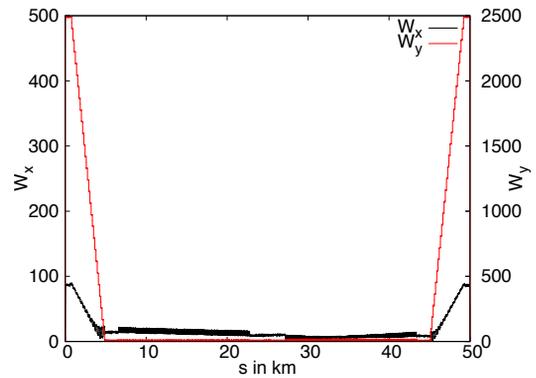


Figure 3: MONTAGUE W functions between IP 1 and IP 2.

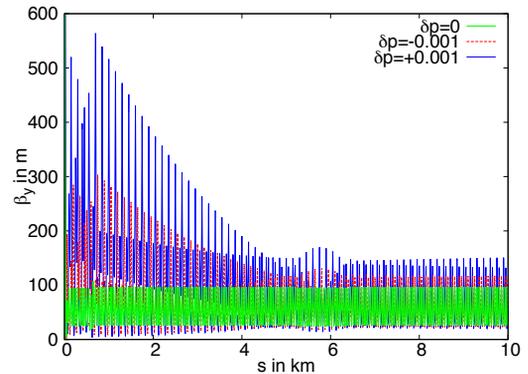


Figure 4: β functions for design energy and chromatic beta-beat at energy deviation of $\delta p = \pm 0.1\%$ ($\beta_y^* = 2\text{ mm}$).

phase space ellipse for energy deviations. The W functions for the first half of the lattice are shown in Fig. 3. Starting from $W = 0$ at the IP, high optics distortions are created by the final doublet quadrupoles increasing W_x to more than 90 and W_y to more than 2500 units. In the mini-arc next to the interaction regions the W functions are iteratively decreased by using mainly the first sextupole family. In the long arcs all sextupole families are used to compensate the linear chromaticity. In the mini-arc before interaction region 2 the W functions are built up again to compensate the final doublet's effect. The effect of the W function's correction on the optics is shown in Fig. 4, which displays the vertical β functions both for design energy and for $\pm 0.1\%$ energy deviation. For $\delta p = +0.1\%$ the betafunction exceeds values 5 times larger than its design value at the beginning of the mini-arc. As the W function is iteratively decreased also this chromatic beta-beat becomes smaller.

Momentum Acceptance Depending on β_y^*

The momentum acceptance obtained by the method described above crucially depends on the vertical β function at the IPs. For $\beta_y^* = 1\text{ mm}$ the natural chromaticity is as mentioned -2059 units. For $\beta_y^* = 2\text{ mm}$ less strong final focus quadrupoles are needed and the overall chromaticity values reduce to about -860 units. As a consequence the tune shift due to chromaticity is considerably smaller and the machine more stable. The tune as a function of relative

energy deviation is shown in Fig. 5. In both cases the vertical tune is dominated by the third order chromaticity term. However, after applying the correction of the W function, the momentum acceptance ranges from -0.21% to 0.07% for $\beta_y^* = 1\text{ mm}$ while stable conditions are obtained in an interval between -0.29% and 0.11% for $\beta_y^* = 2\text{ mm}$. In the $1\text{ mm } \beta_y^*$ case the first order chromaticity in the vertical plane was matched to $Q_y' = +2$. This avoids hitting the half-integer resonance for $\delta p = -0.15\%$.

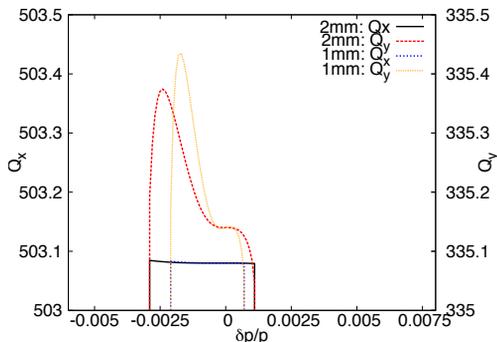


Figure 5: Momentum acceptance after correction of the W functions for two IPs with $\beta_y^* = 1\text{ mm}$ and $\beta_y^* = 2\text{ mm}$. For comparison the following plots have the same scales.

Additional Free Sextupole Pairs

In order to further increase the momentum acceptance additional free parameters in the chromaticity control are taken into account: on top of the correction of the W functions the first six sextupole pairs in the mini-arcs next to the interaction regions were powered individually. The resulting 12 degrees of freedom in the chromaticity correction scheme were optimised by matching routines in MAD-X [6], which flatten the tune dependency both at the edges of the acceptance and globally. In several iterations the momentum acceptance has been increased varying the strengths of the additional free sextupole pairs. Figure 6 shows the result of this optimisation based on corrected W functions using 6 sextupole families per arc per plane. The momentum acceptance could be increased by more than a factor two from $[-0.23\%; +0.29\%]$ to $[-0.49\%; +0.69\%]$. However the vertical tune is still strongly dominated by higher-order terms. Dynamic aperture studies need to prove the stability of this optics.

Downhill Simplex Optimisation

All previous optimisations of the momentum acceptance were done with built-in tools of MAD-X and analytical approximations. For completeness and comparison a numerical approach, the "Downhill Simplex Algorithm" [7] was implemented as well. MAD-X was called by the algorithm to calculate data for the penalty function's evaluation. Sextupole schemes with different numbers of families were optimised: 6 interleaved families per plane, 12 families per plane, and 54 families. The largest momentum acceptance

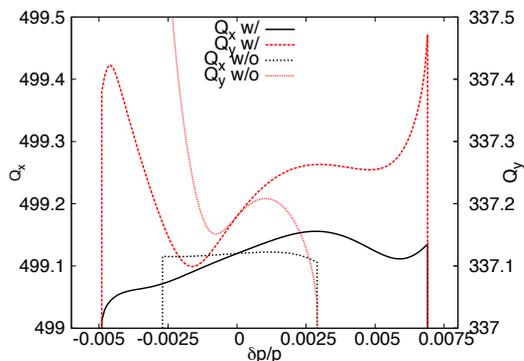


Figure 6: Momentum acceptance after correction of the W functions with six sextupole families per arc per plane with and without additional sextupole pairs in the mini-arcs. A considerable improvement is achieved.

was obtained by the scheme using the same 6 families per plane for all arcs reaching from -0.38% to $+0.54\%$. Figure 7 shows the tune function. Whithin $\delta p = \pm 0.2\%$ the tunes are constant in both planes.

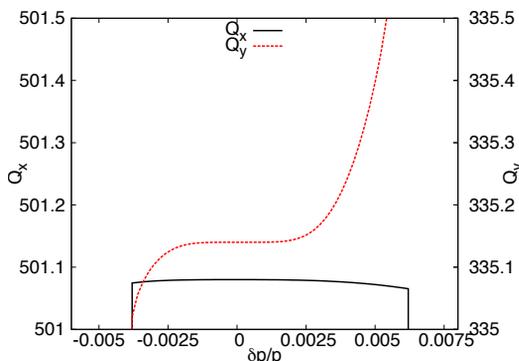


Figure 7: Momentum acceptance after optimisation of six sextupole families per plane using the "Downhill Simplex Algorithm".

CONCLUSION AND OUTLOOK

Considering the very large chromaticity of FCC-ee a global chromaticity correction scheme using the arc sextupoles needs to be joined with a local chromaticity correction system next to the IPs as known from linear collider studies. Still the intensive work on such a global scheme showed that a sophisticated design can improve the momentum acceptance of the lattice by a remarkable amount thus relaxing requirements for the local scheme. In the next steps of this study the combination of schemes will be investigated to obtain the highest achievable performance.

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