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QUANTUM EXCITATION DUE TO CLASSICAL BEAMSTRAHLUNG IN CIRCULAR COLLIDERS*

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Abstract

In the collisions at proposed future circular colliders, like FCC-ee and CEPC, a classical description of beamstrahlung is adequate, since the critical photon energy is much smaller than the particle energy. In the classical regime, for a constant electromagnetic field a simple relation exists between the average photon energy u and the average squared photon energy u^2, which is the same as for standard synchrotron radiation in storage rings. This simple relation only holds if the electromagnetic field is constant and uniform. This is not the case for a beam-beam collision. We here derive an analytical expression for the ratio $\langle u^2\rangle/\langle u\rangle^2$ considering the case of Gaussian-bunch collisions. We compare our result with the photon energies obtained in beam-beam simulation for FCC-ee at beam energies of 45.6 GeV and 182.5 GeV, using the two independent codes BBWS and GuineaPig. Finally, we re-optimize the FCC-ee parameters of a possible monochromatization scheme for direct Higgs production at a centre-of-mass energy of 125 GeV, derived previously, by applying the refined expression for the rms photon energy.

INTRODUCTION

In most electron storage rings operated so far, the equilibrium transverse emittances, energy spread and bunch length are determined by a balance of quantum excitation and radiation damping, both occurring in the accelerator bending magnets [1]. At future high-energy circular colliders, like FCC-ee or CEPC, also the synchrotron radiation emitted during the collision in the electromagnetic field of the opposing beam becomes important. This additional radiation, which is called "beamstrahlung" [2–6], can significantly increase the equilibrium bunch length and energy spread [7–9]. With non-zero dispersion at the collision point, as required for monochromatized collisions [10], beamstrahlung also affects the transverse beam emittance [9, 11]. In addition, the high-energy tail of the beamstrahlung may limit the beam lifetime [12, 13].

The strength of synchrotron radiation is characterized by the parameter Υ, defined as [5,6] $\Upsilon \equiv B/B_c = (2/3)\hbar\omega_c/E_e$, with $B_c = m_e^2c^2/(e\hbar) \approx 4.4$ GT the Schwinger critical field, ω_c the critical energy as defined by Sands [1], and E_e the electron energy before radiation.

For the head-on collision of two 3-dimensional Gaussian bunches of N_b electrons or positrons, respectively, with rms sizes σ_x^*, σ_y^* and σ_z the average Υ is [6]

$$\langle\Upsilon\rangle \approx \frac{5}{6}\frac{r_e^2\gamma N_b}{\alpha\sigma_z(\sigma_x^*+\sigma_y^*)}\,, \quad (1)$$

where α denotes the fine structure constant ($\alpha \approx 1/137$), and $r_e \approx 2.8\times10^{-15}$ m the classical electron radius.

For all proposed circular e^+e^- colliders, $\Upsilon \ll 1$ and $\sigma_x^* \gg \sigma_y^*$ at the interaction point (IP). We can then approximate the average number of photons per collision as [6]

$$n_\gamma \approx \frac{12}{\pi^{3/2}}\frac{\alpha r_e N_b}{\sigma_x^*+\sigma_y^*} \approx \frac{12}{\pi^{3/2}}\frac{\alpha r_e N_b}{\sigma_x^*}\,, \quad (2)$$

and the average relative energy loss as

$$\delta_B \approx \frac{24}{3\sqrt{3}\pi^{3/2}}\frac{r_e^3\gamma N_b^2}{\sigma_z(\sigma_x^*+\sigma_y^*)^2} \approx \frac{24}{3\sqrt{3}\pi^{3/2}}\frac{r_e^3\gamma N_b^2}{\sigma_z\sigma_x^{*2}}\,. \quad (3)$$

The average photon energy normalized to the beam energy, $\langle u\rangle$, is given by the ratio of δ_B and n_γ:

$$\langle u\rangle = \frac{\delta_B}{n_\gamma} \approx \frac{2\sqrt{3}}{9}\frac{r_e^2N_b\gamma}{\alpha\sigma_z\sigma_x^*}\,. \quad (4)$$

QUANTUM EXCITATION

The quantum excitation of oscillations, which gives rise to energy spread and emittance, is the product of the mean square photon energy and the mean rate [1]. In the case of beamstrahlung, the mean rate is simply given by n_γ divided by the average time interval between collisions (half the revolution period, with two interaction points).

In the classical radiation regime and for a constant bending radius ρ, the mean square photon energy $\langle u^2\rangle$ is related to the average photon energy $\langle u\rangle$ via [1]

$$\langle u^2\rangle \approx \frac{25\times11}{64}\langle u\rangle^2 \text{ (constant } \rho). \quad (5)$$

In Ref. [9], by evaluating the quantum-mechanical expressions of the photon spectrum, we demonstrated the validity of (5) for Υ values up to several times 10^{-3}. In Refs. [11, 14, 15], we implicitly employed this relation when we optimized the beam parameters of monochromatized Higgs production. In the case of a Gaussian-bunch collision, the exact relation between $\langle u\rangle$ and $\langle u^2\rangle$ is more complex, however. Indeed, while at constant bending radius ρ, we have [1] $\langle u\rangle = 4/(5\sqrt{3})\hbar c\gamma^3/\rho$, and $\langle u^2\rangle$ is given by (5), in general (5) must be modified as

$$\langle u^2\rangle \approx Z_c\frac{25\times11}{64}\langle u\rangle^2\,, \quad (6)$$

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where the correction Z_c is related to the variation of $1/\rho$ in time and space:

$$Z_c \equiv \left\langle \frac{1}{\rho^2} \right\rangle \bigg/ \left(\frac{1}{\langle \rho \rangle^2} \right) . \tag{7}$$

For head-on collision with a rigid Gaussian bunch, the inverse local bending radius ρ at transverse coordinates (x, y) and time t during the collision (with 0 referring to the centre of the other bunch) can be approximated as [16]

$$\frac{1}{\rho(x,y,t)} = |\Theta(x,y)| \frac{1}{\sqrt{2\pi}\sigma_z/2} \exp\left(-\frac{(ct)^2}{2(\sigma_z/2)^2} \right) , \tag{8}$$

where $\Theta(x, y)$ denotes the integrated deflection angle and σ_z the rms bunch length. The above factorization into transverse and time components is possible if disruption and hour-glass effects are negligible. This is a reasonable assumption for the proposed circular colliders. Under this assumption, for a Gaussian bunch, the deflection angle Θ can be expressed in complex form as [16, 17]

$$\Theta(x,y) = \Delta y' + i\Delta x' = -\frac{2N_t r_e}{\gamma} F(x,y,\sigma^*) , \tag{9}$$

where σ^* denotes the beam covariance matrix at the collision point, N_t the number of particles in the target bunch, and γ the Lorentz factor of the deflected particle. Without any x-y correlations in the matrix σ^*, the function F is [18]

$$\begin{aligned} F(x,y,\sigma) \; = \; & \frac{\sqrt{\pi}}{\sqrt{2}(\sigma_x^{*2} - \sigma_y^{*2})} \times \\ & \left(w\left[\frac{x+iy}{\sqrt{2(\sigma_x^{*2} - \sigma_y^{*2})}} \right] - \right. \\ & \exp\left[-\frac{x^2}{2\sigma_x^{*2}} - \frac{y^2}{2\sigma_y^{*2}} \right] \times \\ & \left. w\left[\frac{(x\sigma_y^*/\sigma_x^* + iy\sigma_x^*/\sigma_y^*}{\sqrt{2(\sigma_x^{*2} - \sigma_y^{*2})}} \right] \right) \end{aligned} \tag{10}$$

where $w(z)$ denotes the complex error function.

Making use of the factorization (8), we write the (total) correction factor (7) as a product

$$Z_c \equiv Z_l Z_t , \tag{11}$$

defining

$$Z_t \equiv \frac{\int_{x,y} |\Theta(x,y)|^2 \rho_t(x,y) \, dx \, dy}{\left(\int_{x,y} |\Theta(x,y)| \rho_t(x,y) \, dx \, dy \right)^2} , \tag{12}$$

where

$$\rho_t(x,y) = \exp\left(-x^2/(2\sigma_x^{*2}) - y^2/(2\sigma_y^{*2}) \right) / (2\pi\sigma_x^*\sigma_y^*)$$

and

$$Z_l \equiv \frac{\int_u \exp(-4u^2)\rho_u \, du}{\left(\int_u \exp(-2u^2)\rho_u \, du \right)^2} , \tag{13}$$

with $\rho_u = \sqrt{2/\pi}\exp(-2u^2)$.

For any Gaussian $Z_l = 2/\sqrt{3} \approx 1.155$, while Z_t depends on the aspect ratio. Figure 1 presents the calculated values of Z_t and Z_c as a function of the aspect ratio $R = \sigma_x^*/\sigma_y^*$. In particular, we obtain $Z_c \approx 1.23$ for a round bunch with $R = 1$ ($R_t = 4(\ln 2 - \ln 3/2)/(\sqrt{2}-1)^2/\pi \approx 1.067$), $Z_c \approx 1.59$ for a quasi-flat bunch ($R \approx 6$), $Z_c \approx 1.77$ for a flat bunch ($R \approx 85$), and $Z_c \approx 1.81$ for an extremely flat bunch ($R > 5000$).

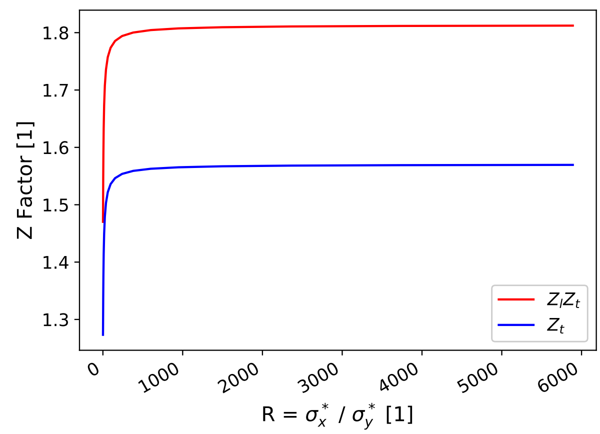

Figure 1: Transverse and total correction factors Z_t and $Z_c = Z_l Z_t$ versus $R = \sigma_x^*/\sigma_y^*$.

BENCHMARKING

To benchmark our analytical results and the various computer codes, we have determined the average energy loss and mean square energy loss in macroparticle simulations of a single beam-beam collision using the strong-strong code GuineaPig [19] and the weak-strong code BBWS [20]. These simulations considered FCC-ee parameters for 45.6 and 182.5 GeV, but without any crossing angle, and with a short bunch (0.5 and 1 mm, respectively), to avoid the hourglass effect. The BBWS and GuineaPig simulations used 10^7 and 10^5 macroparticles, respectively. From these simulations, we infer an empirical correction factor

$$Z_{\rm sim} \equiv \frac{64\langle u^2 \rangle}{25 \times 11 \langle u \rangle^2} , \tag{14}$$

which we may compare with the analytical estimate for Z_c in Fig. 1.

The results are summarized in Table 1. At 182.5 GeV, the analytical estimate lies between the two simulated values, which are quite different from each other. At 45.6 GeV the values from BBWS and GuineaPig are similar, but significantly lower than the analytical estimate. These discrepancies will require further investigation.

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Table 1: Comparison of Simulated and Analytical Correction Factors $Z_{\rm sim}$ and $Z_{\rm c}$ for Two Different Cases.

R	$Z_{\rm sim}$ GuineaPig	$Z_{\rm sim}$ BBWS	$Z_{\rm c}$
225 (45.6 GeV)	1.22	1.21	1.79
560 (183 GeV)	2.19	1.19	1.80

SELF-CONSISTENT EMITTANCE

The beamstrahlung strongly depends on the bunch length. The "total" (equilibrium) bunch length is related to the total energy spread via [1]

$$\sigma_{z,\rm tot} = \frac{\alpha_{\rm C} C}{2\pi Q_s}\sigma_{\delta,\rm tot} , \tag{15}$$

where Q_s denotes the synchrotron tune, C the circumference, and $\alpha_{\rm C}$ the momentum compaction.

In the case of zero IP dispersion, beamstrahlung excites the beam particles only longitudinally, and the total energy spread follows from the self-consistency relation [8, 9]

$$\sigma^2_{\delta,\rm tot} = \sigma^2_{\delta,\rm SR} + \frac{E}{\sigma^2_{\delta,\rm tot}\beta_x^{*\,3/2}\varepsilon^{3/2}_{x,\rm tot}} . \tag{16}$$

For nonzero IP dispersion, the beamstrahlung also increases the transverse emittance. If $D_x^*\sigma_{\delta,\rm tot} \gg \sqrt{\beta_x^*\varepsilon_x}$ (monochromatization), and $\tau_x = 2\tau_E$, where τ_x (τ_E) is the horizontal (longitudinal) damping time, we have [9]

$$\sigma^2_{\delta,\rm tot} = \sigma^2_{\delta,\rm SR} + \frac{E}{D_x^{*\,3}\sigma^5_{\delta,\rm tot}} , \tag{17}$$

$$\varepsilon_{x,\rm tot} \approx \varepsilon_{x,\rm SR} + \frac{2E\mathcal{H}_x^*}{D_x^{*\,3}\sigma^5_{\delta,\rm tot}} . \tag{18}$$

The dispersion invariant $\mathcal{H}_x^*$ is defined as [1]

$$\mathcal{H}_x^* \equiv \left\{\left(\beta_x^* D_x'^{\,*} + \alpha_x^* D_x^*\right)^2 + D_x^{*\,2}\right\}/\beta_x^* , \tag{19}$$

where β_x^*, α_x^*, D_x^* and $D_x'^{\,*}$ denote optical beta and alpha function (Twiss parameters), the dispersion and slope of the dispersion at the collision point, respectively.

In Eqs. (16), (17) and (18) the important coefficient is

$$E \equiv 47\, Z_t Z_l\, \frac{n_{\rm IP}\tau_E}{T_{\rm rev}}\, \frac{r_e^5 N_b^3\gamma^2}{(\alpha_{\rm C}C/(2\pi Q_s))^2} . \tag{20}$$

in which the correction factors (12) and (13) enter.

MONOCHROMATIZATION REVISITED

In a monochromatic collision we introduce IP dispersion of opposite sign for the two beams, so that the spread in the center of mass energy is reduced by the monochromatization (m.c.) factor λ, $(\sigma_w/W)_{\rm m.c.} = \sigma_\delta/\sqrt{2}/\lambda$, with the monochromatization factor $\lambda = \sqrt{D_x^{*\,2}\sigma^2_{\delta,\rm tot}/(\varepsilon_{x,\rm tot}\beta_x^*) + 1}$, where $\sigma_\delta \equiv \sigma_{E_b}/E_b$ is the relative beam energy spread.

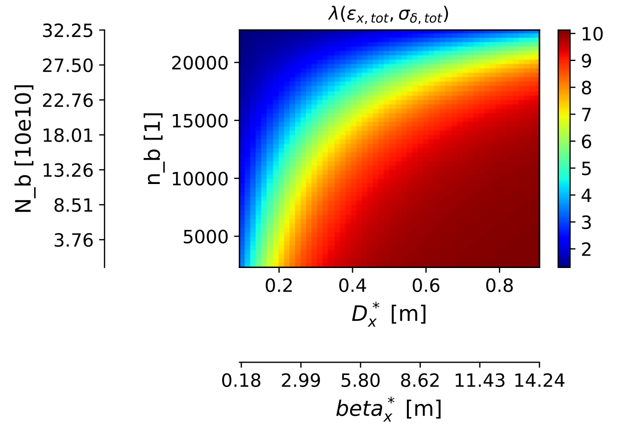

Figure 2: Magnitude of λ including beamstrahlung effects.

Using the modified expression (6), we have reoptimized the monochromatization at 125 GeV, following Ref. [15]. The updated dependence of λ on D_x^* and the number of bunches per beam, n_b, is shown in Fig. 2. Figure 3 presents the maximum achievable luminosity for a given λ.

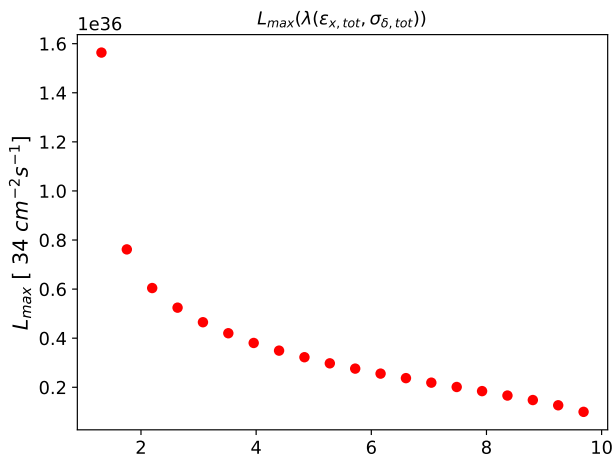

Figure 3: Optimal luminosity as a function of λ.

CONCLUSIONS

A refined model more accurately describes the effect of beamstrahlung on energy spread, bunch length, and transverse emittance. The updated quantum excitation is 20–80% larger than in [9, 15], depending on the aspect ratio. The analytical model was benchmarked against a strong-strong and a weak-strong computer simulation. The overall agreement is at the level of several 10%. Differences are seen not only between the new analytical formula and the simulations, but also between the two simulation codes.

We have updated the parameters of the optimized FCC-ee monochromatization scheme for direct Higgs production at a centre-of-mass energy of 125 GeV. At a monochromatization factor of $\lambda = 10$ ($\sigma_W \approx 5$ MeV) the maximum luminosity is 9×10^{34} cm^{-2}s^{-1}, at $\lambda = 5$ ($\sigma_W \approx 10$ MeV)

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3.1×10^{35} $cm^{-2}s^{-1}$. These numbers are about 20% lower than those obtained previously [15].

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