# CORRECTION OF $v_{r}-v_{z}=1$ RESONANCE IN TRIUMF CYCLOTRON * 

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#### Abstract

The second order linear coupling resonance $v_{r}-v_{z}=$ 1 is driven by an asymmetry in the median plane of the cyclotron due to the presence of a first harmonic in the radial component of the magnetic field. In the TRIUMF cyclotron, this resonance occurs at $\sim 166 \mathrm{MeV}$ and around 291 MeV . When the beam is off-centered radially passing through this resonance, the radial oscillation is converted into vertical oscillation, which can cause beam loss to occur. Although these loss modes do not reduce the machine transmission under normal operation, the spill is sufficient to cause radioactivation. The resonance can be corrected by using the existing harmonic coils. In this paper, we present the results of simulations and measurements that we have performed to correct this resonance.


## INTRODUCTION

The TRIUMF 500 MeV cyclotron has extracted increasingly intense proton beams during the past 43 years since its inception. Over the last 10 years, about $220-270 \mu \mathrm{~A}$ protons are routinely delivered to the users, with occasional demands up to $320 \mu \mathrm{~A}$. For the next 5 -year plan, we shall be adding another $100 \mu \mathrm{~A}$ beam-line [1] for the ARIEL project under construction, thus a capability of $420 \mu \mathrm{~A}$ total extraction shall be required with a total spills less than $1.2 \%$ in the cyclotron to keep activation down.

But very small changes in the circulating beam orbit can induce large oscillations to the vertical centre-of-gravity and size of the beam due to passage through the coupling resonance $v_{r}-v_{z}=1$. In particular, for the particles of extreme positive phases, they are less well centred radially and can have a combined coherent and incoherent amplitude of above $0.5{ }^{\prime \prime}$. The resonance converts some large radial amplitudes into large vertical motions, causing spills in the cyclotron and even in the beam-lines.

As an example, Fig. 1 shows the measured result of vertical beam centroid vs. radius, where the beam has been intentionally mis-centred by about $0.3^{\prime \prime}$ with a $B_{z} 1$ st harmonic coil in the centre region. The fast oscillations occurring between $\sim 215^{\prime \prime}$ and $255^{\prime \prime}$, and between $264^{\prime \prime}$ and $280^{\prime \prime}$ arise from the coupling resonance crossing. If there were no radial-vertical coupling, then the vertical flag, serving as a jaw to restrict the beam height from bottom in the centre region, would be able to reduce vertical beam spill all the way to the extraction. But we observe that it has hardly any such effect. This motivated us to look into this resonance with a view to correcting it.
The coupling resonance occurs at 166 MeV and then again at around 291 MeV , as is shown in the tune diagram Fig. 2.

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Figure 1: The radial probe measured vertical beam centroid vs. radius, showing the fast oscillations of an amplitude $\sim 0.2^{\prime \prime}$ due to the radial-vertical coupling resonance.

In the cyclotron's commissioning period 43 years ago, there


Figure 2: The TRIUMF cyclotron tune diagram, showing three passages through the coupling resonance $v_{r}-v_{z}=1$.
were concerns about this resonance. Also, measurements were taken to investigate the effect of a radial centring error on the height of the beam and on the beam loss [2]. Attempts were made to diminish the effect by altering harmonic coil settings but these did not seem to achieve the goal.

## RESONANCE DRIVING TERM

The coupling resonance is driven by an asymmetry in the median plane of the cyclotron due to presence of the first harmonic in the magnetic field $B_{r}$ component. The Hamiltonian [3] reveals that the driving terms in the magnetic field are something like:

$$
\begin{equation*}
B_{x} \sim\left(\tilde{x} \cos \theta-\tilde{x}^{\prime} \sin \theta\right), B_{z} \sim\left(\tilde{z} \cos \theta+\tilde{z}^{\prime} \sin \theta\right) \tag{1}
\end{equation*}
$$

where $\left(\tilde{x}, \tilde{x}^{\prime}, \tilde{z}, \tilde{z}^{\prime}\right)$ denote normalized phase space coordinates, and $\theta$ is the orbiting angle. The $B_{z}$ component implies that one can use the first harmonic of $B_{z}$ to centre the orbit radially to suppress the coupling. But this is not our 05 Beam Dynamics and EM Fields
objective here, because such a suppression is not a once-and-for-all solution, instead it depends on the amount of centring errors. Our goal is to apply the first harmonic of $B_{r}$ (i.e. $B_{x}$ ) to correct the resonance permanently.

The $B_{r}$ first harmonic error in the base field of the TRIUMF cyclotron is $\sim 0.5 \mathrm{G}$ in the area around the resonance, as shown in Fig. 3. This seems negligible but is still large enough to excite the resonance.


Figure 3: The $B_{r} 1$ st harmonic errors that exist in TRIUMF base field around the coupling resonance (indicated with the vertical dash-lines).

In terms of Joho's formula [3], the maximum vertical amplitude increase per turn is:

$$
\begin{equation*}
\left|\frac{d z_{0}}{d n}\right|_{\max }=\pi k \sqrt{\frac{v_{r}}{v_{z}}} x_{0} \tag{2}
\end{equation*}
$$

where $x_{0}$ and $z_{0}$ are respectively the radial and vertical amplitudes of the oscillations with focusing frequencies $v_{r}$ and $v_{z}$, and $k$ is the so-called critical frequency:

$$
\begin{equation*}
k=\frac{\alpha_{1}}{4} \frac{v_{r}^{2}-v_{z}^{2}}{\sqrt{v_{r} v_{z}}} \tag{3}
\end{equation*}
$$

and $\alpha_{1}$ is the angular tilt of median plane. At 291 MeV , we have $v_{r}=1.323, v_{z}=0.323$ and $\overline{B_{z}}=3.945 \mathrm{kG}$. These give $\alpha_{1}=0.5 / 3945=0.13 \mathrm{mrad}$ for the 0.5 G error amplitude, $k=8.0 \times 10^{-5}$, and $\left|\frac{d z_{0}}{d n}\right|_{\text {max }}=0.00015^{\prime \prime}$ per turn for a $0.3^{\prime \prime}$ radial centring error. This would mean that the beam needs 1000 turns to get $0.15^{\prime \prime}$ vertical amplitude. This growth rate is 100 times smaller than both the measured result and the simulation result as shown in Fig. 1 and in Fig. 6 respectively.

## HARMONIC COILS

We re-investigated this resonance, aiming to correct it by using the existing harmonic coils. TRIUMF cyclotron is equipped with 13 sets of harmonic coils placed at different radii, covering the full energy range; each set is composed of 6 pairs of coils installed in a 6-fold symmetrical manner azimuthally. By powering the coils on top and at bottom in opposite directions, $B_{r}$ component is produced in the geometrical median plane. As an example, Fig. 4 shows the
radial distribution of $B_{r}$ for the harmonic coils \#8 to \#12. It reaches a maximum at inner and outer radii of the coils, and changes sign at nearly the coil centre. What matters to the correction of the coupling resonance is the strength of the $B_{r}$ itself, not its radial gradient.


Figure 4: (Top) $B_{r}$ component in the geometrical median plane due to harmonic coils HC 8 to HC 12 . The 2 vertical dash-lines indicate where the resonance occurs. (Bottom) $B_{r}$ due to $\mathrm{HC10}$ combined with HC 12 , needed for the correction of the resonance occurring at $\sim 291 \mathrm{MeV}$ (the vertical dashline).

We began with orbit simulation for the resonance correction. It turns out that by combining $\mathrm{HC1} 10$ with HC 12 , as shown in Fig. 4, we can correct the resonance which occurs at $\sim 291 \mathrm{MeV}$. Whereas a single coil e.g. HC11 does not work out, though it is spanning the right radial range. This is because the field reverses direction and cancels the effect.

## SIMULATIONS

To begin with, static orbits were simulated. Initially, a single particle of 291 MeV was just sitting on its static equilibrium orbit (SEO) vertically while displaced from the SEO radially by $0.5^{\prime \prime}$. The particle's phase space coordinates were then recorded turn by turn at the starting azimuth for a number of turns. Before correction, the particle traces out a coupled trajectory between the radial and vertical phase spaces, shown in Fig. 5 in the action space $\left(J_{x}, J_{z}\right)$. After correction with $\mathrm{HC10}$ plus HC 12 , the coupling disappears, thus the particle stays on its SEO in both planes.

And then, an accelerated orbit was simulated, starting at 250 MeV , far enough below the resonance energy of 291 MeV . Similarly, the particle was just started from its static equilibrium orbit vertically while displaced from the SEO by $+0.5^{\prime \prime}$ or $-0.5^{\prime \prime}$ radially. The particle's coordinates were recorded turn by turn at the starting azimuth for a number of turns until it gets to 500 MeV extraction. After correction, the vertical oscillation amplitude becomes significantly reduced,


Figure 5: Result of simulation for the static orbit, showing the trajectory of a single particle in the action space turn by g turn for consecutive 400 turns, before and after correction of the coupling resonance. The "energy" exchange between the two planes seems to be agreed with Joho's theory [3] which states that $v_{r} x_{0}^{2}+v_{z} z_{0}^{2}=$ constant. Therefore when all the radial motion of $0.5^{\prime \prime}$ converts into the vertical, we get $z_{0} \simeq 1.0^{\prime \prime}$, namely the z -amplitude is $\sim 2$ times larger than the x -amplitude.
compared with that before correction. See Fig. 6. Should be pointed out that the residual oscillations, even showing up before 260 ", is probably due to the other higher order resonance.


Figure 6: Result of simulation for the accelerated orbit, showing the trajectory of a single particle in the $(r, z)$ space turn by turn for a number of turns until it gets to the extraction, before and after correction of the coupling resonance.

## MEASUREMENTS

Measurements were taken with the radial probe, in which a coherent radial centering error of the beam orbit was introduced intentionally by either detuning the centre region electrostatic deflector's high voltage from the production setting, or by detuning the amplitude and/or phase of $B_{z}$ first harmonic coil HC2 from the production settings. These production settings had been well tuned to minimize the machine spills. As shown in Fig. 7, with correction, the vertical oscillation amplitude is remarkably reduced. Moreover, this correction works well for the oscillations coupled from the other arbitrary centering errors (see Fig. 8). This is exactly the goal we want to achieve.


Figure 7: Result of radial probe measurements, showing the beam vertical oscillation amplitude significantly reduced after correction of the coupling resonance, where a coherent radial centring error of the beam orbit was intentionally introduced by detuning the deflector's high voltage. The flipped phase manifests the correct phase.


Figure 8: Result of radial probe measurements, showing that the resonance correction works well for reducing the amplitude of vertical oscillations coupled from the other centering errors due to detuned HC2's phase (upper) and amplitude (lower).

## SUMMARY

The beam vertical amplitude growth rate due to the coupling resonance, obtained from the measurements and the simulations both, does not agree with Joho formula. Nevertheless, we demonstrated that we can correct the resonance by powering the harmonic coils asymmetrically. As a result of the correction, the machine spills can be further reduced.

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