

# SIMULATION OF INVERSE COMPTON SCATTERING AND ITS IMPLICATIONS ON THE SCATTERED LINEWIDTH

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## Abstract

Compton scattering, though first described some one hundred years ago, has recently experienced a surge of interest due to the search for energy sources that are capable of yielding low emission bandwidths. In particular, the desire for hard x-rays with energies greater than 10 keV has led to increased study of inverse Compton sources. The rise in interest concerning inverse Compton sources has increased the need for efficient models that properly quantify the behavior of scattered radiation given a set of interaction parameters. The current, state-of-the-art, simulations rely of Monte Carlo-based methods, which may fail to properly model collisions of bunches in low-probability regions of the spectrum. Furthermore, the random sampling of the simulations may lead to inordinately high runtimes. Our methods can properly model behaviors exhibited by the collisions by integrating over the emissions of the electrons in the bunch in a lessened amount of time. Analytical simulations of Gaussian laser beams closely verify the behavior predicted by an analytically derived scaling law describing bandwidth of scattered radiation.

## INTRODUCTION

The increasing demand for efficient production of high-energy photons has resulted in interest in the construction of inverse Compton sources [1-4]. In particular, the desire to create hard x-rays from compact devices has propelled many groups worldwide to construct and improve such sources. The primary attraction of these sources is that they are capable of yielding emissions with low bandwidths, which are desirable in various applications.

We propose a new set of codes that can overcome the problems posed by CAIN, namely the inability to properly simulate low-probability regions [5]. While CAIN suffers from poor statistics in these regions, our codes, the improved codes for Compton simulation, overcome this problem by the nature of its formalism: the computed spectra of each individual electron-laser interaction will reflect the probabilities of photon scattering over the entire allowed range, providing statistics in the tails at the same level as in the peaks.

Finally, ICCS can be used to simulate and verify the analytically derived radiation linewidth scaling laws, improving the estimates presented in [6]. We present simulations demonstrating close agreement between predicted and simulated bandwidths of scattered radiation.

## COMPUTATION OF LOW-INTENSITY COMPTON SPECTRA

An earlier version of our codes was described in detail in [7]. The code models the effects of the electromagnetic field through the normalized vector potential and uses it as an input to the simulations. The finite pulse effects possible in a real laser pulse are described properly within a plane-wave approximation. The width and energy of the laser are also given as input. To describe the properties of the incident electrons, the program receives the relative spread in energy, horizontal and vertical emittances  $\epsilon_x$  and  $\epsilon_y$ , and the value of the electron  $\beta$  function evaluated at the interaction points. Finally, the range and location of sampling can be altered to consider various sections of the generated spectra. The algorithm implemented by our code overcomes a serious problem posed when using Monte Carlo integration: As the rarity of low-probability scatterings is reflected exactly in the code, results may suffer from poor statistics. Any electron distribution can be input and easily integrated according to Eq. (33) of [7]:

$$\frac{dU_1}{d\omega'} = \frac{\epsilon_0 c}{2\pi} \int_0^{2\pi} d\phi \int_{\cos\theta_{\max}}^1 |\tilde{E}[\omega(\omega')]|^2 \frac{d\sigma}{d\Omega} \left[ \frac{\omega'}{\omega} \frac{d\omega}{d\omega'} \right] d(\cos\theta),$$

where  $dU_1/d\omega'$  represents the differential scattered spectrum value,  $\phi$  represents the solid scattering angle, and  $\tilde{E}[\omega(\omega')]$  represents the time transform of the electron field. After summing over many particles ( $N_p > 400$ ), an extremely precise spectral distribution is given, closely modelling a continuous probability density function.

A scheme was also devised to improve the sampling of the energies of the electron beam. Previously, distributions of electrons – horizontal, vertical, and longitudinal momenta – have been generated as normally distributed in transverse (horizontal and vertical) and total momenta. A first-degree approximation given the initial energy  $E_0$ , the relative energy spread  $\sigma_E/E$ , and total momenta  $p$  can be

$$\sigma_p = \frac{1}{c} \sqrt{\left[ E_0 \left( 1 + \frac{\sigma_E}{E} \right) \right]^2 - m^2 c^4 - p^2}.$$

used to obtain the spreads in electron momenta  $\sigma_p$ :

This method can be especially advantageous, allowing the user to dictate the energy spread of the electron beam – as is the case in a laboratory setting. Unfortunately, the assumptions made in its derivation may mean that runs at high emittances can deviate from theoretical estimates.

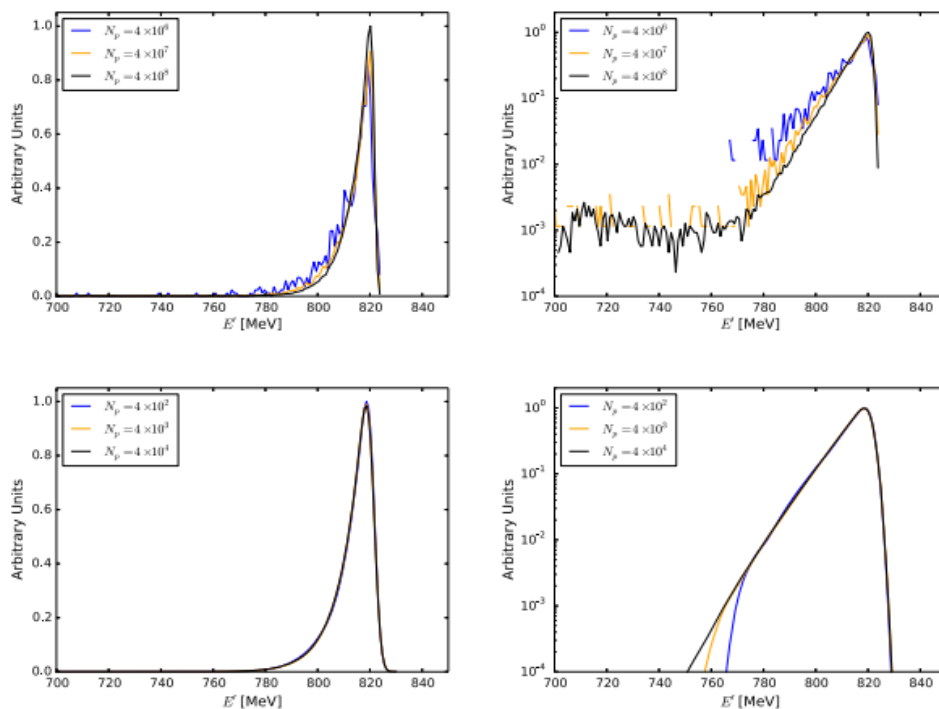


Figure 1: The numerically simulated spectrum for increasing resolutions from CAIN and ICCS for the case given in Table 1. The number of macroparticles  $N_p$  samples the electron beam distribution. Top row: CAIN simulations on the linear (left) and log scale (right). Bottom row: ICCS simulations on the linear (left) and log scale (right).

To combat this issue, ICCS adopts another approach in which distributions of electrons are generated as normally distributed in horizontal, vertical, and longitudinal momenta. In this approach, we specify as input (i) the horizontal and vertical emittances and (ii) the magnitude of the longitudinal momentum  $p_z$  and the associated standard deviation  $\sigma_{p_z}$ , which are used to obtain the longitudinal momentum distribution. The energy of the individual electrons is found from the relativistic energy-momentum relation and then averaged to obtain the mean energy of the electron distribution. However, the drawback of this approach is that the emittances and energy spread of the electron beam cannot both be directly controlled.

Another improvement in the development of these codes has been noted. The original version of the code, described in [7], was originally written in the PYTHON programming language. However, there is a drawback, as the ease associated with its use translates to significantly increased runtimes. ICCS has now been translated to a set of routines implement in C, wrapped in a PYTHON interface [8]. Runtimes can be up to 30 times quicker.

Table 1: Parameters Used as a Standard of Comparison Between the Two Codes (ICCS and CAIN)

| $E_e$ [GeV] | $\lambda$ [nm] | $\varepsilon$ [m]  | $\sigma$ | $\theta_{max}$     |
|-------------|----------------|--------------------|----------|--------------------|
| 7.00        | 1000           | $4 \times 10^{-7}$ | 50       | $5 \times 10^{-6}$ |

A simulation comparing the results of the two was carried out to compare the produced spectra. Three different levels of accuracy were chosen to demonstrate spectral

convergence (400, 4000, and 40000 macroparticles). Figure 1 shows the results produced by the two codes. It can be observed that the simulations with ICCS, compared to those carried out with CAIN, are superior in two ways: (i) Continuous spectra are produced with accurate probabilities, and (ii) low-probability regions (tails) are modelled with a much higher accuracy, as can be seen by comparing the top and bottom right panels of Fig. 1.

## SCALING OF SCATTERED RADIATION LINEWIDTH

Understanding the scaling of scattered radiation bandwidth in Compton sources at high electron energies is a key goal of this investigation. In this section, we present an analytically derived scaling law and test it against the results from numerical simulations of ICCS.

As terms for electron energy spread, photon energy spread, and aperture energy spread are independent, an *rms* value can be used to calculate the relative bandwidth of radiation. The final expression can be given by

$$\frac{\sigma_{E'_{ph}}}{E'_{ph}} = \sqrt{\left(\frac{\sigma_{\theta_{max}} + \sigma_{\epsilon}}{E_{\theta_{max}} + E_{\epsilon}}\right)^2 + \left(\frac{\sigma_L}{E_L}\right)^2 + \left(\frac{\sigma_r}{E_r}\right)^2}$$

where the first two terms represent energy spreads due to aperture and emittance, respectively, while the third and fourth represent energy spread due to laser bandwidth and electron beam energy spread.

There are a couple of important observations to note of this scaling law. First, there is a demonstrated covariance

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between the emittance and aperture effects. Though for small emittances and roughly Gaussian spectra this shift is negligible, it has the potential to skew spectra if significant, subsequently affecting the bandwidth of scattered radiation. Second, the bandwidth due to aperture, emittance, and the width of the laser are suppressed in regimes of high recoil. An important implication thus arises: only the properties of the electron beam dominate the bandwidth of scattered radiation at high electron energies. Such a fact can be used for the design of cost and resource-efficient Compton sources.

Table 2: Parameters Used as a Standard of Comparison Between Predicted Values Dictated by the Scaling Law and Simulated Bandwidth. From top to bottom, the parameters represent electron beam energy spread modulation, laser width modulation, and aperture modulation. The notation “Var.” means the corresponding parameter was varied.

| $E_e$ [GeV] | $\lambda$ [nm] | $\sigma_E/E_e$ | $\sigma$ | $\theta_{max}$     |
|-------------|----------------|----------------|----------|--------------------|
| 7.00        | 0.1            | Var.           | 50       | $3 \times 10^{-5}$ |
| 7.00        | 0.1            | 0              | Var.     | $3 \times 10^{-5}$ |
| 7.00        | 0.1            | 0              | 50       | Var.               |

Simulations of Gaussian laser beams were carried out in order to verify the applicability of the scaling law. Displayed in Fig. 2 are simulations in the deep-Compton regime (recoil factor  $X \approx 1300$ ) that demonstrate close agreement between the predicted and simulated values. Simulations were executed with ICCS, and simulation parameters are displayed in Table 2. Simulations of non-Gaussian laser beams were also tested to gauge emittance effects. As demonstrated in Fig. 2, the covariant model of aperture and emittance dependence predicts much better when the effects of the terms are magnified. Though, as shown in the first panel, a non-covariant model easily suffices for smaller emittance values.

It appears that numerical simulations agree closely with theoretical estimations in all cases. The scaling law presented remains valid for all scatterings in the linear regime, where the effects of ponderomotive broadening are minimal.

## CONCLUSION AND DISCUSSION

This investigation addresses two key issues: (i) the scaling of the scattered radiation linewidth and (ii) the efficient and accurate simulation of inverse Compton scattering spectra in the low-intensity regime.

A creation of an alternative to the widely used CAIN code has been described. Using a linear plane-wave approximation, the resulting scattered spectra can be modeled as a superposition of spectral values for individual electrons [7]. These values are computed by integrating the energy density per angle – given by the product of the time transform of the incident pulse, Compton formula, and cross section – over some defined aperture. In comparison to the Monte Carlo method of evaluating integrals at randomly chosen points, this method proves more accurate.

We have also proposed a model characterizing the bandwidth of scattered radiation given various interaction parameters. We have furthermore computationally demonstrated that there is a dependence between apertures and emittances (at high values). Finally, numerical simulations carried out with many particles verify the greater degree of accuracy of the altered scaling law.

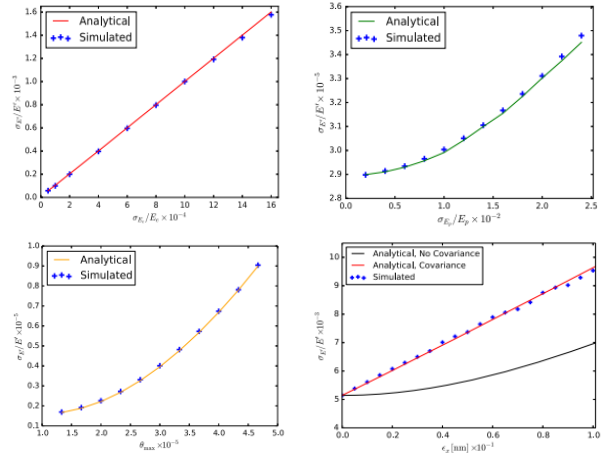


Figure 2: Simulations demonstrating agreement between predicted values and simulated bandwidths. Top row: left panel is electron beam spread modulation; right panel is laser bandwidth modulation. Bottom row: left panel is aperture modulation; right panel is emittance modulation. In the bottom left panels, the covariant and non-covariant models are compared for accuracy.

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