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APPLICATION OF QUAD-SCAN MEASUREMENT TECHNIQUES TO MUON BEAMS IN THE MUON g-2 EXPERIMENT

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Abstract

Determination of the properties of a beam during transport is a vital process for most accelerator-related experiments; for example Fermilab's Muon g-2 experiment requires large numbers of muons to be stored in a storage ring of 7 meter radius, and the transmission fraction has been shown to depend strongly on the properties of the beam, specifically the Twiss parameters. The current equipment in the muon campus beamlines allows only measurement of beam profiles which limits how well propagation can be predicted, however by using the well-studied quad-scan technique it is possible to obtain all of the Twiss parameters at a point using these profiles. Experimental quad-scans of muon beams have not yet been reported, this paper introduces the quad-scan technique and then goes on to discuss the analysis of one such experiment and the results obtained, showing that such a technique is applicable in the muon g-2 experiment to obtain the Twiss parameters without requiring installation of new equipment.

THE MUON g-2 EXPERIMENT

The aim of Fermilab's Muon g-2 experiment is to achieve an unprecedented 140 ppb precision measurement of the anomalous magnetic moment of the muon, to do so it will observe the polarization of muon decays in a precisely designed storage ring [1]. In order to reach the high statistics necessary for such a measurement, this ring must store as many muons as possible, and it has been shown that the Twiss parameters of the beam have a significant effect on the ring acceptance beyond simple beam size. Therefore, accurate measurement of these parameters is of interest to the experiment, although at present beam profile monitors provide the only direct diagnostic measurements in the beamline.

Fermilab's muon campus produces muons with approximately 20 times the statistics of the Brookhaven experiment on which g-2 is based. A beam of pions is produced by collision of a high energy proton beam with an Inconel target, and the produced beam is transported along the M2/M3 line which is designed to capture as many muons of the desired energy as possible. The beam is then injected into the delivery ring around which it travels 4 times so secondary particles such as protons can be removed. After this the beam is transported through the M4/M5 beamlines which end with a set of 5 magnets known as the final focus, through an inflector, and into the storage ring. A model of the final focus is shown in Fig. 1, the inflector is 30 cm downstream of the end of this image. A more detailed description of the muon campus can be found in reference [2].

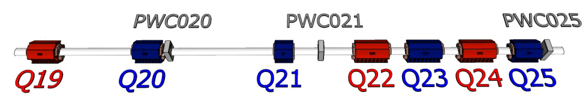

Figure 1: 3D model of the final focus at the end of the M5 beamline. The beam travels from left to right, and the inflector is 30 cm downstream of the end of this image. The image is to scale, with quadrupoles and proportional wire chambers (PWCs) increased by 3 times for clarity.

QUAD-SCAN TECHNIQUE

The foundation of the quad-scan technique discussed in this paper is the application of linear optics to beam transport. It is common to describe a beam by its six-dimensional phase space, and we can consider the transverse properties of a beam in terms of two-dimensional phase spaces (x x') and (y y'), where x is the coordinate relative to the reference beam and x' is the angular displacement of momentum in the x/z plane. The transport of the beam can then be described by the product of two-dimensional matrices, and a simple relationship can be used to obtain a matrix for the change in the Twiss parameters. We can consider a setup as shown in Fig. 2 with a quadrupole of focal length f (in x) followed by a drift of distance d before a profile monitor. The point P_0 is immediately upstream of the quad and is the point at which parameters will be measured, whilst P is the position at which profiles are measured.

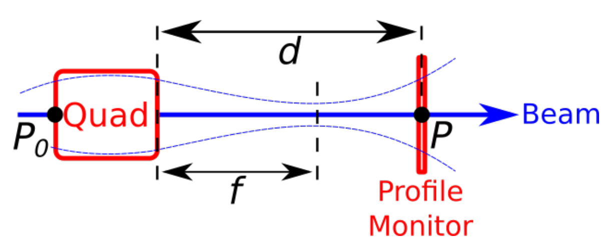

Figure 2: Diagram of the quad scan setup.

In this case, it is a simple task to show the relationship between 2D phase spaces at P and P_0 is described by Eq. (1), and this leads to Eq. (2) for the Twiss parameters [3].

$$\begin{pmatrix} x \\ x' \end{pmatrix} = \begin{pmatrix} 1 - \frac{d}{f} & d \\ -\frac{1}{f} & 1 \end{pmatrix} \begin{pmatrix} x \\ x'_0 \end{pmatrix} \qquad (1)$$

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$$\begin{pmatrix}\beta\\ \alpha\\ \gamma\end{pmatrix}_x = \begin{pmatrix}\left(1-\frac{d}{f}\right)^2 & -2d\left(1-\frac{d}{f}\right) & d^2\\ \frac{1}{f}\left(1-\frac{d}{f}\right) & 1-2\frac{d}{f} & -d\\ \frac{1}{f^2} & \frac{2}{f} & 1\end{pmatrix}\begin{pmatrix}\beta_0\\ \alpha_0\\ \gamma_0\end{pmatrix}_x \quad (2)$$

Note that the sign of f will differ in the horizontal and vertical direction. We can then consider the β parameter, Eq. (3) multiplies this through by the rms emittance ε to obtain the square of the rms beam width, σ.

$$\sigma^2 = \beta\varepsilon = b\left(1-\frac{d}{f}\right)^2 - 2da\left(1-\frac{d}{f}\right) + d^2c \quad (3)$$

Where $a = \alpha_0\varepsilon$, $b = \beta_0\varepsilon$, and $c = \gamma_0\varepsilon$. σ can easily be obtained from a beam profile, however ε is also required in order to convert a, b, and c into the Twiss parameters. Equation (4) relates this to these coefficients, starting from the definition of emittance, $\gamma\beta - \alpha^2 = 1$.

$$\varepsilon^2 = (\beta\varepsilon)(\gamma\varepsilon) - (\alpha\varepsilon)^2 = bc - a^2 \quad (4)$$

From Eqs. (2) and (4) it is clear that Twiss parameters and emittance of the beam can be obtained by fitting a parabola to a plot of $\left(1-\frac{d}{f}\right)$ against σ^2. The above equations assume zero dispersion and conserved emittance (Liouville's theorem). It is also desirable to be able to produce a beam waist over the range of quad strength values for the sake of accurate parabolic fitting.

APPLICATION TO THE FERMILAB MUON g-2 EXPERIMENT

The final focus shown in Fig. 1 allows for a beam waist and is an ideal site for such a scan, the proximity of this site to the inflector makes knowledge of the beam parameters at this point important. For the experiments described in this paper Q21 was turned off, the power to Q20 was varied, and the profiles were collected at PWC21. PWC21 is a proportional wire chamber; two planes of 48 gold plated tungsten signal wires with 10 μm diameter and 2mm spacing in an assembly filled with a gaseous mixture of 80% Argon and 20% Tungsten. The PWCs are capable of measuring beam intensities down to the order of 10^3 particles, producing both horizontal and vertical profiles at the same time.

Initially the quad-scan was simulated using G4Beamline and analysed, which confirmed that the procedure resulted in the expected Twiss parameters immediately upstream of Q20. The experiment was then physically carried out, the focal length of Q20 was varied between 3 m and 7 m (Design value is 3 m). Data collected at the PWC was fitted with a Gaussian (offset to account for noise) to obtain the rms beam width in each case and a parabola was fitted to the appropriate plots. All fitting was performed in Python using SciPy's non-linear least squares fitting algorithm [4], which returns uncertainties on fitted parameters.

RESULTS

Figures 3 and 4 show two parabolas obtained by this process, only profiles which could reasonably be described as Gaussian are included; for some focal lengths the beam was too wide for the PWC.

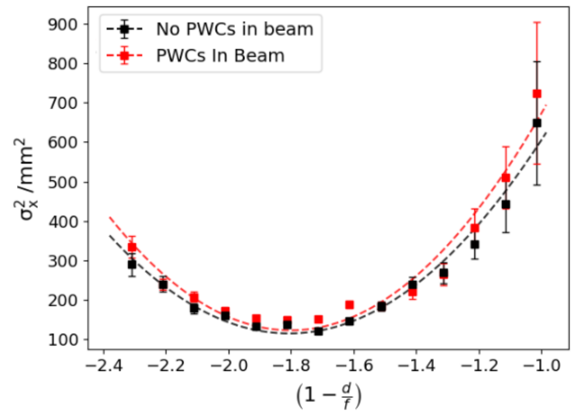

Figure 3: Fitted parabola for horizontal σ.

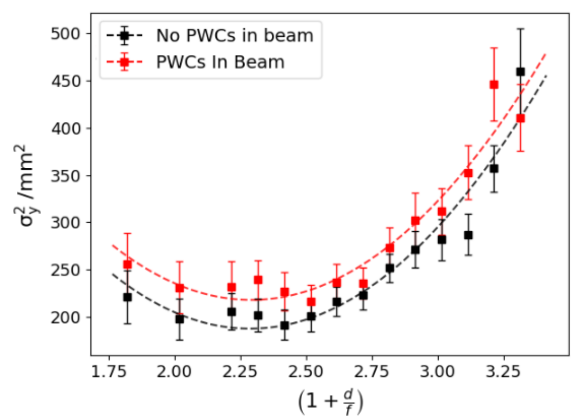

Figure 4: Fitted parabola for vertical σ.

These fits can easily be interpreted for the Twiss parameters and emittance as described above, the black parabolas in Figs. 3 and 4 correspond to horizontal and vertical emittances of 29 μm and 20 μm respectively. The Twiss values are listed in table 1.

Table 1: Measured Twiss Parameters Upstream of Q20

Parameter	Horizontal	Vertical
α	-4.6 ± 0.3	2.4 ± 0.4
β (m)	25.9 ± 1.8	10.7 ± 1.6
γ (m^{-1})	0.86 ± 0.05	0.64 ± 0.10

From inspection of Figs. 3 and 4 it is clear that the vertical fits are less accurate, and this is visible in the uncertainties in table 1; the average horizontal uncertainty is 6.4%, while the average vertical uncertainty is 15.8%. Further investigation into the beamline revealed that due to incorrect current settings in some of the upstream beamline quadrupoles the beam was mismatched and had erratically behaving beta functions in the region of the quad-scan, which may be the

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source of the vertical uncertainty. There also appears to be a slight systematic uncertainty in Fig. 3, this may be a result of non-zero dispersion in the area or some residual field from Q21.

BENCHMARKING

To the authors' knowledge, this is the first reported application of a quad-scan to a muon beam, as such it is important to verify the results obtained against other experimental measurements. One advantage of knowing the Twiss parameters is the ability to propagate them in both directions with linear optics, and since emittance is conserved the moments of the beam and rms width can be predicted, and in this case compared to results measured at other points in the beamline. Figures 5 and 6 show that in the case of the g-2 experiment there is very good agreement between these predictions and the measured values in the horizontal plane, however there is some disagreement in the vertical plane. This disagreement is likely related to the mismatching mentioned in the previous section.

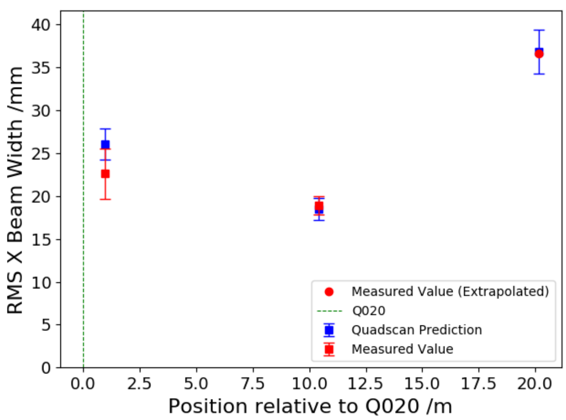

Figure 5: Comparison of measured and (quad-scan) propagated horizontal σ. Furthest value extrapolated from half profile due to noisy wires.

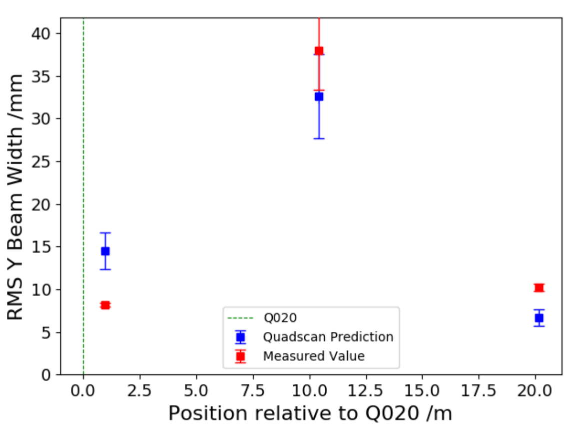

Figure 6: Comparison of measured and (quad-scan) propagated vertical σ.

MEASUREMENT OF PWC SCATTERING

As an example of the power of this technique, it has been used to quantise the effects of the PWCs on the beam. The quad-scan experiment was repeated twice, once with all PWCs upstream of the quad-scan outside of the beam path, and once with PWC301, PWC904, and PWC000 in the path of the beam. Both of these cases are shown in Figs. 3 and 4.

It is expected that the scattering from PWCs will generally increase the rms beam size and uncertainty as is seen, however this technique also provides information on how the Twiss parameters are effected, allowing for propagation further downstream. The technique allows quantification of the effect of PWCs; emittance grows from 29 μm to 32 μm in the horizontal plane and from 20 μm to 21 μm in the vertical plane when the PWCs are added to the beam, which is a notable increase that is difficult to quantify through other methods without additional equipment. Table 2 shows the effect on the Twiss parameters.

Table 2: Effect of PWCs on Beam Twiss Parameters

	Parameter	PWCs	No PWCs
Horizontal	α	-4.8 ± 0.4	-4.6 ± 0.3
	$\beta\ (m)$	26.7 ± 2.2	25.9 ± 1.8
	$\gamma\ (m^{-1})$	0.88 ± 0.06	0.86 ± 0.05
Vertical	α	2.2 ± 0.4	2.4 ± 0.4
	$\beta\ (m)$	9.9 ± 1.4	10.7 ± 1.6
	$\gamma\ (m^{-1})$	0.61 ± 0.09	0.64 ± 0.10

Because the script to analyze the data has already been prepared, the results of this measurement were available near instantly after the data was obtained. Further information on this investigation is available in reference [5].

CONCLUSION

The quad-scan technique has been introduced and carried out on both a simulated and physical beamline. In both cases the results agree, and further comparisons with propagated beam widths support obtained values. This initial quad-scan already proved useful; it identified a discrepancy between expected and obtained results and further investigation into the beamline revealed an issue upstream of the scan.

The horizontal values match other measurements and simulation extremely well, while the vertical values show a slight discrepancy. This emphasizes the stringent requirements of an accurate quad-scan such as linear forces. The technique is extremely useful as it allows Twiss parameters to be measured with a reasonable uncertainty without the need to add any new equipment to the beamline. As well as the position demonstrated here there are several other potential quad-scan locations throughout the muon campus which are being investigated. We are grateful to Andrew Fiedler for his help carrying out the experiment, and Mike Syphers for useful guidance and discussion.

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REFERENCES

[1] J. Grange *et al.*, "Muon (g-2) Technical Design Report", arXiv:1501.06858, (2015)

[2] D. Stratakis *et al.*, "Accelerator performance analysis of the Fermilab Muon Campus", *Physical Review Accelerators and Beams*, vol. 20, 111003, 2017.

[3] M. G. Minty and F. Zimmermann, *Measurement and Control of Charged Particle Beams*, Springer, Berlin, 2003.

[4] SciPyDocs, "scipy.optimize.curve_fit", https://docs.scipy.org/doc/scipy/reference/generated/scipy.optimize.curve_fit.html

[5] D. Stratakis, "Quad-Scan", *Muon g-2 Internal Document*, GM2-doc-6764-v1.