SOME PROBLEMS OF THE BEAM EXTRACTION FROM CIRCULAR ACCELERATORS

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Abstract

In this article some problems of optimizing the output beam of particles from the circular accelerator are discussed. In particular, we consider some problems of matching the booster, Nuclotron and collider in the NICA project. The main attention is paid to matching of the extraction beam systems. The proposed approach allows providing qualitative and quantitative analysis of the impact of various factors of the corresponding control systems.

INTRODUCTION

The problems of long time evolution of particles in cyclic accelerators arise not only in the implementation of the extraction of the particles. As an example, we should mention the problem of injection of particles into the accelerator, and also the problem of long time evolution (over millions of revolutions) of the particle beam in the storage rings and colliders. Let us look at the main types of problems that arise in similar types of tasks:

• construction of closed orbits in ideal and non-ideal machines;
• the problem of stability of the closed orbits in the framework of the linear and non-linear approximation of the control fields including possible deviations from the ideal parameters;
• problems of the beam injection and extraction from circular accelerators

It should be noted that when modeling of the long-time evolution (more than one billions revolutions) it is necessary to perform huge number of steps of integration and guarantee preserving both the energy of the particles and property of the symplecticity of corresponding mapping.

It is known that knowledge is information about control elements allows you to find the stable fixed points and areas (islands) with steady evolution in its neighborhoods. Knowing the location of these areas and their characteristics allows managing (using additional controls) during of extraction or injection processes using the information about topology of the corresponding closed orbits. Controlling by the corresponding classes of stable orbits can be carried out regardless. In particular, it allows not to use septum-magnets, which are traditionally used for extraction of the beam (see, eg, [**?**]). Note that the trend of development of modern circular accelerators leads to the need to include among the objects of control nonlinear control field with increasingly nonlinearity. The transition to an essentially nonlinear dynamics leads to the need for new and effective methods of mathematical modeling of long-term evolution of the beam in the accelerator channel. Increasing order nonlinear effects on the one hand allows you to "improve the quality of the beam" but to a significant complication of corresponding mathematical models and as a consequence corresponding algorithms and programs.

The purpose of this paper is to present the basic principles that not only form the basis of the proposed approach but and realized as special software and demonstrated effectiveness for a number of tasks [2]. The problems of extraction and injection of beam particles in cyclic accelerators included in the NICA complex, should be consider accurately enough, due to the peculiarity of the complex [3].

A study of the dynamic system in this case is carried out in terms of the map $\mathcal{M}_k$, generated by the control elements on the k-th step iteration $\mathbf{X}_{k+1} = \mathcal{M}_k \circ \mathbf{X}_k$, where $\mathbf{X}_k$ is the phase vector on k-th step of the iterative process, where $\mathbf{X}_k$ is phase vector on k-th step of the iterative process wich generated by the periodic dynamic system, where $\mathcal{M}_k$ is an operator of the evolution of a dynamical system corresponding to k-th period.

If $\mathcal{M}_k = \mathcal{M}$, then for any k we can talk about periodic mapping. In this case, the full map for k-th turns (for periodical channel) one can write

$$\mathcal{M}(s+kL|s) = \underbrace{\mathcal{M} \circ \ldots \circ \mathcal{M}}_{k \text{ times}} = \mathcal{M}^k,$$

where $\mathcal{M} = \mathcal{M}(s+L|s)$.

Before constructing the computational process for evolution of the beam, we should not only carefully examine the properties of the map $\mathcal{M}$, but also to ensure the fulfillment of these conditions in all stages of the computational experiment. Here we have in mind the preservation of the properties of symplecticity for the map $\mathcal{M}$ throughout interval solutions of the problem In particular, namely similar approach is implemented in a rather numerous modern works on the generation of symplectic difference schemes or integrators for Hamiltonian systems. In a number of previous works (see for example [4]) we considered the method of symplectification of block matrices included into the matrix solutions of Hamiltonian dynamical equations. Similar approach allows not only use unified mathematical tools, but create efficient algorithms and derive interpretable results.

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ISBN 978-3-95450-181-6

MATHEMATICAL MODELS

The Main Types of the Tasks for Long-Term Evolution

Let us point out the following (related) types of beams physics problems arising in the study of the evolution of the long beam in cyclic accelerators:

1) search periodic orbits of the beam control systems and studying of their stability;
2) study of resonances (especially nonlinear): search resonance regions, the separatrices construction , the determination of their configuration, and so on;
3) the construction of the dynamic aperture, its maximization by introducing non-linear correctors;
4) the study of the conditions of formation of the beam halo and its characteristics.

These and other tasks require the creation of efficient methods, algorithms and computer codes. First of all we should pick out several levels of increase of efficiency of computer modeling (namely computer modeling as symbolic and numerical, is the main means of solving the problems stated above). Methods of the *first level* are based on the use of effective (from computational point of view) mathematical methods. Methods of the *second level* are based on algorithms that efficiently implement the selected mathematical methods. From the point of view of the used matrix formalism, it is primarily the various accelerators" matrix algebra. We also should use parallel algorithms and computation distribution, intended for realization on high-performance computers. The *third-level methods* are based on the use of various types of data and knowledge bases, allowing the use of the knowledge gained in the study of similar systems. Of great importance at this stage of play the methods and means of artificial intelligence, such as computer algebra, graph theory, computational intelligence and so on. It should be noted that this ideology should be included in the computer simulation circuit at an early stage selection of mathematical methods and appropriate software.

The Equations of Motion

Before derivation of the equations it is necessary to do some number of assumptions. In particular, in this article we will suggest: 1) absence of any interaction between the particles in the beam;
2) for external fields we limit ourselves to magnetic fields and and will consider the adiabatic nature of the magnetic field changes with time;
3) the development of resonance occurs in the median plane, as evidenced by preliminary calculation non-linear equations taking into account experimental data along the reference trajectory.

The computational experiments demonstrated that the initial transverse phase portraits circulating beam in the horizontal plane x, x' and the vertical plane y, y' are images that differ little from the ellipse. But in the process resonance the phase portrait of the beam is in the horizontal plane x, x' radically deformed, although in the vertical plane the deformation is negligible. Thus, the resonant members practically no effect on the phase portrait of the beam in the plane y, y'. This is confirmed by calculations carried out in view of the experimental data. All our assumptions correspond to the real processes taking place at resonant multi-turn (slow) extraction, and thus provide sufficient adequacy of the approximating model under consideration. Given the above assumptions the equations of motion of particles in a magnetic field up to terms of the second order have the form

$$x'' + (1-n)h^2 x = (2n-1-\beta)h^3 x^2 + h' x x' + \frac{h}{2}(x')^2 + \frac{1}{2}(h'' - nh^3 + 2\beta h^3)y^2 + h' y y' - \frac{1}{2}h(y')^2 + \mathcal{O}(3), \quad (1)$$

$$y'' + nh^2 y = 2(\beta - n)h^3 xy + h' x y' - h' x' y + h x' y' + \mathcal{O}(3), \quad (2)$$

where $' = d/ds$, s is the independent variable which measured along the reference orbit, $\mathcal{O}(3)$ are members of the third and higher orders on the variables x, x', y, y'. Here we are considering the monoenergetic beam and that the equilibrium trajectory is a reference orbit. In equations (1) and (2) used standard for this class notation tasks.

N o t e 1. The curvature of the base curve (equilibrium trajectory in our case) is h and the maximum value of x, defined by the position of the septum magnet. We should note that In our case the members of the third order of smallness give about a few percent compared to the second order.

The Investigation of Motion Equations

Preliminary there was carried out computing experiment using for several hundreds of turns in two planes (that ensures a reasonable global error under using conventional numerical methods, such as Runge-Kutta methods). The results showed that resonance phenomena develop mainly in the plane $\{x, x'\}$, so henceforth consider the equation for beam evolution only in this plane:

$$x'' + (1-n)h^2 x = (2n-1-\beta)h^3 x^2 + h' x x' + \frac{h}{2}(x')^2 + \mathcal{O}.$$

We should note that the terms containing x' no effect on development of resonance, and only lead to slight deformation of the beam phase portrait during slow extraction (in particular, there is a certain rotation around a center point). Therefore, these members may also be discarded. The complete equations in the plane x, x' can be written in the matrix form

$$\mathbf{X}(s) = \mathbb{M}^2(s|s_0)\mathbf{X}_0^2 = \mathbb{M}^{11}\mathbf{X}_0 + \mathbb{M}^{12}(s|s_0)\mathbf{X}_0^{[2]}, \quad (3)$$

where $\mathbb{M}^2(s|s_0)$ is the full matrix of the evolution of nonlinearities up to second order, $\mathbb{M}^{11}$ is a matriciant of lin-

ISBN 978-3-95450-181-6

earized equation, $\mathbb{M}^{12}$ is the matrix corresponding to nonlinear second-order fields. Adiabatic changes of coefficients ν^2 and α_1 allow replacing them by piecewise constant functions. Then on the entire interval of permanence for the equation (3) we can obtain a solution manually or by using one of the systems computer algebra codes (in this paper were used Reduce system codes, Maple V and Wolfram Mathematica). In the calculations we also used another (smoothed) representation of the motion equations. The results showed that a smooth approximation is a very workable model, while the computing speed many times more than the speed of calculations with using other models. We note that usage of other models for computation of the full cycle is almost impossible due to a time-consuming and the unacceptable exactness of the numerical calculations [2]. Thus, the desired solution can be represented in the form

$$\mathbb{M}^{11} = \begin{pmatrix} C_1 & S_1/\nu \\ -\nu S_1 & C_1 \end{pmatrix},$$

$$\mathbb{M}^{12} = \frac{\alpha_1}{6\nu^4}\begin{pmatrix} \varkappa^1_{11} & \varkappa^1_{12} & \varkappa^1_{13} \\ \varkappa^1_{21} & \varkappa^1_{22} & \varkappa^1_{23} \end{pmatrix} + \frac{\alpha_2}{\nu_1}\begin{pmatrix} \varkappa^2_{11} & \varkappa^2_{12} & \varkappa^2_{13} \\ \varkappa^2_{21} & \varkappa^2_{22} & \varkappa^2_{23} \end{pmatrix}.$$

Here, the matrices $\mathbb{M}^{11}$ and $\mathbb{M}^{12}$ can becalculated in , in particular, symbolical form (see, [4]) that allows you to not only significantly reduce the computing time, but also to realise study of the effect of control parameters on the beam dynamics. We should note that there occur resonance phenomena, however it can be shown that the elements of the matrix $\mathbb{M}^{12}$ have a finite value on the resonance, but there appear secular terms. In a periodic structure of cyclic accelerators we should for each turn to repeat calculations very large number of times (in particular more than 10^9 times). But the procedure of exponentiation allows (given the structure of the accelerator) significantly reduce the time required of computing. The matrix $\mathbb{M}$ is an upper triangular block-matrix consisting of $\mathbb{M}^{11}$, $\mathbb{M}^{12}$ and $\mathbb{M}^{22} = \mathbb{M}^{11^{[2]}}$, and is responsible for translating the beam for a single turn. Since $10^9 = ((10)^3)^3$ then the number of required operations is significantly reduced. We note to further speed up the computation we can calculated the matrix $\mathbb{M}^(3)$ in advance in a symbolical form, to store in a special database and then use on demand for numerical computation of the corresponding matrices.

According to the [3] the optical structure of the booster has FODO periodicity and consists of four superperiods, each of which consists of 5 regular periods and a interval which does not contain the dipole magnets. Regular period includes quadrupole lenses, two dipole magnets and four free period, designed to accommodate multipole correctors, collimators and special diagnostic equipment. The beam injection system from the Nuclotron into the accelerator complex consists of two parts. The first part provides output of the beam from the booster into the matching channel and then the beam is injected into the Nuclotron. The matching channel (represented in Fig. 1, see [5]), and performs the beam focus using six quadrupoles. The turns in the horizontal and vertical directions are realized using dipole magnets. The channel also has elements of exercising "stripping" beam and separators for spurious charge states.

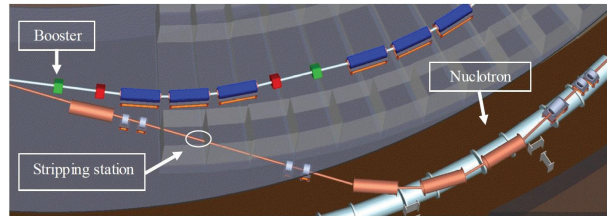

Figure 1: Injection system layout and beam orbits and envelopes: 1 - injected beam, 2 - bumped one, 3 - circulated beam after first turn injection, 4 - after accumulation; BM $1 \div 3$ - magnets for the bump formation, SM - injection septum magnet.

The second variant of the beam output (slow extraction) used for medical-biological and applied research on the beams from booster. This problem also makes special demands on the beam. In this case the operating point of the booster shifts into the area of nonlinear resonance. It should be noted that this process takes place over a large number of turns. Thus, both types of output beam makes it necessary to ensure the implementation of restrictions of the beam at the stage of its evolution from booster. We note that similar restrictions also lead to the need for detailed investigation of the formation of the beam in the case of multiturn evolution.

CONCLUSION

In this study we conducted a testing of different regimes of beam extraction from the particle accelerator. Results of the study demonstrated high efficiency from a computational point of view and the correctness of the results.

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ISBN 978-3-95450-181-6