STUDY OF BEAM LENGTH MEASUREMENT BASED ON TM010 MODE

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Abstract

Beam length measurement in frequency domain is a familiar method, and the resolution is seriously limited by the system signal-noise-ratio (SNR) and the beam length measured. Usually this method can only obtain the resolution about ~10*ps* with beam length ~30*ps* when using signal from button or stripline BPM. But in FEL case, the beam length is the *ps* or sub–*ps* order. The paper discusses the probability of beam length measurement based on the TM010 mode in FEL case. When adopting High Order Mode(HOM) reject and system gain control, the system SNR can arrive at 112*dB* and the resolution can achieve 30*fs* with beam length *ps* or sub-*ps* order.

INTRUDUCE

The measurement area and resolution of beam length measurement in frequency domain is seriously limited by the system SNR. So obtaining the system SNR up to 100*dB* is the decisive factor of beam length measurement in frequency domain in FEL case. It is well known that cavity BPM can achieve the position measurement resolution up to ~10*nm* because the signal from TM011 mode having a very high R/Q. The amplitude ratio of TM010 mode and TM011 mode of the same cavity can be written as:

$$K_r = \frac{(R/Q)_{010}^{0.5}}{(R/Q)_{110}^{0.5}} = \sqrt{\frac{2f_{110}}{f_{010}}} \cdot \frac{J_0(rk_{010})J_0(R_c k_{011})}{J_1(rk_{011})J_1(R_c k_{010})} \quad (1)$$

Where f is the harmonic frequency of TM_{010} mode and TM_{011} mode, and J_0 and J_1 is the 0-order and 1-order Bessel function respectively, r is the beam position from the cavity centre, R_c is the cavity's radius, k is the wave number of TM_{010} mode and TM_{011} mode. It is reported that the cavity BPM's resolution can achieve 0.05μm when beam charge is 1nC [1][2]. It means that the measurement system noise equals the output TM_{011} mode signal with beam condition 0.05μm@1nC. In the case, K_r is up to 4×10^5. So the system SNR of TM_{010} mode can be inferred up to 112*dB*.

When the beam is a Gaussian distribution bunch, the signal amplitude of TM_{010} mode can be written as:

$$V = \pi q \cdot f_{010} \cdot \sqrt{\frac{Z}{Q_L}\left(\frac{R}{Q}\right)_{010}} \exp\left(-\frac{2\pi^2 f_{010}^2 \cdot \sigma_L^2}{c_0^2}\right) \quad (2)$$

Where q is the beam charge, Z is output impedance, Q_L is the quality factor, σ_L is the beam length, $(R/Q)_{010}$ is the normalized shunt impedance of TM_{010} mode. So if used two cavities which working as TM_{010} mode at different frequency, the output signal can be written as:

$$V_1 = \pi q \cdot f_{010,1} \cdot \sqrt{\frac{Z}{Q_{L,1}}\left(\frac{R}{Q}\right)_{010,1}} \exp\left(-\frac{2\pi^2 f_{010,1}^2 \cdot \sigma_L^2}{c_0^2}\right) \quad (3)$$

$$V_2 = \pi q \cdot f_{010,2} \cdot \sqrt{\frac{Z}{Q_{L,2}}\left(\frac{R}{Q}\right)_{010,2}} \exp\left(-\frac{2\pi^2 f_{010,2}^2 \cdot \sigma_L^2}{c_0^2}\right) \quad (4)$$

So the coupling of beam charge can be eliminated by the outputs division. Then the beam length can be getting as:

$$\sigma_L^2 = \frac{c_0^2}{2\pi^2(f_{010,2}^2 - f_{010,1}^2)}\left(\ln\left(\frac{V_1}{V_2}\right) + \ln\left(\frac{f_{010,2}}{f_{010,1}}\right) + 0.5\ln\left(\frac{Q_{L,1}(R/Q)_2}{Q_{L,2}(R/Q)_1}\right)\right) \quad (5)$$

In the Eq.(5), the second and the third term is constant decided by the two cavities, so the sum of the two terms can be defined as $\ln(R_1/R_2)$, then:

$$\sigma_L = \sqrt{\frac{c_0^2}{2\pi^2(f_{010,2}^2 - f_{010,1}^2)}} \cdot \sqrt{\ln\left(\frac{V_1 R_2}{V_2 R_1}\right)} \quad (6)$$

And when the beam deviates the Gaussian distribution, the *rms* beam length can get from a similar equation [3]. When the beam length is very shorter than c/f_{010}, the resolution can be written as:

$$\Delta\sigma_L \simeq \frac{c_0^2}{2\pi^2(f_{010,2}^2 - f_{010,1}^2)} \cdot \frac{SNR_V}{\sigma_L} \quad (7)$$

Where SNR_v is the signal-noise-ratio of the TM_{010} mode. From the Eq.(7), it can be seen that the resolution of the beam length measurement is seriously limited by the SNR of diagnostic system and beam length itself. Supposing the SNR 112*dB*, $f_{010,1}$ and $f_{010,2}$ equal 3*GHz* and 8*GHz* respectively, Fig.1 shows the relation between the resolution and beam length. It can be seen that the resolution can achieve 3.5μm@1nC even the beam length is only 0.1*ps*.

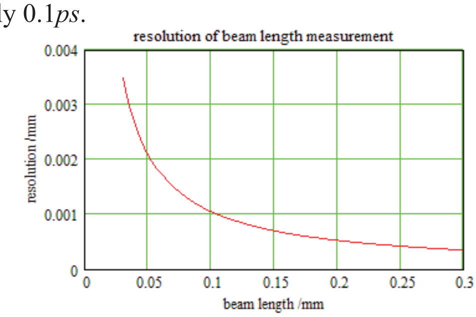

Figure 1: Relation between the resolution and bunch length.

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SIMULATION

Based on the theory analysis, some simulations are done with MAFIA simulation code when considering Shanghai SDUV FEL case. The working frequency of the cavity BPM of the SDUV FEL is designed at 4.7*GHz*, and the TM_{010} mode's frequency of the position cavity works at 3.6*GHz*. Fig.2 and Fig.3 are the waveforms in time domain and frequency domain from the reference cavity with beam charge 1*nC* and beam length 10*ps*. Three cases are simulated which the beam charge is 1nC and the beam positions are 0.25*mm*, 1.25*mm* and 5.3*mm* respectively. Fig.4 is the output amplitudes of TM_{010} mode when beam locates at different positions. It can be seen that the 'invariable' TM_{010} mode signal changing when beam position changing. The SNR is only about 40*dB* if the deviations between the output amplitudes are seen as diagnostic noise. Fig.5 is the simulation result of the bunch length measurement in frequency domain in the case.

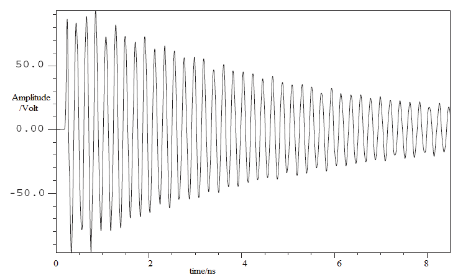

Figure 2: Output TM010 signal of reference cavity.

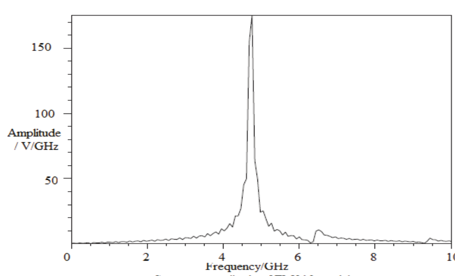

Figure 3: Spectrum of reference cavity.

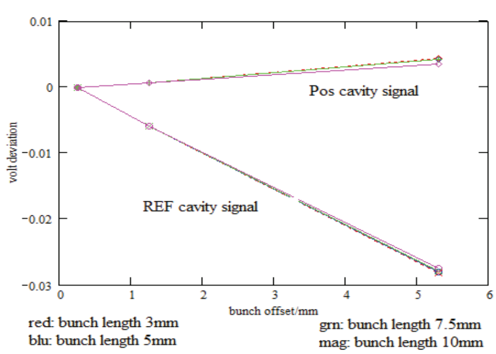

Figure 4: Amplitudes deviations when beam position changing.

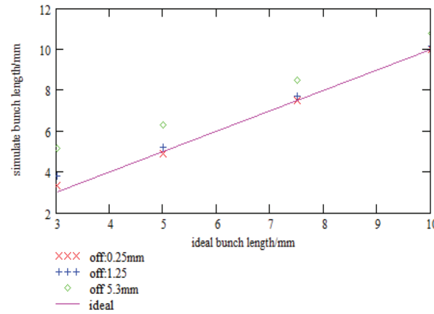

Figure 5: Beam length simulation result.

Because the amplitude difference is increasing when position offset increasing, it can be supposed that some position mode signals such as TM_{011} mode are coupling into the TM_{010} mode's output. The output structure has to be improved for avoiding the coupling. Fig.6 and Fig.7 show the coupling improvement after the structure improvement. Fig.8 and Fig.9 shows the result after this improvement. The SNR increases to 60*dB* and the resolution is about 0.3*ps* when beam length 10*ps*.

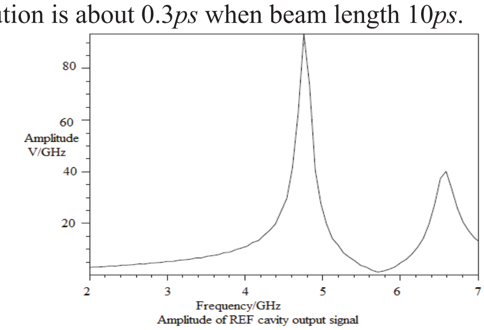

Figure 6: The coupling between the TM_{010} mode and TM_{011} mode.

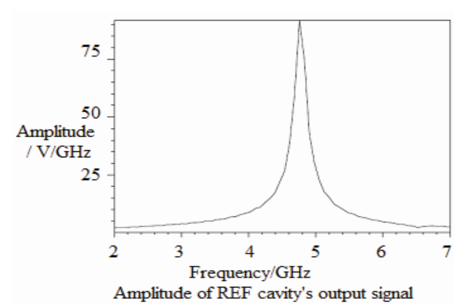

Figure 7: Improvement of modes coupling.

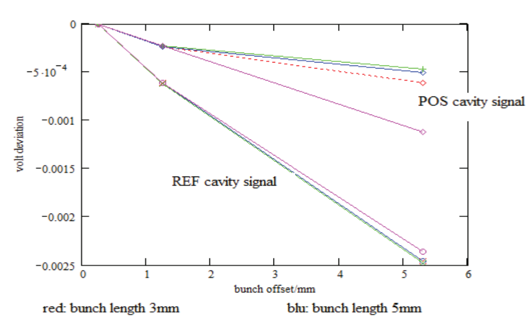

Figure 8: Amplitudes difference after improvement.

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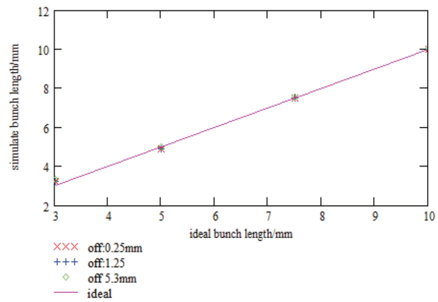

Figure 9: Beam length simulation result after improvement.

From the equation 7, it can be seen that the resolution is in verse proportion to the difference of the square of working frequencies. So the resolution will increase when adding the difference. Fig.10 and Fig.11 is the result when changing the working frequencies to 4.7*GHz* and 5.8*GHz* respectively. The resolution is less 0.3*ps* now.

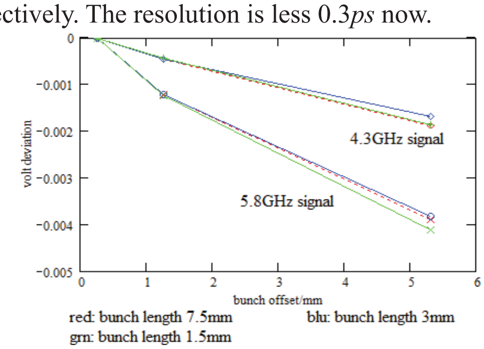

Figure 10: Amplitudes after changing the working frequencies.

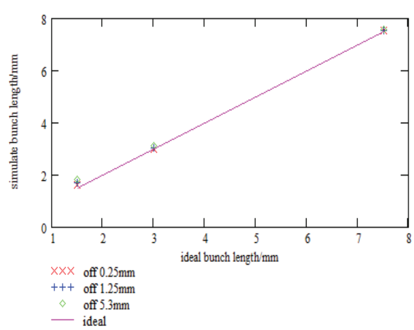

Figure 11: Result after frequencies changing.

From the Fig.8 and Fig.10, it can be seen that the absolute deviation of output amplitudes is increasing in proportion to the working frequency. It means that there is a factor which in proportion to the frequency acting on the TM_{010} mode's output amplitudes. From the equation 1 and 2, it has to consider the effecting of (R/Q) which is determined by the term $J_0(rk_{010})$. Fig.12 and Fig.13 shown the improved simulation result of Fig.10 and Fig.11 after considering the effect of the term $J_0(rk_{010})$. The deviation of the output amplitude is decreasing to 0.1%, and the beam length measurement resolution is changing better than 100*fs*. Considering the minimum time step is about 200*fs* when simulation, the attainable and reasonable resolution should be about 100*fs* in the simulation.

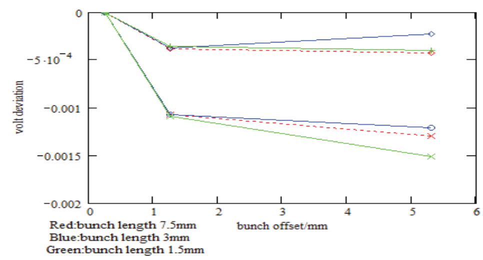

Figure 12: Amplitudes deviation after adjusting the $J_0(rk)$ term.

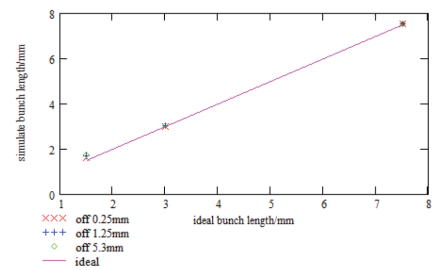

Figure 13: Result after $J_0(rk)$ term adjusting.

At last, the thermal noise should be considered. From Fig.2, the loaded shunt impedance is larger than 100*Ohm* in the FWHM bandwidth. If considering bunch-by-bunch beam length diagnostic, the bandwidth maybe is about 500*MHz*, the peak of the signal amplitude in the bandwidth is about 70*V* in this simulation case. The peak voltage of the thermal noise in this bandwidth is only about 0.12*mV*. It means that the system SNR will be up to 134*dB* if only considering the thermal noise.

CONCLUTION

The beam length measurement in frequency domain based on TM_{010} mode has a very high signal-noise-ratio. So the anticipative resolution will be about to 10*fs* when beam length is only about to 100*fs*. A series simulation based on Mafia had done and proved that the attainable beam length resolution was up to the limit of the simulation and accorded with anticipation.

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