INVERSE RESPONSE MATRIX COMPUTATION FOR THE STORAGE RING SLOW ORBIT FEEDBACK CONTROL: SYNTHESIZED TOPOLOGICAL INVERSE COMPUTATION

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Abstract

For the storage ring orbit feedback control, the inversed SVD (Singular Value Decomposition) method is normally working under stabilized orbit situation, but less effective at relatively large orbit fluctuation. To overcome such numerical drawback, we investigated the alternative feedback control based on the solution tracking algorithm. Using our novel STIC (Synthesized Topological Inverse Computation), we simulated the formation of residual tune orbit under the closed orbit dynamics response matrix, measurable relationship between BPM-data readout and corrector-MPS setting. By placing empirical evidence of beam based alignment measurement, we achieved remarkable numerical fidelity on our STIC feedback algorithm, especially ascribing the topological importance of inverse response matrix computation. We demonstrated the STIC-inherent triggering behaviour and adaptive pattern-notched feedback stability.

INTRODUCTION

Practically useful for SOFB (slow orbit feedback) with assistance of FOFB (fast orbit feedback), the inversed SVD manipulation [1] is not fully acceptable because a type of consecutive instability noise irreversibly accumulates in the beam trajectory deviation. On the other hand, a novel numerical algorithm – emerging from a topological math approach – can lead to numerical self-consistency and solution tracking, dramatically suppressing ill-posed instability problems associated with numerical truncation residuals. This approach, known as a singularity regularization method, makes it feasible to compute a feature-invariant inverse matrix and system-matched de-noising filter. For deep investigation of the closed orbit feedback control, we applied our novel recipe of inverse computation, namely STIC(Synthesized Topological Inverse Computation) [2]. Fundamentally, this Orbit-STIC feedback is similar to the numerical computation of the inner origin function ascribing a type of complex systems. In our other study [2], we described the singularity inhibitor algorithm with essential topological nature engaged in the symplectic transformation (e.g. S-matrix, satisfying A_3=SA_1 with S^2=-I). Because this inverse math algorithm deals with the phase-component feature, commonly required is the complete phase-space relationship, e.g. the horizontal and vertical coupling term in the closed orbit response matrix.

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Even though LOCO (linear optics closed orbit) configurations are so complicated, entire physics of beam dynamics feature can be extracted from the measured response matrix. In this study, we introduced the PLS-II response matrix, even not so much matured since upgrading commission year 2012. [3] Proper matrix refinement can be made both numerically and empirically, according to our self-consistent manner of filtering out the uncertainty of measurement errors escaping from beam dynamics constraints. We are deeply investigating our advanced feedback algorithm, Orbit-STIC, practically in collaboration with world-wide accelerator community.

NUMERICAL PROCEDURES

As depicted in Fig. 1, the closed orbit feedback algorithm can be expressed with the following equations:

$$\Delta \boldsymbol{x}_{BPM} = \Delta \boldsymbol{x}_i - \Delta \boldsymbol{x}_o = \mathbf{R}\,(\Delta \boldsymbol{g}_i - \Delta \boldsymbol{g}_o) \quad (1)$$

$$\overline{\Delta \boldsymbol{g}} = \Delta \boldsymbol{g}_o + (\Delta \boldsymbol{g}_i - \mathbf{R}^{-1}\,\widetilde{\Delta \boldsymbol{x}_i}) \quad (2)$$

$$\overline{\Delta \boldsymbol{x}} = \mathbf{R}\,\Delta \boldsymbol{g}_o = \mathbf{R}\,\mathbf{R}^{-1}\,\Delta \boldsymbol{x}_0 \quad (3)$$

$$\overline{\Delta \boldsymbol{g}} = \mathbf{R}^{-1}\,\overline{\Delta \boldsymbol{x}} + (\mathbf{I} - \mathbf{R}^{-1}\mathbf{R})\,\mathbf{R}^{-1}\,\widetilde{\Delta \boldsymbol{x}_i} \quad (4)$$

$$\overline{\Delta \mathbf{g}_s} = \gamma \mathbf{U}_s^{(0)}\,\overline{\Delta \boldsymbol{x}} + \mathbf{U}_s^{(\gamma)}\,\widetilde{\Delta \boldsymbol{x}_i} \quad (5)$$

$$\mathbf{U}_{stic}^{(\gamma)} = [\mathbf{R}_{stic}^{-1}\mathbf{R}]_r^{-1}\left(\mathbf{I} - \gamma\,\mathbf{R}_{stic}^{-1}\mathbf{R}\right)\mathbf{R}_{stic}^{-1} \quad (6)$$

$$\mathbf{U}_{svd}^{(\gamma)} = \left(\mathbf{I} - \gamma\,\mathbf{R}_{svd}^{-1}\mathbf{R}\right)\mathbf{R}_{svd}^{-1} \quad (7)$$

$$\begin{aligned}\overline{\Delta \mathbf{g}_{svd}} &= \alpha\,\mathbf{U}_{svd}^{(0)}\,\Delta \boldsymbol{x}_p + \mathbf{U}_{svd}^{(\alpha)}\,\Delta \boldsymbol{x}_p \\ &= \left((1+\alpha)\mathbf{I} - \alpha\,\mathbf{R}_{svd}^{-1}\mathbf{R}\right)\mathbf{R}_{svd}^{-1}\,\Delta \boldsymbol{x}_p \end{aligned} \quad (8)$$

Eqs (1)-(4), include the basic recursive formula for the feedback control. Eqs (5)-(7), are the modified formalism based on Eq(4). This is efficient and convenient way of stabilizing the iteration process. For the STIC feedback in Fig. 2, as $\boldsymbol{\gamma} \to \mathbf{1}$, it is proper to get the stabilized BPM-solution (Δxo). In contrast, for the SVD method, as $\boldsymbol{\alpha} \to \mathbf{1}$, the feedback speed is retarded and eventually extinguished. (Refer to Table 1.) Typically, when the particular orbit solution is given as Δx_p, the corrector-MPS setting is simply computed according to Eq (8). Two

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numerical approaches, STIC and SVD, are significantly different: (i) STIC algorithm works at the solution tracking mode with valid Δx_o-seed value in Fig. 1. If the initial Δx_o-setting is largely deviated from the real orbit situation, prompt feedback is jeopardized. (ii) SVD algorithm computes Δg_o directly from BPM-readout (current orbit). Even uncertainty of Δx_p is not totally rejected under α-parameter adjustable feedback speed. However, under the SVD feedback control, the orbit will be excursive without smart feedback such as STIC solution tracking. More specification will be described later with our numerical simulation results in Fig. 3, relevant to quantitative basis in Fig. 4 and Table 1.

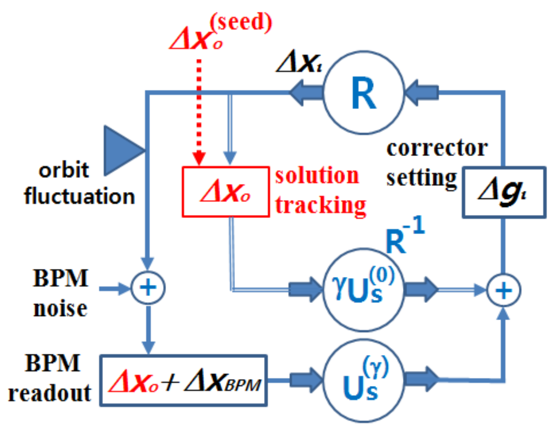

Figure 1: Algorithm diagram of STIC feedback process. The inverse process ($\mathbf{R}^{-1}$) is represented as the equivalent inverse algebra using two conjugate operators ($\mathbf{U}^{(0)}_{\mathrm{stic}}, \mathbf{U}^{(\gamma)}_{\mathrm{stic}}$) in Eqs (5)-(6). The solution tracking operator ($\mathbf{U}^{(0)}_{\mathrm{stic}}$) keeps the solution orbit stabilized, while the filtering operator ($\mathbf{U}^{(\gamma)}_{\mathrm{stic}}$) plays a nullifying role against the amplification of noise components along repeated feedback loop.

RESULT AND DISCUSSION

As shown in Fig. 2, numerical simulation results indicate that the residual tune orbit - corresponding to Δx_o in Fig 1 - exists in real beam orbit, as well as numerical final solution converged via numerical iteration. This residual tune orbit can be taken as an average of statistical fluctuation as numerically simulated. Its amplitude was compared to the so-called reference orbit, empirically measured through BBA(Beam Based Alignment) method [4], as plotted in Fig. 2 (b). Numerical counterpart is apparently coincident to the empirical pattern, in case that the numerical inversion ($\mathbf{R}^{-1}$) is reasonably close to our STIC result. At least, it is a good indicator to recognize the plausible betatron oscillation - the lattice tune arrangement coupled with quadrupole magnet to BPM position involving beam based offset errors. Normally, the betatron oscillation is damping slowly after every top-up injections, and is fluctuating within the bounded stability in beam dynamic aperture.

Our numerical simulation results, illustrated in Fig. 3, are deserved to develop the hybrid algorithm connecting STIC and SVD. Within numerical simulation, it is possible to trigger the effect of STIC solution feedback. When triggered, this solution is surely valid. The resultant STIC Δx_o-solution is stabilized better compared to the SVD-case. This is a case of triggering at the extremely stabilized orbit – statistically reachable instantaneously.

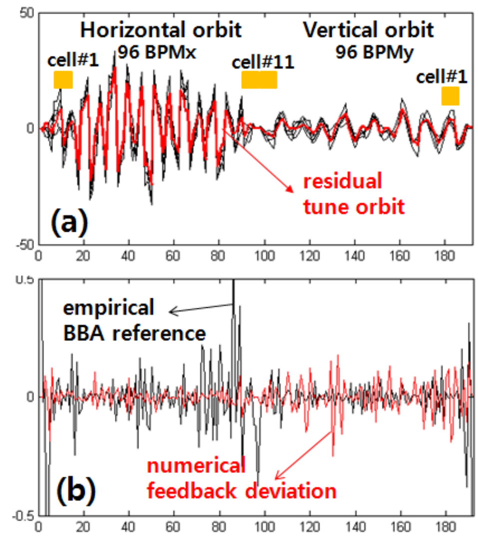

Figure 2: (a) Simulated residual tune orbit, statistical average of random-number generated orbit fluctuation. Horizontal/vertical tune orbit indicates the operational status (ν_x=15.28, ν_y=9.18) of PLS-II. (b) Empirical evidence is observed from typical beam based offset measurement, the BPM arrangement data presumed as $\Delta \mathbf{x}_o^{(\mathrm{ref})} = (\mathbf{I} - \mathbf{R}\mathbf{R}^{-1})\, \Delta \mathbf{x}_o^{(\mathrm{tune})}$.

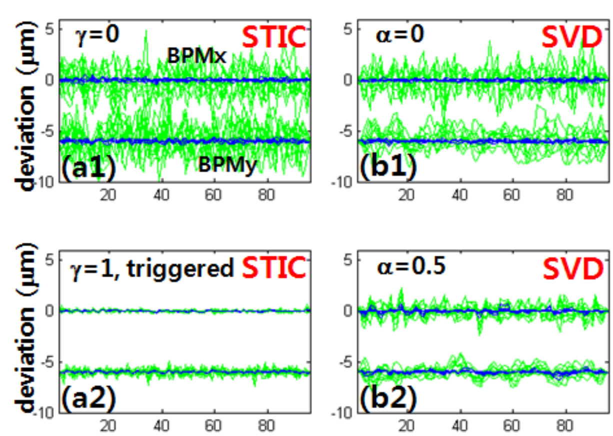

Figure 3: Comparison between STIC and SVD feedback simulation. In the case (a2), the STIC solution tracking is performed when the seed setting (Δx_0 in Fig. 1) is taken from other feedback orbits, STIC-(a1) or SVD-(b1). Appropriate γ–parameter can control the triggering threshold, associated with statistically distributed orbit stability (blue color). Up to complete orbit tracking situation, the extremely stabilized orbit can be established via such an adaptive triggering operation.

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Whenever STIC process confirms the stabilized BPM-readout orbit at good confidence, we can put appropriate setting values for corrector magnets with sufficient safety. In Fig. 3 (b), eventually Δx_o-solution can be obtained at the minimal deviation - indicating submicron stability desirable. In this way, it is possible to reach the maximum stability with safety and quickness, and meanwhile orbital drift problem will be removed.

In Fig. 4, showing the feature of inverse matrices, it is clearly manifested how STIC feedback works differently from SVD feedback process. In case of SVD, $\mathbf{U}^{(0)}_{\mathbf{svd}}$ is only effective, but $\mathbf{U}^{(1)}_{\mathbf{stic}}$ actually vanishes as indicated, i.e. $\|\mathbf{Q}\| = \mathbf{0}$ in Table 1. This belongs to a single pattern style of feedback kernel. In contrast, STIC is a homology case - combing doublet patterns ($\mathbf{U}^{(0)}_{\mathbf{stic}}$ and $\mathbf{U}^{(1)}_{\mathbf{stic}}$) into one matrix. With such distinctiveness, this STIC process is more potential than SVD for theoretical computation and modelling. As we computed the residual tune orbit in Fig. 2, we can achieve a great numerical confidence regardless of noise fluctuation. Indeed, the entire details of closed beam orbit can be evaluated from the measured response matrix, e.g. a theme of accelerator simulation works. Of course, as we promised, our STIC process will be prominent for a practical implementation of on-tracking solution, especially fast feedback process competing FOFB system. If so, our feedback system may have a great benefit to establish top-up operation mode safely and quickly – optimizing a fast betatron oscillation damping. Also, we believe, this smart and effective algorithm can be interfaced with RF tune frequency adjustment. Subsequent feedback under noise statistics can be traced with sensitively evaluating the disparity covariance (maximum entropy) across orbital instability situation, as pointed out in other study [2].

Table 1: Characteristics of Inverse Matrix

Inverse Operator $\mathbf{A}^{-1}$	Norm of operator algebra: $\|\mathbf{Q}\|$	$\|\mathbf{Q}^2 - \mathbf{Q}\|$	$\|\mathbf{QA} - \mathbf{A}\|$	$\|\mathbf{P}^2 - \mathbf{P}\|$
$\mathbf{R}_\mathbf{r}^{-1}$	1.0	0.23	0.051	23.27
$\mathbf{R}_\mathbf{stic}^{-1}$	1.0	8e-7	0.004	4e-7
$\mathbf{U}^{(0)}_{\mathbf{stic}}$	1.23	0.30	0.002	0.25
$\mathbf{U}^{(1)}_{\mathbf{stic}}$	0.54	0.27	1.0	0.35
$\mathbf{R}_\mathbf{svd}^{-1}$	1.0	2e-14	0.018	2e-14
$\mathbf{U}^{(0)}_{\mathbf{svd}}$	1.0	2e-14	0.018	2e-14
$\mathbf{U}^{(0.5)}_{\mathbf{svd}}$	0.5	0.25	0.5	0.25
$\mathbf{U}^{(1)}_{\mathbf{svd}}$	0 (meaningless, featureless)			
pinv(R) @matlab	1.0	7e-13	2e-14	1e-12

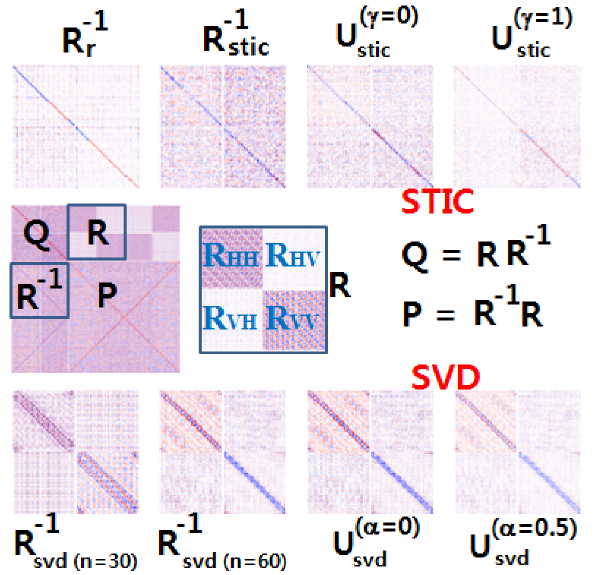

Figure 4: Color-map representation for the storage ring orbit response matrix, [$\mathbf{R_{HH}}$,$\mathbf{R_{HV}}$; $\mathbf{R_{VH}}$,$\mathbf{R_{VV}}$]. For $\Delta \boldsymbol{x_i} = \mathbf{R}\,\Delta \boldsymbol{g_i}$ system, [**Q**,**R**; $\mathbf{R}^{-1}$,**P**] consists of complete Hilbert four-space set with **R** (forward), $\mathbf{R}^{-1}$ (inverse), **Q**=$\mathbf{RR}^{-1}$ (Δx-space projection) and **P**=$\mathbf{R}^{-1}\mathbf{R}$ (Δg-space projection). With $|\text{intensity}|^{0.65}$ adjusted image, red/blue color indicates positively/negatively valued. Refer to numerical feature characteristics in Table 1.

Our simulated numerical distinctiveness is so remarkable and thinkable for practical utility of feedback system with modern computers of massive parallel architecture. Our STIC result – phase retrieval solution - is basically dependent on math approach, which is being revealed as a category of Bayesian statistics and maximum entropy theory. Like a hologram, STIC matrix itself possesses the phase-retrieval feature and its tracking (lock-in) stability. Inversed SVD matrix is improper physically, losing such crucial topology-invariant features. A new math approach is emerging nowadays to think about a new instrumentation and challenging.

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