NEW METHOD TO MONITOR THE CURRENT AND POSITION OF ONE OR TWO PARTICLE BEAMS

M. J. Hagmann, NewPath Research L.L.C., P. O. Box 3863, Salt Lake City, UT 84110, U.S.A.

Abstract

A group of sinusoidally-wound coaxial toroidal coils can be used to determine the magnitudes, phases, and locations of one or two time-dependent currents through their common aperture. A single current filament requires one uniformly-wound coil and two others having turn densities proportional to the sine and cosine of the azimuthal coordinate. Three simple algebraic equations give the magnitude, phase, and location of the current in terms of the voltages induced on the three coils, and there is no ill-conditioning. Two current filaments require two additional toroids with turn densities proportional to the sine and cosine of two times the azimuthal coordinate, and the solution is obtained using the method of steepest descent. Solutions for more than two currents become numerically unstable. Numerical tests were made by specifying the magnitudes, phases, and locations of the currents, calculating the induced voltages, adding Gaussian noise to model measurement errors, and then using the algorithms to calculate the currents and their locations. These simulations confirm that this method may be used with one or two currents

INTRODUCTION

Others have used a variety of different techniques to monitor the location of a single beam in an accelerator, including arrays of capacitive pickups, resistive wall gap monitors, electrostatic monitors, split-plate monitors, split-cylinder monitors, button monitors, longitudinal transmission lines, resonant cavities, and reentrant cavities [1]. Other techniques include secondary emission monitors, wire scanners, multi-wire chambers, gas curtains or jets, residual gas monitors, scintillator screens, scrapers and measurement targets, and synchrotron radiation [2], as well as the deflection of a probe beam of electrons [3]. Three groups have described work that is directly related to this paper. Two used four identical coils to determine the current and its location [4,5]. Murgatroyd and Woodland [6] made a short note that two coils with turn densities varying as sin(θ) and cos(θ) could measure the location of a single current, but they gave no analysis or experimental results and these authors could not be reached.

A Rogowski Coil is a non-ferrous current probe formed by bending a uniformly wound helical coil to follow a closed curve having arbitrary shape [7-9]. When a time-dependent current passes through the aperture that is enclosed by the bent helix a voltage is induced on the coil which is independent of the location of the current. However, currents that are located outside of the aperture do not induce a voltage on the coil. Deviations from a uniform winding are carefully avoided because they cause the induced voltage to depend on the location of the current within the aperture, but it will be shown that a group of coils having a specific type of nonuniformity may be used to accurately determine the current and its location.

ANALYSIS

Figure 1 is a diagram used for deriving expressions for the open-circuit voltage induced on a non-ferrous toroidal coil that may have a nonuniform winding. The toroid has a mean radius R, and the cross-sectional area of the tube of the toroid is A. Consider the induction in an incremental winding of length Rdθ that is centered at (R,θ) or equivalently (X_1,Y_1), which is caused by a filament with current $I = I_{10}e^{j\omega t}$ that intersects the X,Y plane at point $P(X_2,Y_2)$. The dashed line L_1 is directed normal to the increment of winding. Dashed line L_2 is parallel to the magnetic field, and dashed line L_3 is parallel to the X-axis.

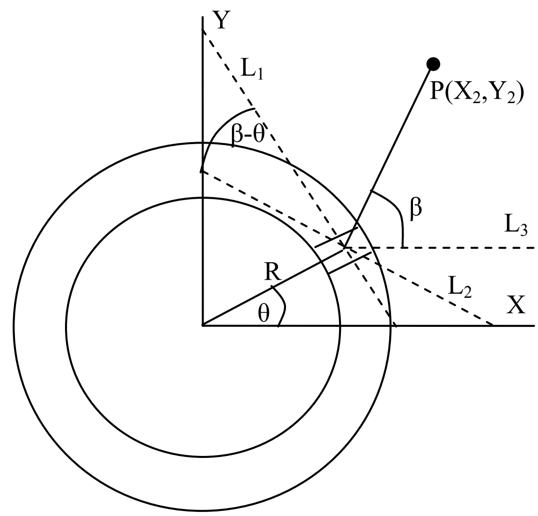

Figure 1: Diagram for analysis.

Let N’(θ) be the number of turns per unit length of the coil, as measured on a circle with radius R. For example, with a toroidal coil having a uniform winding, $N' = N_0' = N_T/2\pi R$, where N_T is the total number of turns. The number of turns in an increment of the winding is $dN = N'(\theta)\, R\, d\theta$. Thus, if the height and width of the tube are much less than R, the open-circuit voltage on the entire winding is given by

$$V_{OC} = -\int_0^{2\pi} \frac{j\omega\mu_0 N'(\theta) RAI \cos(\beta-\theta)\, d\theta}{2\pi\sqrt{(X_2-X_1)^2+(Y_2-Y_1)^2}} \quad (1)$$

Using trigonometry to obtain an expression for cos(β - θ), Eq. (1) simplifies to give the following:

$$V_{OC} = \frac{j\omega\mu_0 AI}{2\pi}\int_0^{2\pi}\frac{N'(\theta)\left[1-\frac{X_2}{R}\cos\theta-\frac{Y_2}{R}\sin\theta\right]}{\left[\left(\frac{X_2}{R}-\cos\theta\right)^2+\left(\frac{Y_2}{R}-\sin\theta\right)^2\right]}d\theta \quad (2)$$

The expressions for N' are chosen from the following set, which constitutes the basis for a Fourier series:

$$N'(\theta)\in\left\langle N_0',\sum_{J=1}^{\infty}N_{JC}'\cos(J\theta),\sum_{J=1}^{\infty}N_{JS}'\sin(J\theta)\right\rangle \quad (3)$$

where the N_{JC}' and N_{JS}' are coefficients as is N_0'. By substituting the set in Eq. (3) into Eq. (2), and evaluating the integral, the open-circuit voltage that is induced on each coil is given by the corresponding term of the following set:

$$V_{OC}\in j\omega\mu_0 AI_{10}\left\langle \begin{array}{c} N_0',\sum_{J=1}^{\infty}\frac{N_{JC}'}{2}\left(\frac{R_2}{R}\right)^J\cos(J\theta_1), \\ \sum_{J=1}^{\infty}\frac{N_{JS}'}{2}\left(\frac{R_2}{R}\right)^J\sin(J\theta_1)\end{array}\right\rangle \quad (4)$$

where $R_2 < R$ so that the current must be located within the aperture

Expressions for a Single Current Filament

If there is a single current filament, Eq. (4) may be used to show that the current and its coordinates may be uniquely determined from measurements that are made using three coils, by the following three equations:

$$I_{10}=\frac{-jV_{OC0}}{\omega\mu_0 AN_0'} \quad (5)$$

$$X_2 = R_2\cos(\theta_2)=\frac{2RN_0'V_{OC1C}}{N_{1C}'V_{OC0}} \quad (6)$$

$$Y_2 = R_2\sin(\theta_2)=\frac{2RN_0'V_{OC1S}}{N_{1S}'V_{OC0}} \quad (7)$$

Here V_{OC0}, V_{OC1C}, and V_{OC1S} are the open-circuit complex voltages induced on the three coils, for which the respective number of turns per unit length is N_0', N_{1C}'cos(θ), and N_{1S}'sin(θ). The derivation of Eqs. (5)-(7) implicitly assumes that $R << \lambda$, so these equations require that V_{OC1C}, and V_{OC1S} are in phase with each other and in phase quadrature with the current.

If the errors in the three measured voltages are small, but much greater than the effects of imperfections in the fabrication and placement of the coils, Eqs. (6) to (8) require that the fractional errors in the calculated current and its location are related to the fractional errors in the voltage measurements as follows:

$$\frac{\delta I_{10}}{I_{10}}=\frac{\delta V_{OC0}}{V_{OC0}} \quad (8)$$

$$\frac{\delta X_2}{X_2}=\frac{\delta V_{OC1C}}{V_{OC1C}}-\frac{\delta V_{OC0}}{V_{OC0}} \quad (9)$$

$$\frac{\delta Y_2}{Y_2}=\frac{\delta V_{OC1S}}{V_{OC1S}}-\frac{\delta V_{OC0}}{V_{OC0}} \quad (10)$$

Equations (6) to (8) show that the fractional error in the calculated current is equal to the fractional error in measurement with the uniformly wound coil. If the fractional errors in measuring the three voltages are comparable, then the fractional errors in the calculated coordinates have expectation values that equal the square-root of 2 times the fractional error in the calculated current. There is no ill-conditioning for the case of a single current filament.

Expressions for Two Current Filaments

If there are two current filaments, $I_1 = I_{10}e^{j\omega t}$ at (R_1, θ_1) and $I_2 = I_{20}e^{j\omega t}$ at (R_2, θ_2), Eq. (4) shows that the open circuit voltages on the first five coils from the set will be given by

$$V_{OC0} = j\omega\mu_0 AN_0'[I_{10}+I_{20}] \quad (11)$$

$$V_{OC1C} = j\omega\mu_0 A\frac{N_{1C}'}{2R}\left[I_{10}R_1\cos(\theta_1)+I_{20}R_2\cos(\theta_2)\right] \quad (12)$$

$$V_{OC1S} = j\omega\mu_0 A\frac{N_{1S}'}{2R}\left[I_{10}R_1\sin(\theta_1)+I_{20}R_2\sin(\theta_2)\right] \quad (13)$$

$$V_{OC2C} = j\omega\mu_0 A\frac{N_{2C}'}{2R^2}\left[I_{10}R_1^2\cos(2\theta_1)+I_{20}R_2^2\cos(2\theta_2)\right] \quad (14)$$

$$V_{OC2S} = j\omega\mu_0 A\frac{N_{2S}'}{2R^2}\left[I_{10}R_1^2\sin(2\theta_1)+I_{20}R_2^2\sin(2\theta_2)\right] \quad (15)$$

It may be seen that this set of 5 equations in 6 unknowns is a determined system by considering that the currents and voltages are complex variables requiring both phase and magnitude, but the phases of the measured voltages must have a common reference. It appears that Eqs. (11)-(15) cannot be solved directly. Thus, a cost function is defined as the sum of the squares of the residuals in these five equations, and the method of steepest descent is used to determine the values of I_{10}, R_1, θ_1, I_{20}, R_2, and θ_2 for which the cost function has a minimum.

Equations (11)-(15) were also used to derive two simultaneous equations having only the variables θ_1 and θ_2, so these two equations may be solved and then the other 4 unknowns may be determined. However, this procedure has been shown to have much lower numerical stability than in using the method of steepest descent as just described.

NUMERICAL TESTS OF ALGORITHMS

Numerical tests were made by specifying the magnitudes, phases, and locations of the currents, calculating the induced voltages, adding Gaussian noise to model measurement errors, and then using the algorithms to calculate the currents and their locations. These results were compared with the specified values to find the errors, so that the range of convergence and numerical stability could be determined. Highlights of the results of these tests are as follows:

- For one current filament, Eqs. (5)-(7) may be used for an explicit solution, or the method of steepest descent may be used to determine the solution by the minimization of residuals. Three coils are required, and both procedures are highly accurate with errors that are consistent with Eqs. (8)-(10).

- For two current filaments, Eqs. (11)-(15) may be used with the method of steepest descent. Five coils are required, and the errors are consistent with the Gaussian noise that is introduced to model the measurement errors. An expression for the condition number of the matrix has not been determined, but the fractional errors are several times the fractional errors in the measurements.

- For two current filaments, two simultaneous equations in the variables θ_1 and θ_2 may be solved and then the other 4 unknowns may be determined from these two variables. However, this procedure has much lower numerical stability than the method of steepest descent as just described. Furthermore, there is a narrow range of convergence and the errors in determining the remaining 4 unknowns are much greater than the errors in θ_1 and θ_2.

- It would appear to be possible to use the method of steepest descent with the measurements from 2M +1 coils to determine the magnitudes, phases, and locations of M currents. However, the solutions are numerically unstable with more than 2 currents.

SUMMARY AND CONCLUSIONS

- Three coaxial toroidal coils may be used to determine the magnitude, phase, and location of one time-dependent current that passes through their common aperture. These three coils should have the number of turns per unit length constant, and proportional to the sin(θ), and the cos(θ), respectively, where θ is the azimuthal coordinate.

- Five coaxial toroidal coils may be used to determine the magnitudes, phases, and locations of two time-dependent currents that pass through their common aperture. These five coils should have the number of turns per unit length constant, and proportional to the sin(θ), cos(θ), sin(2θ), and cos(2θ), respectively.

- The magnitude, phase, and location of one current are determined from measurements of the magnitude and phase of the voltages that are induced on three coils, either by an explicit solution or by the method of steepest descent, with fractional errors that are approximately equal to the fractional errors in the measurements.

- The magnitudes, phases, and locations of two currents are determined from measurements of the magnitude and phase of the voltages that are induced on five coils by the method of steepest descent, with fractional errors that are several times the fractional errors in the measurements.

- It does not appear to be practical to use this method to determine the magnitudes, phases, and locations of more than two currents.

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