# A STATISTICAL ANALYSIS OF THE BEAM POSITION MEASUREMENT IN THE LOS ALAMOS PROTON STORAGE RING*! 

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#### Abstract

The beam position monitors (BPMs) are the main diagnostic in the Los Alamos Proton Storage Ring (PSR). They are used in several applications during operations and tuning including orbit bumps and measurements of the tune, closed orbit (CO), and injection offset. However, the BPM data acquisition system makes use of older technologies, such as matrix switches, that could lead to faulty measurements. This is the first statistical study of the PSR BPM performance using BPM measurements. In this study, 101 consecutive CO measurements are analyzed. Reported here are the results of the statistical analysis, tune and CO measurement spreads, the BPM single turn measurement error, and examples of observed data acquisition errors.


## INTRODUCTION

A PSR BPM is a collection of four stripline electrodes located in the beam pipe and situated top, bottom, left, and right. The BPM diameters are $4^{\prime \prime}$ and $6^{\prime \prime}$. They are tuned to the 201.25 MHz longitudinal structure of the beam and are a 201.25 MHz quarter wavelength long, $\sim 37$ cm . A mechanical relay matrix switch (MUX) selects the BPM for measurement. There are four MUXs to interface with each of the four BPM electrodes. Beam signals from the selected BPM pass the MUX to the analog front end (AFE) where the signals are converted from AM to PM. The AFE outputs a voltage proportional to the power ratio deposited on opposing BPM electrodes [1]. The voltages are digitized by a 12 -bit analog to digital converter (ADC). The intrinsic BPM resolutions (defined as the BPM diameter divided by 2 to the power of the bit depth, $\mathrm{d} / 2^{\mathrm{b}}$ ) are .0124 mm and .0186 mm for the $4^{\prime \prime}$ and $6^{\prime \prime}$ BPMs respectively. The ADC is triggered to digitize data by a beam present trigger. The digitized voltages are read to the input/output controller (IOC) where they are converted back to beam positions using geometric coefficients and AM-PM theory [2,3]. The position data is lastly read by EPICS.

There are 18 real and two "missing" BPMs in the PSR. The missing BPMs do not exist, but data is still collected. The missing BPMs are included in the analysis because they possess information about the data acquisition errors. For the convenience of analysis, an orbit response matrix (ORM) BPM naming convention is employed. This convention distinguishes the two dimensions of a single BPM, dividing a bi-directional BPM into two different

[^0]BPMs, a horizontal and vertical BPM. The convention then gathers all BPMs of the same direction and numbers them consecutively. Thus, the 20 BPMs in the PSR are divided into 40 different BPMs. BPMs 1-20 are the horizontal BPMs and BPMs 21-40 are the vertical BPMs such that BPM 1 and BPM 21 are the horizontal and vertical division of SRPM01, BPMs 2 and 22 are the horizontal and vertical parts of SRPM02, and so on. The missing BPMs are indexed as BPMs $10,15,30$, and 35.

## MEASUREMENT SETUP AND DATA ANALYSIS

The PSR was set up for single turn injection and after $\sim 1800$ turns, extraction to the tune up beam stop (TUBS) at 20 Hz . The vertical bump magnets and the harmonic buncher that respectively allow phase space painting in the vertical and keep the beam bunched longitudinally were turned off. The CO was centered. The beam was injected near-on-axis ( $[-0.72 \mathrm{~mm}, 0.31 \mathrm{mradian}]$ in the horizontal and [ $1.99 \mathrm{~mm},-0.168$ mradian] in the vertical) to avoid scraping and BPM saturation. Injecting near-onaxis allowed for 40 turns of turn-by-turn beam positions to be collected before the 201.25 MHz longitudinal structure of the beam washed out due to momentum variation in the mircopulses. The energy of the beam was corrected using the time of flight for 1100 turns.
One hundred one consecutive CO measurements were taken for the same PSR configuration. Each CO measurement consists of 40 turns of turn-by-turn beam position data at each BPM. Since a MUX is employed to select the BPMs, data from only one BPM can be recorded per machine cycle. The CO is measured at each BPM on a different pulse with slightly different central momentum (different CO ) due to the pulse-to-pulse momentum variations in the linac. It took 7.5 minutes to take 101 CO measurements.

As the beam circulates around the ring, it performs harmonic (betatron) oscillation about the CO. Thus, the turn-by-turn BPM data is then fit to a cosine wave,

$$
\begin{equation*}
y_{n}=A \cos (2 \pi m+\phi)+O_{f f s e} \tag{1}
\end{equation*}
$$

where $\mathrm{y}_{n}$ is the turn-by-turn BPM data, $n$ ranges from 1 to $40 ; A, v, \phi$, and $O_{f f s e t}$ are the amplitude, betatron tune, phase, and CO respectively. A nonlinear least squares fitting routine was used to fit for $A, v, \phi$, and $O_{f f s e t}$. The sum of squares of residuals per degree of freedom (SSR/DOF) was used as the goodness of fit quality factor. A maximum likelihood (ML) error analysis was applied to calculate the fitting error on the fitting parameters, the single turn BPM measurement error, and the covariance and correlation matrices relating the fitting parameters. Aside from the tune and the phase, the fitting parameters were found to be uncorrelated. The tune and the phase


Figure 1: (Color) A typical scan. The blue circles are the BPM data, the green squares are the cosine wave fit, and the red line is the value of the extracted CO from the cosine wave fit of Eq. (1).
had a correlation of $\sim-.86$. This correlation comes from the fact that the first derivatives of the fitting function, Eq. (1), with respect to the tune and phase differ by a constant $2 \pi n$. The ML error analysis assumes mean zero random errors. The average residual for each fit was found to be $<6 \mathrm{e}-8 \mathrm{~mm}$, satisfying the ML requirement. A typical scan and fit are shown in Fig. 1.

When the turn-by-turn BPM data is fit using Eq. (1), it is assumed that there is no nonlinear motion induced by higher order multipole magnets (such as sextupole and octupole), or nonlinearities in the BPM measurement. This assumption is justified because the beam is injected near-on-axis where the nonlinearities are the smallest. Nonlinearities in the beam position measurement are observed in BPM data when the beam is injected at production injection offsets ( $\sim[-3 \mathrm{~mm}, .6 \mathrm{mradian}]$ in the horizontal and $\sim[17 \mathrm{~mm}, 3$ mradian $]$ in the vertical) for phase space painting.

## DATA ACQUISITION ERRORS

Although the turn-by-turn data is expected to follow Eq. (1), sometimes there are errors in the way that the BPM data is collected or handled and the recorded beam positions are not sinusoidal. Eleven different data acquisition errors were observed in the PSR BPMs [4]. The three most common errors are reported here.

## BPM Selection Errors

BPM selection errors look like cosine waves, and fit cosine waves very well because they are cosine waves! BPM selection errors are the most insidious errors. However finding them is not difficult. This error is $x-y$ symmetric i.e. if the error occurs in the horizontal, it also occurs in the vertical.

Normally the ADC buffer, where the digitized voltages are stored, is cleared between machine cycles. When the ADC buffer is not cleared, the IOC reads the same


Figure 2: The SSR/DOF comparing the power ratios from one scan with the preceding scan for all scans at all BPMs. The horizontal line shows the threshold value of $10^{-10}$. The vertical line divides horizontal and vertical BPMs. Note that some scans from the missing BPMs (BPMs 10, 15, 30, and 35) also have BPM selection errors.
digitized voltages multiple times. However, the IOC "thinks" the data is from the MUX selected BPM which is not the BPM where the digitized voltages originated. The IOC analyzes the same digitized voltages using the different coefficients of the MUX selected BPM. The same data can be used four times and even in different CO measurements.

BPM selection errors are identified and removed from the data set by comparing the power ratios of a scan with the scan taken immediately before it. If the ADC buffer is not cleared, the digitized voltages are used again to calculate the same power ratios for both scans. An SSR/DOF of the difference of the power ratios for both scans can be computed. This value for all scans is plotted in Fig. 2. An SSR/DOF of less than $10^{-10}$ indicates a BPM selection error.

Of the 3636 scans taken for the CO reproducibility measurement, 78 scans were removed because they possessed a BPM selection error (2.15\%), and eight CO measurements of the 101 in the reproducibility dataset had BPM selection errors, $7.95 \%$. This error does not affect the tune, but does yield a fitted phase equal to previous BPM. It is most likely that a BPM selection error will yield faulty amplitude and offset results.

## Flat Line Errors

At first glance, flat line errors are obvious errors, and thus easily identified and removed from the dataset. This error is also $\mathrm{x}-\mathrm{y}$ symmetric. A typical flat line error is shown Fig. 3.

Flat line errors have only been observed in BPMs that immediately follow the missing BPMs. The scan structure of Fig. 3 is consistent with all flat line errors, six or seven points of random noise with significant amplitude and the rest of the data equal to zero. The noise is similar to BPM
data collected with both MUX outputs unplugged and unterminated.
The zeros are easy to understand. The IOC will "fill" the scan with zeros if it receives less digitized turns from the ADC than the user requested number of turns [5].

To understand why there are always six or seven turns of random noise, one needs to look at data recorded from the previous, missing BPM. Data from the missing BPMs always consists of one or two points of random noise. The first point can be attributed to a digitization triggered by noise from the ADC disarm trigger, which fires $\sim 1 \mathrm{~ms}$ after $T_{0}$, long after the beam has been extracted from the PSR [6]. If there is a second point, it seems to be a digitization $\sim 100 \mu$ s after the disarm trigger. This could be a result of the software flushing the ADC pipeline (where three turns of digitized data are stored before being saved to the buffer to increase the ADC sampling speed) before reading the buffer. The power ratios of the first (and/or second) non-zero point from the missing BPM are the same as the power ratios of the first one or two points of the scan with the flat line error. This means that the ADC buffer did not clear properly and the same data is used for both scans. The addition of five points of random noise to the data from the missing BPM is presumably a digitization triggered by noise from the ADC arm and disarm triggers and three points from flushing the pipeline.

Flat line errors are identified as scans that have matching power ratios and more points of turn-by-turn BPM data compared the previous scan. Flat line errors were found in ten of the 3636 scans (.28\%) and in five of the 101 CO measurements, $4.95 \%$. If left in the data set, a flat line error will result in bad measurements for all of the fitted parameters.

## Missing Turn Errors

A missing turn error occurs when the ADC digitizes the AFE output voltage as normal, but then for some turn, the beam present trigger does not trigger the ADC to digitize data. The beam passes by the BPM and no data is taken. The following beam present triggers prompt the ADC to digitize data like normal.
A typical scan with a missing turn error is plotted in Fig. 4. The missed turn is observed by comparing the BPM data with the initial cosine wave fit guess. Although the initial cosine guess is not perfect, it does have a constant frequency that in the beginning of the scan matches up well with the BPM data. At turn 24, the cosine guess indicates a maximum in the oscillation, but the BPM data goes down from the previous turn. After turn 24, the turn-by-turn data is one turn ahead of the initial cosine guess. However, if the BPM data was delayed a turn for turns after 24, all traces lie on top of each other until a turn is missed, and then the BPM data with the added turn and the initial cosine guess match for the rest scan.

In the CO reproducibility measurement, 106 turns were missed out of 145440 (. $073 \%$ ), 96 scans out 3636 were removed from the data set because they had missing turn


Figure 3: A typical flat line error.


Figure 4: (Color) A typical missing turn error. The blue circles are BPM data, the green squares are points for an initial cosine wave guess fit, and the red x's are BPM data with a turn added at turn 24.
errors $(2.64 \%, 10$ scans were observed to have two missing turns), and 42 out of 101 CO measurements contained a scan with a missing turn error, $41.58 \%$. If left in the data set, the missing turn error will compromise the tune measurement and the phase through correlation in the cosine fit, but the amplitude and offset are unaffected.

## RESULTS

After the BPM data is fit to a cosine wave and all data acquisition errors have been identified and removed, the resulting good scans are left describing the reproducibility of the beam position measurement. The results for the fitted tune and offset parameters and the BPM single turn measurement error are summarized in this section.

## The Fitted Tune

The tune is the frequency of the cosine wave fit. The tune fit is better with more turns of data. Since the tune is

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Figure 5: Histogram of final horizontal tune distribution.


Figure 6: Histogram of final vertical tune distribution.
$\sim .2,40$ turns of data is eight betatron oscillations and yields a good fit. The tune is multiplied by $2 \pi$ in Eq. (1), so only the fractional tune can be fit. The integer part of the tune will only contribute to other betatron oscillations in between turns and will not be observed in the discrete time measurement of the $\mathrm{BPM}, \cos 2 \pi=\cos 2 \pi i$ where $i=$ $1,2,3 \ldots$

All of the BPMs in one direction measure the same tune, so the final quoted tune measurement is actually the average of all scans in that direction. Histograms of the total horizontal and vertical fitted fractional tune distributions are shown in Figs. 5 and 6. The rms standard deviation of these profiles is about 10 times smaller than the expected error using the current operations method, $4.2 \mathrm{e}-4(.22 \%)$ in the horizontal and $3.4 \mathrm{e}-4(.17 \%$ of the fitted fractional tune) in the vertical. The tune averages are .191432 in the horizontal and .197938 for the vertical.

## The Fitted Offset

The fitted offset is the CO. The fitting error on the offset calculated from the ML error analysis for each scan is fairly constant across all BPMs, Fig. 7. The fitting error


Figure 7: (Color) The fitting error on the offset parameter for all scans without data acquisition errors (blue circles) and the measurement spread at each BPM, red squares.
on the offset is only slightly larger than the intrinsic resolution of the BPM. The fitting errors for the vertical BPMs match the spread in the measurement distribution. Thus the precision of vertical CO measurement is limited by the intrinsic resolution of the BPM measurement. The difference between the horizontal fitting error and the offset rms measurement spread is a clue pointing to the pulse-to-pulse momentum variations influencing the horizontal offset measurement spread. The horizontal measurement spread is $\sim 5$ times larger than the fitting error. The pulse-to-pulse momentum variations do not influence the vertical CO measurement


Figure 8: (Color) The square of the CO measurement spread (blue circles) fit (red line) to $\sigma_{C O}^{2}=\sigma_{B P M}^{2}+2\left\langle\varepsilon_{\delta}, \varepsilon_{B P M}\right\rangle D+\sigma_{\delta}^{2} D^{2}$, where $\sigma$ is an rms error, $\varepsilon$ is the absolute error, $<\ldots>$ is a covariance, $D$ is the measured dispersion function, and BPM and $\delta$ indicate errors due to the BPM measurement and pulse-to-pulse momentum variations respectively [4], shows the dependence of the CO measurement spread to the pulse-to-pulse momentum variations.

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because the vertical dispersion function is small. Interestingly, the horizontal CO measurement spread tracks the measured dispersion function well, yielding a calculated correlation of .97 . This is because the CO change at a BPM is equal to the relative momentum change multiplied by the dispersion function, $\Delta \mathrm{x}_{C O}=\mathrm{D} \delta$. The dependence of the CO measurement spread on the dispersion is shown in Fig. 8. The fit in Fig. 8 is derived from assuming that there are only two contributions to the CO measurement error: an error in the BPM measurement and changes to the CO due to errors in the momentum, the pulse-to-pulse momentum variations.

The error on the average CO can be calculated using the offset measurement spreads and Eq. (2). The error on the average CO for 101 CO measurements is less than the intrinsic resolution of the BPMs. At this scale the discrete nature of the digitization in the ADC is prominent. Thus the accuracy of the CO measurement is limited by the intrinsic resolution of the BPM measurement.

$$
\begin{equation*}
\sigma_{a v e}=\frac{\sigma_{\text {meas }}}{\sqrt{M}} \tag{2}
\end{equation*}
$$

## The Single Turn Measurement Error, Sigma

The single turn measurement error (sigma) is the rms standard deviation of the gaussian residual distribution.
Every turn of data taken at one BPM should have the same measurement error. The single turn measurement error for all scans is shown in Fig. 9. All BPMs have about the same measurement error, between .1 and .2 mm . This is much smaller than expected for the PSR BPMs. Analyzing the residuals as a function of turn it becomes apparent that the main contributor to sigma is a constant offset drift across all scans of turn-by-turn BPM data. The average drift of the offset in a scan of 40 turns is $\sim .4 \mathrm{~mm}$. This creates a residual distribution with a larger rms standard deviation.

## CONCLUSIONS

One hundred one consecutive CO measurements were taken for the same PSR configuration. A CO measurement consists of 40 turns of turn-by-turn BPM data at each of the 20 BPMs. The turn-by-turn BPM data was fit to a cosine wave using Eq. (1). Three of the 11 data acquisition errors found were discussed. These errors yield bad fitted parameters. Two hundred sixty four scans out of the 3636 scans in the CO reproducibility measurement were observed to have data acquisition errors, $7.26 \%$. But 62 CO measurements of the 101 in the dataset possessed a data acquisition error, $61.38 \%$. After these scans were removed, the fitted parameters from the remaining good data scans were analyzed.

All scans of one dimension were used to calculate the overall tune value. The tune measurement was [.191432, .197938] with an rms measurement spread of $4.2 \mathrm{e}-4$ $(.22 \%)$ in the horizontal and $3.4 \mathrm{e}-4(.17 \%$ of the fitted fractional tune) in the vertical.

The CO measurement spreads were found to be $\sim .1 \mathrm{~mm}$ in the horizontal and $\sim .02 \mathrm{~mm}$ in the vertical. The CO Instrumentation


Figure 9: (Color) The calculated single turn measurement error (sigma) for all scans without data acquisition errors (blue circles) and the average with one rms standard deviation, red squares.
measurement spread in the vertical is minimum and limited by the intrinsic resolution of the BPM measurement. The pulse-to-pulse momentum variations are the cause of the much larger CO measurement spread in the horizontal. The correlation between the horizontal CO measurement spread and the measured dispersion function is .97 . The error on the average CO for 101 CO measurements is less than the intrinsic resolution of the BPMs. Thus the accuracy of the CO measurement is also limited by the intrinsic resolution of the BPM measurement.

The single turn measurement error (sigma) was found to be between .1 and .2 mm . A constant offset drift in each scan is the main contributing factor to sigma.

## ACKNOWLEDGEMENTS

Special thanks to S.Y. Lee of Indiana University.

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[^0]:    *Work supported by the US DOE under contracts DE-AC52-
    06NA25396 and DE-FG02-92ER40747 and the NSF under contract
    NSF PHY-0852368.
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