# CALCULATIONS OF ELECTRON BEAM MOTION IN ELECTRON COOLING SYSTEM FOR COSY

M.I. Bryzgunov, V.M. Panasyuk, V.B. Reva (BINP, Novosibirsk)

*Abstract*

Results of calculations of electron beam motion in cooler for COSY (Juelich, Germany) are shown. The aim of the calculations is to study excitation of the beam galloping due to magnetic and electric field ripples, imperfection of bending field in toroids, transverse electric field in the end of accelerating tube etc. Dependences of the beam temperature on different parameters of magnetic and electrostatic systems are presented. Methods of correction of electron beam motion in order to decrease its transverse velocity are shown.

## INTRODUCTION

In low energy electron coolers temperature (transverse velocity) of electron beam is not very important parameter because motion of electrons is adiabatic along full length of coolers from electron gun to collector and there is no significant excitation of transverse velocity. But in electron cooling systems for high energy, longitudinal Larmor length for electrons is high. Because of this, motion is not adiabatic and strong excitation of transverse temperature of the beam in possible. In such case investigation of electron motion in order to prevent strong increasing of electron transverse temperature is important task.

There are several one-particle effects which can increase transverse temperature of electron during the flight in magnetic field of the electron cooler:

1) magnetic field ripples,
2) electric field ripples,
3) excitation on transverse electric field in the end of the acceleration tube,
4) excitation on transverse magnetic field in transition between different values of magnetic field,
5) entry and leaving from bending parts.

In linear approximation, transverse beam motion in the system can be divided to for modes. First one is transverse shift of the beam relatively the reference line of the system without transverse velocity. Second one is dipole mode, in which all electrons of the beam with the same longitudinal coordinate move in transverse direction synchronously in the same direction, i.e. centre of mass of the beam moves, but there is no motion of electrons relatively the centre of mass. Third mode is synchronous motion of electrons relatively the centre of mass, but centre of mass doesn't move. In literature such motion is called galloping. And the fourth mode appears in systems where quadruple component of transverse magnetic field exists. This mode changes transverse profile of the beam by compressing it in one direction and stretching in perpendicular direction.

In systems with axial symmetry only galloping (third mode) can exist. Since in points 1-4 from the list above we suppose axial symmetry, only galloping can be excited there. In point 5 excitation of all modes is possible.

Influence of quadruple component on the temperature of the beam is very weak and it can be adjusted by field index in bends of the cooler. According to this, we need only two different types of additional correctors: dipoles and axial lenses, for correction of beam motion.

## ELECTRON COOLER FOR COSY

Electron cooler for COSY (Juelich, Germany) is constructed to work in wide range of operating electron energy: 25 keV (injection) – 2 MeV (maximum energy for the ring) [1]. Magnetic field in cooling section is 2 kG. Cooling time ~10 sec.

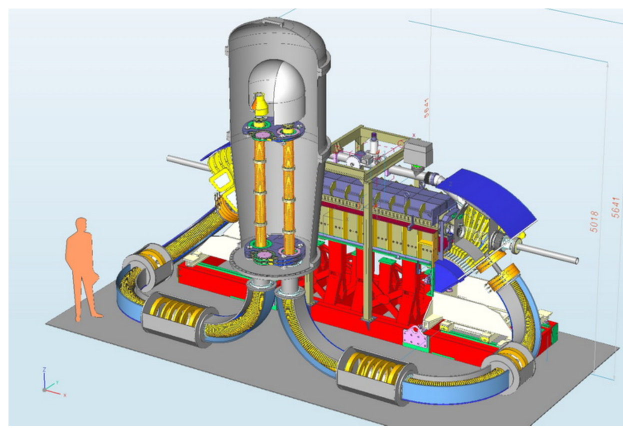

Figure 1: Electron cooler for COSY.

Strict requirements on size of the cooler, forces us to make its shape so complicated (fig. 1). In such shape acceleration and deceleration tubes are placed in one high voltage vessel. Length of acceleration tubes is 2 m. Along full trajectory from gun to collector electrons move in longitudinal magnetic field.

Magnetic system of high voltage vessel consists of two sets of identical coils, producing longitudinal magnetic field in acceleration and deceleration tubes. Transport line between high voltage vessel and cooling section consists of three 90° bends, one 45° toroid, two straight sections for technical purposes and two sections with variable profile of magnetic field for minimization of excitation of transverse motion in transition between different values of magnetic field. Cooling section consists of special coils with possibility to adjust straightness of magnetic force line by rotating each coil independently [2]. Bending field in 90° bends (82 G for 2 MeV electrons) is made with special coils (fig. 2). Each bend includes 4 coils. We are planning to change current in every coil independently, so one can regulate both value of the field and field index.

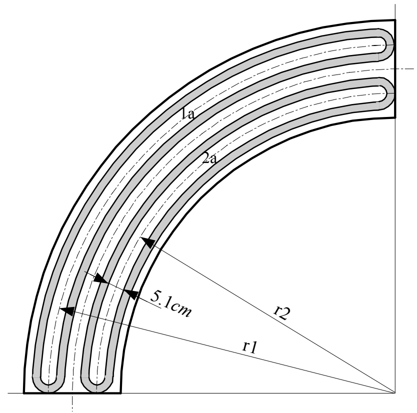

Figure 2: Bending coils in 90° bends.

## HEATING ON DIFFERENT ELEMENTS

All main effects which can heat electron beam were calculated. We suppose that strong heating of the beam should be avoided but small excitation will be compensated with special correctors.

### *Magnetic Ripples*

The dynamic of electron in electrostatic column can be described with equations:

$$\begin{cases} x'' + \dfrac{\gamma'}{\gamma\beta^2}x' - \dfrac{qB}{\gamma\beta m_e c^2}y' - \dfrac{qB'}{2\gamma\beta m_e c^2}y = 0 \\ y'' + \dfrac{\gamma'}{\gamma\beta^2}y' + \dfrac{qB}{\gamma\beta m_e c^2}x' + \dfrac{qB'}{2\gamma\beta m_e c^2}x = 0 \end{cases} . \quad (1)$$

Here we suppose that there is only longitudinal electric field and no transverse electric field. Transverse magnetic field is calculated in paraxial approximation of the first order. Making substitutions $\eta = x + iy$, $k = qB/(m_e c^2)$, $\eta = \eta_1(w)\cdot\exp\left(-\frac{i}{2}\int_0^w \kappa(w')dw'\right)$, $w = \int \frac{dz}{\gamma(z)\beta(z)}$, one can rewrite the system in form of equation for complex amplitude $\eta_1(w)$

$$\frac{d^2\eta_1}{dw^2} + \frac{\kappa(w)^2}{4}\eta_1 = 0 . \quad (2)$$

From the theory of Mathieu equation there is a condition on resonance. In terms of $w$ variable it means that frequency of changing of $k$ is equal to $2k$. Higher order resonances are weak and can be neglected. In terms of $s$ variable it means that period of magnetic system is equal to the length of Larmor spiral. During the acceleration, Larmor length is changing and resonance occurs only in small region of acceleration part. Calculation of trajectory shows that increasing of temperature of the beam on magnetic ripples during acceleration is small (less than $5\cdot10^{-6}$ cm/s) and can be neglected.

### *Electrostatic Tube*

In the end of acceleration tube, strong defocusing transverse electric field appears. This field can increase transverse temperature of the beam. Transverse velocity of electron after transition through the thin final lens can be estimated using formula:

$$V_T = \frac{eE}{2p}r , \quad (3)$$

here $E$ – electric field in tube, $r$ – radius of electron, $p$ – momentum of the electron.

Such estimations give good coincidence with precise calculations of electron trajectory calculated with SAM program (fig. 3). For energy $U$=2 MeV transverse velocity after transition for electron with $r$=1.5 cm is about $9\cdot10^7$ cm/s . In longitudinal magnetic field 500 G it corresponds to transverse Larmor radius 0.5 mm.

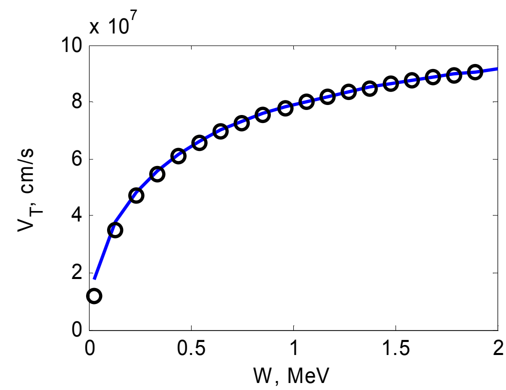

Figure 3: Transverse velocity of the electron started with $r$=1.5 cm after transition through the end of acceleration tube (solid line – estimation using (3), circles – calculation).

Since electric field for acceleration of beam is made with separated electrostatic plates with different potential, resulting electric field is not homogenous and transverse electric field can increase transverse temperature of the beam. To estimate influence of this effect, electric field in acceleration column was calculated using SAM program (BINP, Novosibirsk). These calculations show that the highest effect in electric filed ripples introduces flange region between different sections. But for chosen construction, influence of such ripples is very weak in comparison with effect in the end of tube.

### *Bends*

In the general case, magnetic force line which starts in center of acceleration column doesn’t coincide with centre of the bend. Because of this, transverse magnetic field can appear in transition from liner part to the bend. To decrease heating of the beam after transition through a bend, the length of the bend should be equal to integer number of Larmor length. In such a case kick on entry to bend in compensated by kick on leaving (fig. 4).

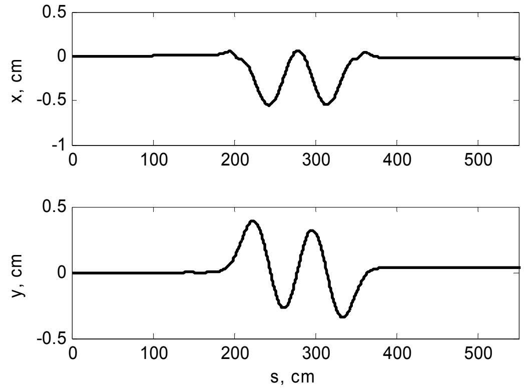

Figure 4: Trajectory of electron with energy 2 MeV in 90° bend. Bend starts in *s*=200 cm, radius *R*=100 cm.

The worst situation, which one should avoid, occurs when length of bend is equal to $n+1/2$ of Larmor length. In such a case two kicks are added and resulting transverse velocity of electrons is very high.

### *Matching Section*

Transition between two different values of magnetic field leads to increase of transverse temperature of the beam. In transition from 1 kG to 2 kG for 2 MeV electrons transverse velocity increases for $3{\cdot}10^9$. To avoid this, special matching is needed. There are different methods of matching (for example see [3-5]). In this work it is made using special profile of magnetic field.

To calculate magnetic field profile one can transform (2) to equation on envelope *W*:

$$W'' + \frac{k^2}{4}W - \frac{1}{W^3} = 0\,. \qquad (4)$$

Supposing that acceleration is absent, one can turn back from variable *w* to *s* in (4) just replacing *w* by *s* and *k* by $k_N$ which is equal to $k_N = \frac{k}{\gamma\beta}$. From (4) value of *W* function without oscillations is

$$W = \sqrt{\frac{2}{k_N}}\,. \qquad (5)$$

Any envelope function can be set and shape of magnetic field can be restored

$$B(z) = \frac{\gamma\beta mc^2}{e}k_N(z) = \frac{2\gamma\beta mc^2}{e}\sqrt{\frac{1}{W^4} - \frac{W''}{W}}\,. \qquad (6)$$

Using this method any *W* function which haven't oscillations in the beginning and in the end of matching section and equal there to the values calculated by (5) can be set and shape of magnetic field can be restored.

The section (fig. 5) is made as an addition insertion in beam line between acceleration tube and first bend (and between final bend and deceleration tube). The insertion includes some additional coils with one current source for every coil. Current in every coil is set in order to make shape of magnetic field calculated from (6).

In fig. 5 the profile is made for 2 MeV electrons using 11 additional coils. Form of envelope function is

$$W(s) = W_1 + \frac{W_2 - W_1}{2}\left(erf\left(\frac{s}{a}\right) + 1\right), \qquad (7)$$

with *a*=15 cm. Values of magnetic field in the beginning and in the end are 500 G and 1000 G correspondingly.

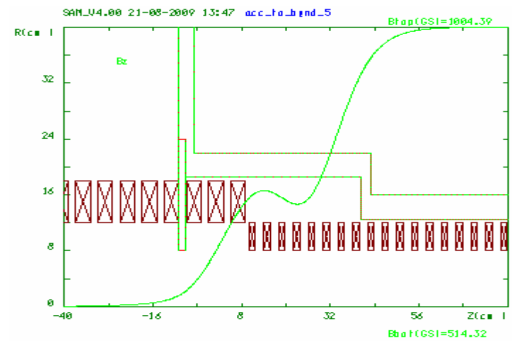

Figure 5: View of the matching section and magnetic field shape for electrons with energy W=2 MeV.

It was said above, that the cooler works in wide range of electron energy. Therefore we need to make its own shape of magnetic field in the matching section for every value of energy. Because of this,, the section consists of so many coils.

Since it is impossible to make ideal shape, there is small galloping of the beam after transition, which should be corrected.

Similar matching section is used in transition between 3-rd 90° bend and 45° toroid with field 2 kG.

### *Straight Section*

There are two straight sections between 90° bends (fig. 6).

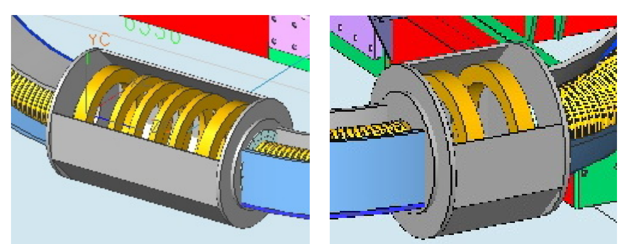
Figure 6: Straight sections.

The purpose of the insertions is to provide easy access to vacuum system in order to: install vacuum pumps, provide assembly and disassembly of vacuum system, simplify output from pickup electrodes. Therefore the insertions consist of three types of relatively big coils. Two types for producing of magnetic field in the beginning and in the end of insertion and one type in center. Using the coils, linear parts with appropriate uniformity of magnetic field can be made.

## CORRECTION

It was said that we need two types of correctors to decrease transverse motion of electron beam in the cooling section. One type for correction of dipole motion of the beam and another type for correction of beam galloping.

First one is supposed to be made of two independent dipoles. Every dipole consists of two coils connected in series. Using the corrector one can decrease both $V_x$ and $V_y$ components of transverse velocity independently.

In order to correct beam galloping the same coils which are installed in the matching section are supposed to be used. Changing current in one coil one can make axial magnetic lens. Using several coils, beam galloping can be decreased.

Supposing that beam diagnostic system allows us to measure transverse Larmor radius with resolution 0.1 mm [5], resulting transverse angle for 2 MeV electrons is about $2.5\cdot10^{-3}$. It corresponds to temperature of the beam 1 eV.

## ENERGY RANGES

It is clear that it is possible to set one value of energy of electron beam and to adjust cooler optics for this energy in order to avoid high temperature of the electron beam in cooling section. But wide range of operating electron energy means that optics of the cooler must be easily adjustable for all energies. One method of realization of such system is to adjust optics manually for every value of energy. Number of parameters and correctors in our system allow us to do this, but this method is very time consuming. There is easier method which was proposed for this system. The idea of the method is to change magnetic field in the cooler synchronously with beam energy. For example: if magnetic system was adjusted for 2 MeV electron beam, then after decreasing of energy for value $U$, we must decrease magnetic field (longitudinal and transverse) in $\alpha$ times where $\alpha$ is equal to ratio of momentums for 2 MeV and for $U$:

$$\alpha = \frac{p_{2MeV}}{p_E} \approx \sqrt{\frac{(2+0.511)^2 - 0.511^2}{(U[MeV]+0.511)^2 - 0.511^2}} \,. \qquad (8)$$

It is impossible to use this formula for full range of energy. Because of this we divided the range for 4 smaller ranges (fig. 6).

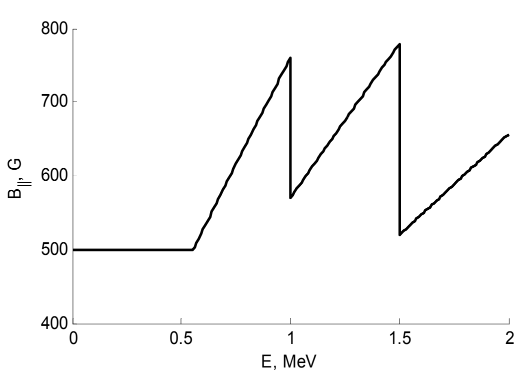


Figure 7: Longitudinal magnetic field in bends for different energies.

From the fig. 6 one can see that there are three regions with independent adjustment of optics (0.55~1.0 MeV, 1.0~1.5 MeV, 1.5~2.0 MeV). The idea of the division is to have integer number of Larmor length in a 90° bend. The numbers are 4, 3 and 2 for regions 0.55~1.0 MeV, 1.0~1.5 MeV and 1.5~2.0 MeV correspondingly. After adjustment, magnetic field in the cooler must be changed synchronously with energy. For energy lower than 0.55 MeV Larmor length is small, because of this motion of electrons is adiabatic and excitation of transverse motion of the beam is small.

## CONCLUSION

So complicated shape of the cooler and so wide range of operating energy force us to adjust optics of the cooler in order to make transverse temperature of electron beam in the cooling section as small as it is required.

In some elements (bends, matching sections, straight sections) resulting transverse temperature is decreased adjusting some parameters of the elements. Otherwise, after transition of these elements resulting temperature can be very high. In other elements, in which resulting temperature is not very high, special adjustment isn't used.

Resulting transverse motion of the beam is sum of dipole motion and beam galloping. In order to correct it two types of correctors (two perpendicular dipoles and axial magnetic lens in matching sections) are used.

## ACKNOWLEGMENTS

Authors thank Dr. J. Dietrich from COSY (Juelich, Germany) and head of electron cooling laboratory V.V. Parkhomchuk (Novosibirsk, Russia) for useful consultations. Also thank to V.K. Gosteev for help in printing of the article.

## REFERENCES

[1] J. Dietrich, M.I. Bryzgunov, A.D. Goncharov, V.V. Parkhomchuk, V.B. Reva, D.N. Skorobogatov, “Prototype of the high voltage section for the 2 MeV electron cooler at COSY-Juelich”, EPAC’08, Genoa, Italy, p. 3467.

[2] V. Bocharov, A. Bubley, S. Konstantinov, V. Panasyuk, V. Parkhomchuk, “Precision measurements and compensation for the transverse components of the solenoids magnetic field”, Instruments and Experimental Techniques, Vol. 48, Number 6, 2005, p. 772.

[3] V. Parkhomchuk, I. Ben-Zvi, “Electron cooling for RHIC”, C-A/AP/47, http://www.agsrhichome.bnl.gov/AP/ap_notes/ap_note_47.pdf, April, 2001, p. 59.

[4] Ya.S. Derbenev, “Advanced optical concepts for electron cooling”, ECOOL’99, Uppsala, Sweden, p. 223.

[5] “HESR electron cooler. Design study”, http://www5.tsl.uu.se/hesr/public/D32_HESR_EC.pdf, Uppsala, Sweden, 2009.