IMPLEMENTATION OF LONGITUDINAL DYNAMICS WITH BARRIER RF IN BETACOOL AND COMPARISON TO ESME

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Abstract

The barrier bucket RF system is successfully used on Recycler storage ring at Fermilab. The special program code ESME [1] was used for numerical simulation of longitudinal phase space manipulations. This program helps optimizing the various regimes of operation in the Recycler and increasing the luminosity in the colliding experiments. Electron and stochastic cooling increases the phase space density in all degrees of freedom. In the case of a small phase space volume the intrabeam scattering introduces coupling between the transverse and longitudinal temperatures of the antiproton beam. For numerical simulations of the cooling processes at the Recycler, a new model of the barrier buckets was implemented in the BETACOOL code [2]. The comparison between ESME and BETACOOL codes for a stationary and moving barrier buckets is presented.

This article also includes an application of the barrier bucket numerical model for simulation of the luminosity distribution for RHIC colliding experiments. These simulations take into account the specific longitudinal distribution of the bunch and the vertex size of the detector.

BARRIER BUCKET MODELS IN THE BETACOOL PROGRAMM

Currently, the BETACOOL code has three algorithms that describes the synchrotron motion of the particles and which can be used for the simulation of the barrier bucket (BB) models. The first algorithm solves the standard equations of motion in the longitudinal phase space($s-s_0$, $\delta=\Delta p/p$). The equations are:

$$\begin{cases} \dfrac{d(s-s_0)}{dt} = |\eta|\beta c\delta \\ \dfrac{d\delta}{dt} = -\dfrac{ZeV(t)}{Cp_0} \end{cases} \quad (1)$$

where βc is the ion velocity, η is the ring off-momentum factor, *Ze* – the particle charge, *V(t)* – the dependence of RF voltage on time, *C* – the ring circumference and p_0 – the momentum of the particles.

In the context of this algorithm the longitudinal motion any arbitrary RF voltage shape can be simulated. However, the problem of this algorithm is the calculation time because the integration step should be much smaller than the synchrotron period.

To avoid this problem, the analytical solution of the longitudinal motion between two square barrier buckets was introduced. In this case, the integration step can be independent on the synchrotron period. When the ion passes through the cavity gap at voltage $\pm V_0$ it gains (losses) an equal amount of energy ZeV_0, i.e.

$$\frac{d(\Delta E)}{dt} = \pm\frac{ZeV_0}{T_0} \quad (2)$$

where ΔE is the energy deviation from the synchronous one, T_0 – the revolution period. The ion trajectory in the longitudinal phase space ($t-t_0$, ΔE) inside the bucket can be written in the following form:

$$(\Delta E)^2 = \begin{cases} A_E^2, \quad if\ |t-t_0| \le T_2/2 \\ A_E^2 - \left(|t-t_0| - \dfrac{T_2}{2}\right)\dfrac{2\beta^2 E_0 ZeV_0}{T_0|\eta|}, \\ \qquad if\ T_2/2 \le |t-t_0| \le (T_2/2)+T_1 \end{cases} \quad (3)$$

where A_E is the maximum energy deviation from the synchronous energy E_0, V_0 is the voltage height, T_1 is the pulse width, T_2 is the gap duration. The phase space trajectory is composed of a straight line in the RF gap region and a parabola in the square RF wave region

The analytical model has static potentials for the barrier bucket with a rectangular shape which is resolved analytically in the longitudinal phase space. However, using of the analytical model is very difficult for the case of a moving bucket with an arbitrary shape.

A numerical model of the RF bucket is implemented in the BETACOOL code where the motion of one particle through each barrier is calculated independently. After crossing of the barrier the particle energy can increase (Fig.1a), decrease (Fig.1b) or the particle can be reflected by the barrier (Fig.1c).

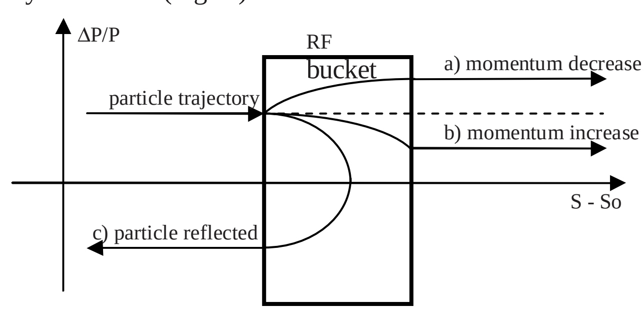

Figure 1: Particle trajectories trough a RF barrier in longitudinal phase space.

For the description of the individual synchrotron motion of each particle one can use a series of the barriers and numerical integration over the longitudinal phase

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space. If the integration step is larger than the synchrotron period, we can assume that for the static barriers the synchrotron period is random.

COMPARISON WITH ESME

The first example shows a comparison of longitudinal particle dynamics for stationary BB without any cooling or heating processes. The initial distribution of particles was generated by the ESME program (Fig.2) and translated into the BETACOOL units (Fig.3). In BETACOOL (Fig.3), the red line is the barrier distribution in units of momentum spread and the blue line is the average momentum spread over each barrier length.

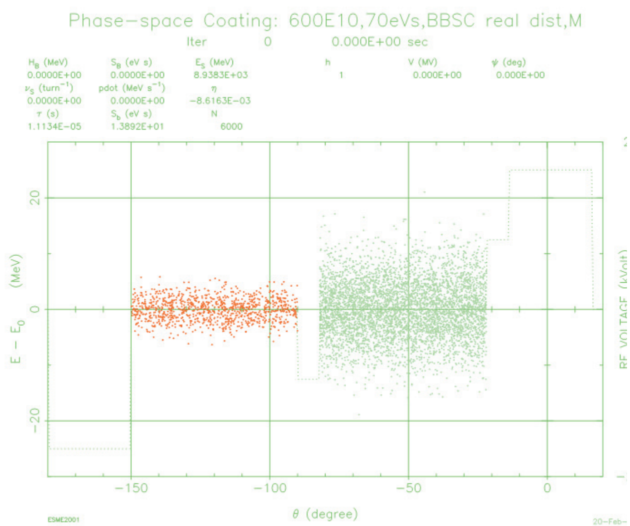

Figure 2: Longitudinal distribution of model particles in ESME.

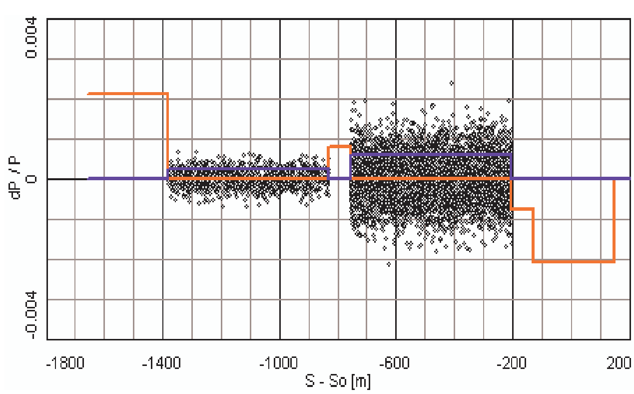

Figure 3: Longitudinal distribution of model particles and barrier bucket in momentum space in BETACOOL.

The beam profile along the longitudinal coordinate shows the particle density normalized to the line density of a coasting beam (Fig.4). This value is used for the simulation of IBS over each barrier length. The black line is the integral of particles normalized to the maximum value of the longitudinal profile.

The simulation consisted in letting evolve the initial distribution which was not in equilibrium, without any other effects. The final distributions after 15 sec (real time) have good agreement with ESME results (Fig.5,6). For BETACOOL, two sets of data with different integration steps are shown.

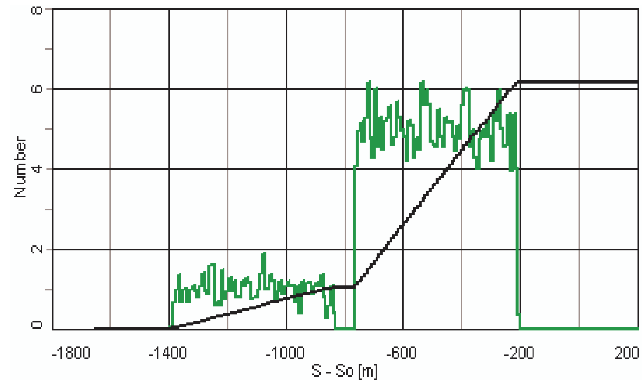

Figure 4: Longitudinal beam profile along longitudinal coordinate in BETACOOL.

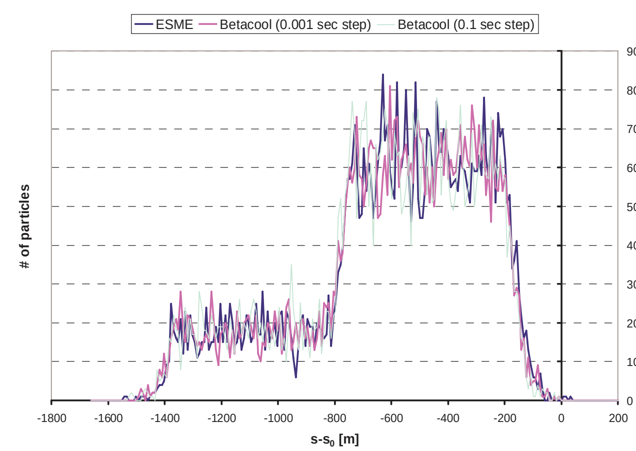

Figure 5: Longitudinal beam profiles along the longitudinal coordinate.

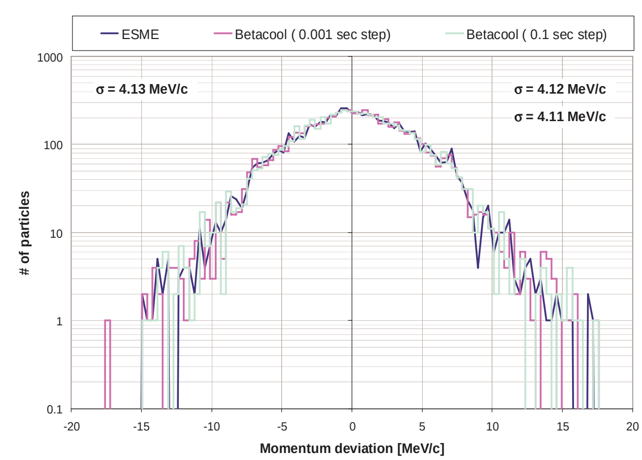

Figure 6: Momentum spread distributions. Final distributions.

The moving barrier option can be used for manipulation of particles in the longitudinal phase space in the case of adiabatic expansion or compression of the bunch. Input files for moving barriers can have any number of columns: the first column is the time in seconds; each other pairs define the position of the barriers in circumference units and the amplitude in volts. Consecutive rows determine the initial and final conditions (for both the position and the amplitude). Increments of the position and amplitude for each integration step are linear.

The initial distribution of particles was generated by the ESME program and was translated into BETACOOL (Fig.7).

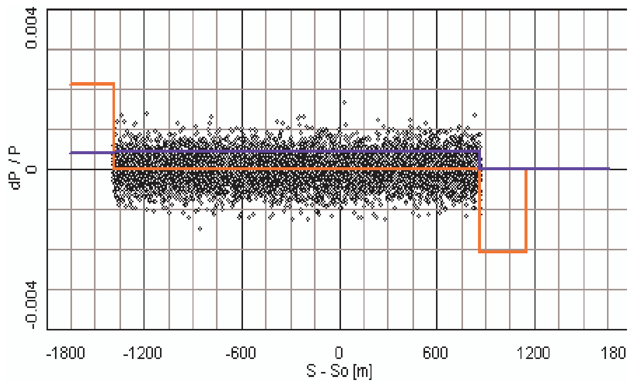

Figure 7: Longitudinal distribution of model particles and barrier buckets in momentum space in BETACOOL. Initial distribution.

In this example, the simulation entailed reproducing a ‘squeeze’, where the beam bunch length is reduced relatively slowly. The time sequence was the following: wait for 5 seconds (from initial distribution), squeeze for 5 seconds, and wait for 10 seconds. Longitudinal beam profiles and momentum spread distributions in the final state are shown in Fig.8.

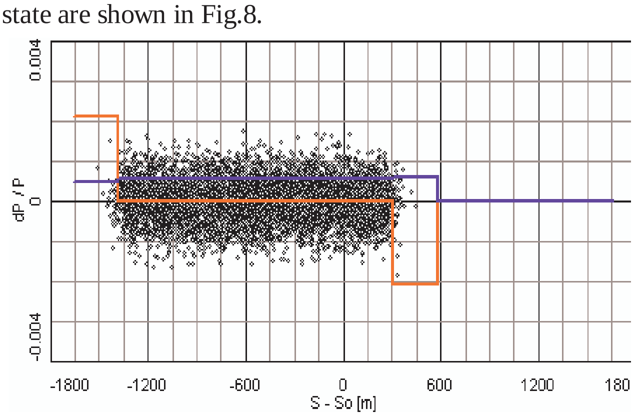

Figure 8: Longitudinal distribution of model particles and barrier bucket in momentum space in BETACOOL. Final distribution after 25 sec.

Although the agreement appears to be pretty good, a slightly more quantitative analysis shows bigger discrepancies than for the stationary bucket case. The bunch length (defined as twice the rms) is 1438 m in ESME while it is 1479 m in BETACOOL (a ~3% relative difference). This is actually visible in Fig.9 in which one can see that the edges of the beam fall more rapidly in the BETACOOL case than in the ESME simulation, hence extending the ‘flat-top’. Also, even though the momentum distributions overlap very well in Fig.10, the rms momentum spreads are somewhat different: 4.67 MeV/c in ESME and 4.49 MeV/c in BETACOOL. The relative difference here is 4%, while the momentum spread agreed almost exactly for the stationary bucket case.

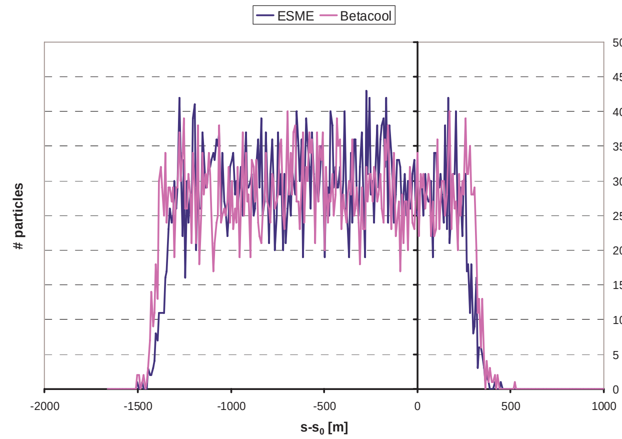

Figure 9: Longitudinal beam profiles along the longitudinal coordinate. Final distribution.

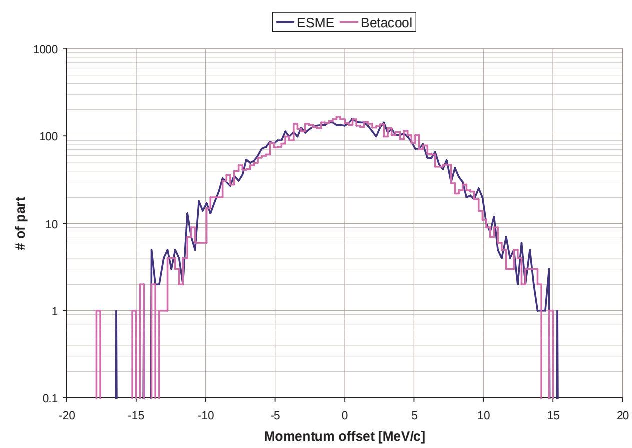

Figure 10: Momentum spread distributions. Final distribution.

The two examples above illustrate that the implementation of longitudinal dynamics in BETACOOL is appropriate and agrees with the well established (and more advanced) code ESME, even in the case of moving barriers, which is more difficult to reproduce correctly (for instance, it is important to make sure that the integration step size is much smaller than the synchrotron period).

USING BARRIER BUCKET MODEL FOR RHIC COLLIDING EXPERIMENT

The longitudinal synchrotron motion realized in the BETACOOL program can be used for different tasks. The specific particle distribution in RHIC colliding experiments is formed when the initial momentum spread is much higher than the amplitude of the RF voltage. Note that in real RHIC experiments satellites are not as strongly pronounced as in Fig. 11 [3]. In Fig. 11, the distribution was intentionally strongly cooled to produce clearly pronounced satellites for illustration purposes.

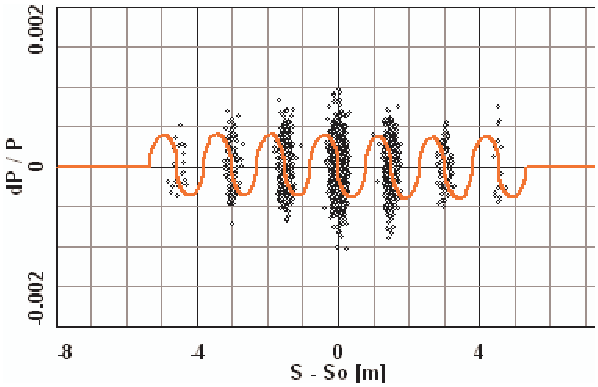

Figure 11: Particle distribution with satellites in longitudinal phase space.

Additional satellites are produced in neighbouring potential wells are due to the sum of two harmonics of the RF system (Fig.12). The luminosity calculation for such a specific particle distribution was implemented in the BETACOOL program (Fig.13). The time dependence can be calculated via the particle velocity. Note that the interaction region is twice smaller than the total bunch length.

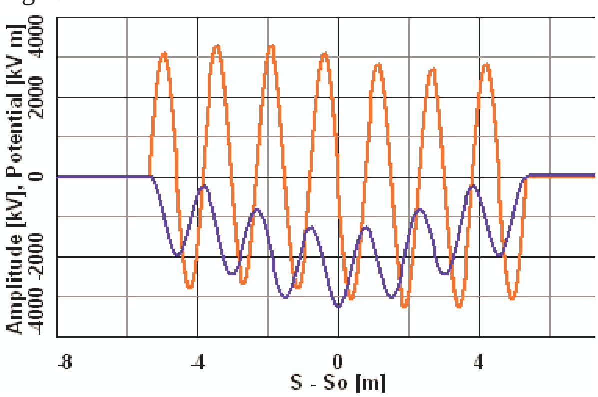

Figure 12: Distribution of the RF voltage as the sum of low and high RF harmonics (red line) and corresponding potential distribution (blue line).

The vertex cut defines as the interaction region where the colliding events can be registered (Fig.14). This algorithm permits to numerically calculate the hourglass effect for an arbitrary particle distribution in the longitudinally phase space. For example for Fig.13 and Fig.14 the hourglass effect factors are 0.76 and 0.75, respectively.

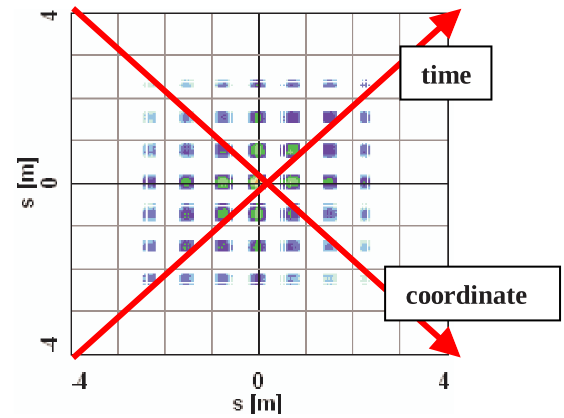

Figure 13: Luminosity distribution for colliding experiments without vertex cut.

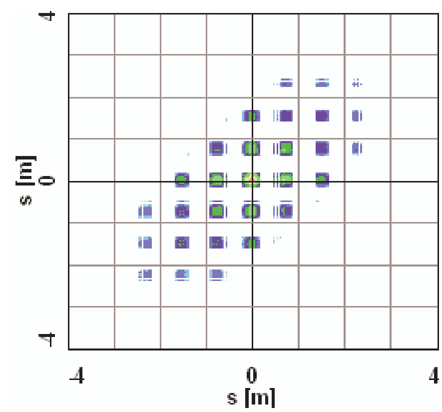

Figure 14: Luminosity distribution for colliding experiments with vertex cut (±100 cm).

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