STUDY OF COLLECTIVE EFFECT IN IONIZATION COOLING *

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Abstract

As a charged particle passes through a non-gaseous medium, it polarizes the medium and induces wake fields behind it. The interaction with wake fields perturbs the stopping power for the beam particles that follow. The perturbation strongly depends on the densities of both the incident beam and the medium. To understand this collective effect, detailed studies have been carried out. Both analytic and simulation results are obtained and compared.

INTRODUCTION

The study of the physics of a charged particle passing through a non-gaseous medium is of long history [1, 2, 3]. For a single particle, if its momentum is high enough, it will lose energy through both ionization process and density effect. The latter has been systematically studied. For a beam consisting of a large number of particles, the interaction among the beam particles should also be taken into account in order to describe the process correctly.

Essentially, the density effect is introduced by the polarization of the medium. The electric fields from the polarized medium molecules generate wake fields behind the incident particle, which perturb the motion of the beam particles following. If the particle density of the beam is high enough, the wake will enhance the stopping power for beam particles, and may possibly increase the rate of ionization cooling.

In this article, we derive the expressions for the wake electric field introduced by a single incident charged particle and its perturbation on the stopping power. This is extended to a two-particle system and a multi-particle system with various distributions. The comparison with simulations is next demonstrated. Finally, the damping mechanism on the wake is discussed, and its effect on the stopping-power enhancement is found to be important.

WAKE ELECTRIC FIELD

First, let us focus on a single particle of charge e moving with velocity v in the z direction, where $-e$ is the electron charge. The particle is at longitudinal position $z = z_1$ at time $t = 0$. Cylindrical coordinates are used with $\vec{\rho}$ denoting the transverse directions. The scalar potential generated by both the incident particle and polarized medium in the Coulomb gauge is given by

$$\phi(\vec{r},t) = \frac{e}{\pi v}\int d\omega \int \frac{\kappa d\kappa J_0(\kappa\rho)}{\kappa^2+\omega^2/v^2}\frac{e^{i\frac{\omega}{v}(z-z_1-vt)}}{\varepsilon(k^2,\omega)}, \quad (1)$$

where J_0 is the Bessel function. The wave number vector is denoted by $\vec{k} = (\vec{\kappa}, k_z)$ and the frequency by ω. In the above, the integration over k_z and $\vec{\kappa}\cdot\vec{\rho}$ have already been carried out. The polarization of the medium is described by the dielectric constant, which in a dispersive medium takes the form

$$\varepsilon(k^2,\omega) = 1 - \omega_p^2 \sum_j \frac{f_j}{\omega^2 - \omega_j^2 + i\omega\Gamma_j}, \quad (2)$$

where f_j is the fraction of bound electrons that oscillate with the bound frequency ω_j and damping rate $\frac{1}{2}\Gamma_j$, with $\sum_j f_j = 1$. In the above, $\omega_p = \sqrt{4\pi n_e e^2/m_e}$ is the plasma frequency, where m_e is the electron mass and n_e the electron density. We make the assertion that ω_p is much larger than the bound frequencies and damping rates.[1] Then $\frac{1}{2}\Gamma_j$ can be replaced by the infinitesimal positive number ϵ, leading to

$$\frac{1}{\varepsilon} = \frac{\omega^2}{(\omega+i\epsilon)^2-\omega_p^2} = \frac{\omega^2}{(\omega-\omega_p+i\epsilon)(\omega+\omega_p+i\epsilon)}. \quad (3)$$

Contour integration over ω can now be performed giving

$$\phi(\vec{r},t) = e\int d\kappa \frac{\kappa^2 J_0(\kappa\rho)}{\kappa^2+\omega_p^2/v^2} e^{-\kappa|z-z_1-vt|} + \frac{2e\omega_p}{v}\int d\kappa \frac{\kappa J_0(\kappa\rho)}{\kappa^2+\omega_p^2/v^2}\sin\frac{\omega_p}{v}(z-z_1-vt)\theta(z_1+vt-z). \quad (4)$$

The limits of integration are from $\kappa = 0$ to

$$\kappa = \frac{\omega_p}{v}\sqrt{x_m^2-1} \quad \text{with} \quad x_m = \frac{2\gamma m_e v^2}{\hbar\omega_p}, \quad (5)$$

which corresponds to the maximal momentum transfer in a collision. In the above, $\gamma = 1/\sqrt{1-v^2/c^2}$, c is the velocity of light, and $\hbar$ is the reduced Planck constant. The second term in Eq. (4) is the potential coming from the polarization of the medium, and the first term is the *medium-modified self-field* of the incident charged particle. The longitudinal and transverse electric fields derived from the second term vanish in front of the particle and are therefore the wake fields. Behind the particle, they take the form:

$$E_z^w(\vec{r},t) = -\frac{2e\omega_p^2}{v^2}\int d\kappa \frac{\kappa J_0(\kappa\rho)}{\kappa^2+\omega_p^2/v^2}\cos\frac{\omega_p(z-z_1-vt)}{v},$$
$$E_\rho^w(\vec{r},t) = +\frac{2e\omega_p^2}{v^2}\int d\kappa \frac{\kappa^2 J_1(\kappa\rho)}{\kappa^2+\omega_p^2/v^2}\sin\frac{\omega_p(z-z_1-vt)}{v}. \quad (6)$$

Evaluating at the particle location ($z = z_1 + vt$, $\rho = 0$), we obtain the longitudinal field on axis,

$$E_z^w = -\frac{2e\omega_p^2}{v^2}\ln x_m. \quad (7)$$

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[1] We believe the bound frequencies are one order of magnitude smaller than ω_p in liquid hydrogen.

The corresponding energy loss per unit time or stopping power is

$$\frac{dW}{dt} = evE_z^w = -\frac{2e^2\omega_p^2}{v}\ln x_m. \quad (8)$$

Behind the particle, the electric wake can be very well approximated by extending the upper limit of the κ integrations to infinity, resulting in

$$E_z^w = \frac{2e\omega_p^2}{v^2}K_0\left(\frac{\omega_p\rho}{v}\right)\cos\left[\left(\frac{z-z_1}{v}-t\right)\omega_p\right],$$
$$E_\rho^w = \frac{2e\omega_p^2}{v^2}K_1\left(\frac{\omega_p\rho}{v}\right)\sin\left[\left(\frac{z-z_1}{v}-t\right)\omega_p\right], \quad (9)$$

with $K_{0,1}$ the modified Bessel functions of the second kind.

The vector potential contributes only to the medium-modified self-field when bound frequencies are neglected. The electric field derived from it consists of two parts: one part cancels the medium-modified stationary self-field from the scalar potential in Eq. (4), while the other represents the medium-modified self-field of a moving charge. The total self-field can be written as

$$E_z^s = e\int d\kappa\,\frac{\kappa^3 J_0(\kappa\rho)}{\kappa^2+\frac{\omega_p^2}{v^2}}e^{-\gamma\sqrt{\kappa^2+\frac{\omega_p^2}{c^2}}|z-z_1-vt|},$$
$$E_\rho^s = e\int d\kappa\,\frac{\gamma\kappa^2 J_1(\kappa\rho)}{\kappa^2+\frac{\omega_p^2}{v^2}}\sqrt{\kappa^2+\frac{\omega_p^2}{c^2}}\,e^{-\gamma\sqrt{\kappa^2+\frac{\omega_p^2}{c^2}}|z-z_1-vt|}.$$

In the absence of the medium ($\omega_p = 0$), it reduces to the familiar pancake self-field,

$$E_z^s = \frac{e\gamma Z}{(\rho^2+\gamma^2Z^2)^{3/2}},\quad E_\rho^s = \frac{e\gamma\rho}{(\rho^2+\gamma^2Z^2)^{3/2}}, \quad (10)$$

where $Z = z - z_1 - vt$. In the presence of the medium, the self-field decays very much faster with respect to Z. For a bunch with longitudinal and transverse radii $\gg v/\omega_p$, the self-field has almost no influence compared with the wake fields. Therefore we ignore it in the rest of our discussions.

TWO-PARTICLE SYSTEM

Now let us discuss the stopping power for a two-particle system. The particles are denoted by 1 and 2, respectively, with the charge density

$$\rho(\vec r,t) = e\left[\delta(\vec r-\vec r_1-\vec v_1 t)+\delta(\vec r-\vec r_2-\vec v_2 t)\right]. \quad (11)$$

The electric field at $(\vec r,t)$ is

$$\vec E(\vec r,\,t) = -\frac{i2e}{4\pi^2}\int d^3k\int d\omega\,\frac{\vec k e^{-i\omega t}}{\varepsilon k^2}\times$$
$$\times\left[e^{i\vec k\cdot(\vec r-\vec r_1)}\delta(\vec k\cdot\vec v_1-\omega)+e^{i\vec k\cdot(\vec r-\vec r_2)}\delta(\vec k\cdot\vec v_2-\omega)\right]. \quad (12)$$

The energy gained per unit time by the two particles are

$$\frac{dW_{1,2}}{dt} = -\frac{i2e^2}{4\pi^2}\int d^3k\frac{\vec k\cdot\vec v_j}{k^2\varepsilon(k^2,\vec k\cdot\vec v_{1,2})}\times$$
$$\times\left[1+e^{\pm i\vec k\cdot(\vec r_1-\vec r_2)\pm i(\vec k\cdot(\vec v_1-\vec v_2)t)}\right]. \quad (13)$$

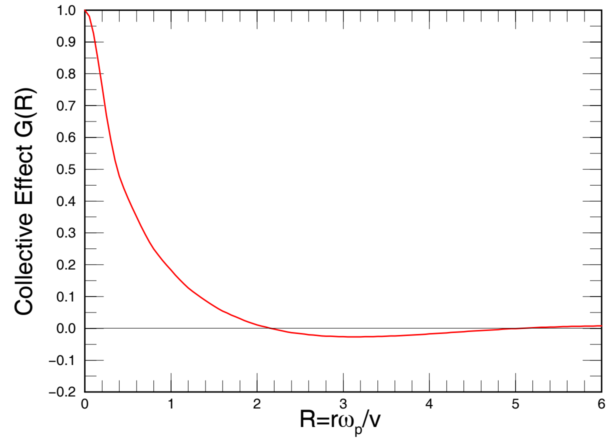

Figure 1: Stopping power enhancement due to collective wake effect on a two-particle system as a function of $R = r\omega_p/v$, where r is the separation between the two particles.

For the special case where $\vec v_1 = \vec v_2 = \vec v$, we have the *average* energy loss per particle per unit time or stopping power

$$\frac{dW}{dt} = -\frac{ie^2}{2\pi^2}\int d^3k\,\frac{\vec k\cdot\vec v}{k^2\varepsilon(k^2,\vec k\cdot\vec v)}\left[1+\cos(\vec k\cdot\vec r)\right],$$

where $\vec r = \vec r_1 - \vec r_2$. Here, only the imaginary part of $1/\varepsilon$ contributes. After averaging over all orientations of this two-particle system, we arrive at

$$\left\langle\frac{dW}{dt}\right\rangle_{\rm angles} = -\frac{e^2\omega_p^2}{v}\ln x_m\left[1+G(R)\right], \quad (14)$$

where the correlation function or stopping power enhancement is defined as

$$G(R) = \frac{-\frac{\sin Rx_m}{Rx_m}+\frac{\sin R}{R}-\mathrm{Ci}(Rx_m)+\mathrm{Ci}(R)}{\ln x_m}, \quad (15)$$

and $\mathrm{Ci}(x) = -\int_x^\infty dy\,\cos y/y$ is the cosine integral. Here, the distance of separation of the two particles, $R = r/a_I$, has been normalized to $a_I = v/\omega_p$ or the *interaction length*. The correlation function [4] as depicted in Fig. 1 shows that $G(R) = 1$ at $R = 1$ and decreases rapidly when $R \gg 1$ with oscillation period equal to the plasma wavelength $\lambda_p = 2\pi a_I$. It is interesting to point out that the averaging over all orientations cancels out all self-field contribution.

VARIOUS BEAM DISTRIBUTIONS

In this section we will discuss the stopping power enhancement for a particle in the center of a bunch as a result of the polarization wake. Various bunch distributions are used and their importance analyzed.

Uniformly Distributed Sphere

Let us start with a uniformly distributed spherical N_b-particle bunch of radius r_0. The extra collective stopping power G_t received by the particle at bunch center is obtained by integrating the two-particle correlation function $G(R)$ of Eq. (15) over all the particles in the bunch. We get

$$G_t = \frac{3N_b}{R_0^3}\int_0^{R_0} R^2 G(R)dR = N_b \frac{f(R_0) - f(R_0 x_m)}{\ln x_m}, \quad (16)$$

with $R_0 = r_0\omega_p/v$, the reduced bunch radius and

$$f(u) = \left(\frac{1}{u^3} + \frac{1}{u}\right)\sin u - \frac{\cos u}{u^2} - \mathrm{Ci}(u). \quad (17)$$

Since x_m is usually a very big number, the above can be readily approximated as

$$G_t \approx \frac{3N_b \sin R_0}{R_0^3 \ln x_m}. \quad (18)$$

As an example, consider a $\gamma = 2.2$ bunch containing $N_b = 1\times 10^{12}$ muons going through liquid hydrogen of density $\rho_{H_2} = 0.07099$ g/cm^{-3}. The electron density is $n_e = \rho_{H_2} N_A = 4.275\times 10^{28}$ m^{-3}, where N_A is Avogadro's number. The plasma frequency is therefore $\omega_p = 1.166\times 10^{16}$ s^{-1} leading to $x_m = 2.323\times 10^5$. If bunch is a uniformly distributed sphere of radius $r_0 = 1$ mm, $R_0 = 4.369\times 10^4$ and the envelope of G_t is ± 0.0029. However, since the bunch edge can never be made sharper than the interaction length $a_{_I} = 2.289\times 10^{-8}$ m, the rapid oscillation of G_t with R_0 with period λ_p implies the enhancement of stopping power from correlation is essentially zero.

Cylindrical Bunch

Let us consider next distributions having the separable form $f(z,\rho) = f_z(z) f_\rho(\rho)$. One example is a bunch with uniform distribution in the transverse direction, but that is tapered at both end longitudinally, or

$$f_z(z) = \frac{A_n}{\hat z}\left(1 - \frac{z^2}{\hat z^2}\right)^n, \quad f_\rho(\rho) = \frac{2\pi\rho}{\pi\hat\rho^2}, \quad (19)$$

where $A_n = \Gamma(n+\frac{3}{2})/[\sqrt{\pi}\Gamma(n+1)]$ for any $n > -1$, $\hat\rho$ is the transverse radius of the bunch, and $\pm\hat z$ are the longitudinal extents of the bunch. We can no longer apply the expression of the all-orientation-averaged correlation function $G(R)$ of Eq. (15), because the distribution is now different in different directions. Instead, we start from Eq. (9) to compute G_t, the collective stopping power enhancement for a particle at the center of the bunch, by the integration,

$$G_t \ln x_m = N_b \int_0^{\hat z} dz\, f(z,\rho)\cos\left(\frac{z}{a_{_I}}\right)\int_0^{\hat\rho} d\rho f_\rho(\rho) K_0\left(\frac{\rho}{a_{_I}}\right), \quad (20)$$

and obtain easily

$$G_t \ln x_m = \frac{\sqrt{\pi}A_n}{(\omega_p\hat\rho/v)^2}\left(\frac{2v}{\omega_p\hat z}\right)^{n+\frac{1}{2}} J_{n+\frac{1}{2}}\left(\frac{\omega_p\hat z}{v}\right). \quad (21)$$

We see clearly that there is an oscillation in the Bessel function $J_{n+\frac{1}{2}}$ which gives positive or negative enhancement depending very sensitively on the half bunch length $\hat z$. In fact, when n is an integer, the Bessel function reduces to sine and cosine with period λ_p, and the result will be similar to that of the uniform distribution in a sphere discussed earlier. We learn from Eq. (21) that the longitudinal and transverse beam sizes behave very differently, and it is the longitudinal that introduces the oscillations. To avoid oscillations, we need to go to a distribution without finite longitudinal boundaries.

Lorentzian Distribution

Let us keep the transverse distribution to be finite and uniform, but let the longitudinal distribution be

$$f_z(z) = \frac{z_1}{\pi}\frac{1}{z^2 + z_1^2}, \quad (22)$$

where z_1 is the half longitudinal length at half maximum. The correlation stopping power enhancement G_t for a particle at the bunch center is found to be

$$G_t \ln x_m = \frac{N_b e^{-z_1/a_{_I}}}{(\hat\rho/a_{_I})^2}. \quad (23)$$

If $\hat\rho = z_1 = 1$ mm, $\hat\rho/a_{_I} = z_1/a_{_I} = 4.37\times 10^4$. Then the stopping-power enhancement is $G_t = 45.3\times 10^{-19000}$, which is essentially zero. In order to have a more reasonable effect at the bunch intensity of 1×10^{12}, we require $\hat\rho/a_{_I} = z_1/a_{_I} = 20$. Then $G_t = 0.417$. If $\hat\rho/a_{_I} = z_1/a_{_I} = 25$, $G_t = 0.0018$ instead. However, one must remember that at $a_{_I} = v/\omega_p = 2.289\times 10^{-8}$ m, these examples correspond to $z_1 = 4.6\times 10^{-7}$ and 5.7×10^{-7} m, or bunches of sub-micron lengths. At this moment, the most aggressive cooling scheme proposed by Neuffer [5] is to have an eventual bunch length of 30 cm and transverse radii $r_0 = 50$ μm.

Tri-Gaussian Distribution

Let

$$f_z(z) = \frac{e^{-z^2/(2\sigma_z^2)}}{\sqrt{2\pi}\sigma_z} \quad \text{and} \quad f_\rho(\rho) = \rho\frac{e^{-\rho^2/(2\sigma_\rho^2)}}{\sigma_\rho^2}. \quad (24)$$

The stopping power enhancement can be integrated readily to the exponential function. However, when $\sigma_\rho/a_{_I} \gg 1$, it can be neatly reduced to

$$G_t \ln x_m = \frac{N_b e^{-(\sigma_z/a_{_I})^2/2}}{(\sigma_\rho/a_{_I})^2}, \quad (25)$$

which is much smaller than the case of the Lorentzian distribution. Here, we again see that the transverse and longitudinal behave every differently. Although both are Gaussian distributed, the longitudinal decay of the collective stopping power enhancement is Gaussian, while the transverse decay is $(a_{_I}/\sigma_\rho)^2$, which is very much milder.

Exponential Distribution

In order to achieve a larger stopping power enhancement, the distribution must roll off very rapidly from the center of the bunch. A possible distribution is the exponential

$$f_z(z) = \frac{e^{-|z|/z_1}}{2z_1} \quad \text{and} \quad f_\rho(\rho) = \frac{e^{-\rho/\rho_1}}{\rho_1}. \quad (26)$$

When $z_1/a_{_I} \gg 1$ and $\rho_1/a_{_I} \gg 1$, G_t is given by

$$G_t \ln x_m \approx \frac{\pi N_b}{8(\rho_1/a_{_I})(z_1/a_{_I})^2}. \quad (27)$$

Or $G_t = 3.81\times 10^{-4}$ at $\rho_1 = z_1 = 1$ mm. If either the bunch sizes are each reduced 5 times or the bunch intensity is increased by a factor of 125, the collective effect enhancement will become 4.8%, and thus more significant.

In Neuffer's scheme [5] with transverse bunch radii $\rho_1 = 50\ \mu$m and bunch length $z_1 = 30$ cm, we get a stopping power enhancement of $G_t = 8.47 \times 10^{-8}$, which increases rapidly to 7.62×10^{-3} when the bunch length is reduced to $z_1 = 1$ mm. Here, again we notice that the longitudinal bunch length is the most important factor that determines the enhancement. For example, if we divide a bunch up longitudinally into bunchlets of shorter lengths but with the particle density unchanged, the stopping power enhancement for the bunch particles will be increased.

COMPARISON WITH SIMULATION

OOPICPro [6] developed by Tech-X Corporation is able to simulate a charged particle beam passing through matter. We simulate a $\gamma = 2.2$ tri-Gaussian muon bunch with rms radii 1 mm traveling through a plasma medium of electron density $n_e = 4.28 \times 10^{18}$ m^{-3} (which is very much less than that in liquid hydrogen). The peak beam density is 5×10^{19} m^{-3}, corresponding to total bunch intensity $N_b = 0.787 \times 10^{12}$. The longitudinal wake is shown in Fig. 2. The same wake can be computed by integrating our derived wake in Eq. (9) over all the particles in the bunch in a fake liquid hydrogen medium of the same low density. The field patterns in both the longitudinal and transverse directions agree with the simulation results. For example, the oscillation wavelength is $\lambda_p = 1.43$ cm corresponding to the plasma frequency of $\omega_p = 1.17 \times 10^{11}$ s^{-1}. However, OOPICPro gives the peak longitudinal wake electric field 1.02×10^8 V/m, while the computed one in Fig. 3 gives 4×10^8 V/m. The discrepancy may come from the fact that OOPICPro treats the medium as a fully ionized plasma while we treat the medium as a liquid with polarized molecules. This difference will be examined next.

RELAXATION AND DAMPING

We need to address the question of relaxation or damping of the plasma oscillation to determine whether the oscillatory wake can be established and sustained.

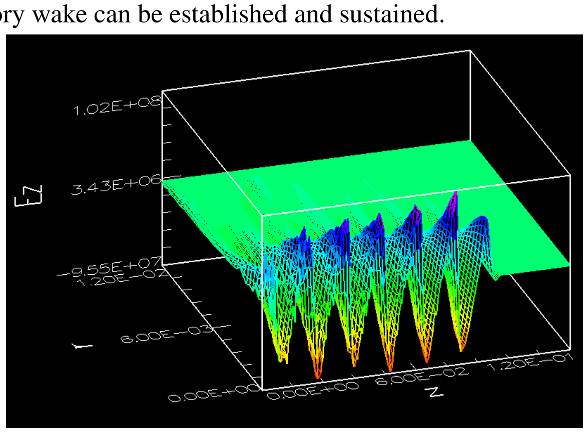

Figure 2: (Color) The longitudinal wake electric field behind an incident muon bunch simulated by OOPICPro with electron density 4.28×10^{18} m^{-3} and peak particle density 5×10^{19} m^{-3}. The bunch is tri-Gaussian distributed of rms radii 1 mm in all directions with $\gamma = 2.2$ muons. Both the longitudinal and transverse axes (z and r) are in m while E_z is in V/m.

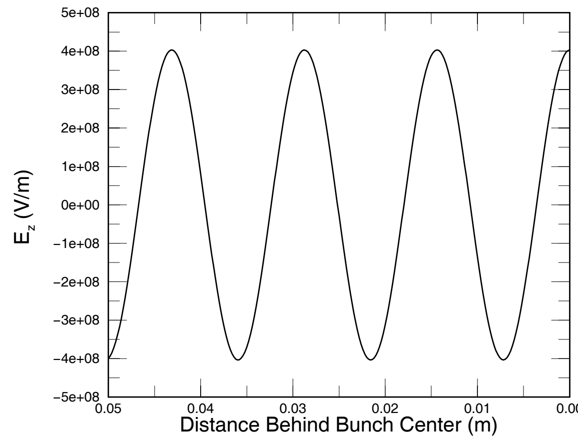

Figure 3: Computed longitudinal wake behind a tri-Gaussian bunch. Bunch sizes and medium density are the same as in the OOPICPro simulation in Fig. 3. The bunch center is at the origin and 2×10^5 macro-particles have been used.

Cold Plasma

In a fully ionized plasma, electrons are free to move around as a thermal gas. In the presence of the incident muon beam, the electrons are driven into oscillations about the background ions at plasma frequency. At the same time these electrons collide with the ions. If a collision takes place within one period of plasma oscillation, the plasma oscillation will be disturbed. Thus collision with ions serves as a damping mechanism. The collision frequency of an electron with the ionic background is given by [7] $\nu_e = 2.9\times10^{-9} n_i T_e^{-3/2} \ln\Lambda$ s^{-1}, where n_i is the ionic density in m^{-3}, $\ln\Lambda \approx 10$ is the cutoff logarithm, and T_e is the thermal temperature of the electrons in eV. For the above OOPICPro simulation, we substitute $n_i = 4.28\times10^{18}$ m^{-3} and $T_e = 1.72\times10^{-3}$ eV (corresponding to 20 K) to obtain $\nu_e = 1.74\times10^{12}$ s^{-1}, which is comparable to the plasma frequency of $\omega_p = 1.17\times10^{11}$ s^{-1}. So the wake will be heavily perturbed. We do see some damping of the simulated wake in Fig. 2, but not as heavy as estimated. The discrepancy may come from the relatively higher temperature of the ionized electrons, which may not even be in thermal equilibrium. Since the collision frequency increases as n_i while the plasma frequency increases with $\sqrt{n_i}$, the damping of plasma oscillation will become more severe as the plasma density increases. Such a simulation has been carried out using OOPICPro with the plasma density increased 10^4-fold to $n_i = 4.28 \times 10^{22}$ m^{-3} while all other parameters remain unchanged. The result in Fig. 4 shows that the on-axis longitudinal wake is damped almost immediately as soon as it is generated. The maximum $|E_z|$ is less than 8.5×10^5 V/m as compared with 1×10^8 V/m in Fig. 2.

Liquid Hydrogen

In producing Fig. 3, we employed the wake expression without consideration of damping and the electron density lowered to $n_e = 4.28\times10^{18}$ m^{-3} so as to compare with simulation. Now let us come back to the case of real liquid hydrogen where the bound-electron density is

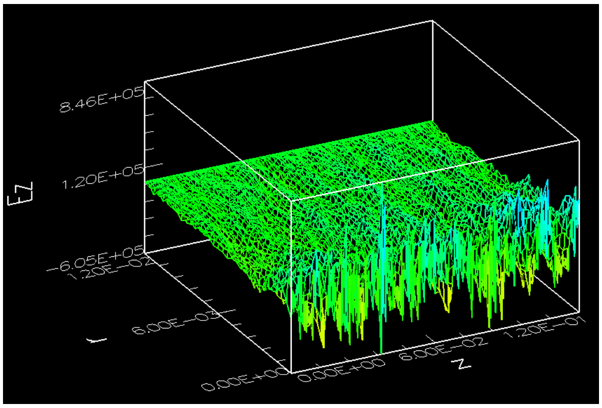

Figure 4: (Color) OOPICPro simulation for a muon beam in a plasma. All parameters are the same as in Fig. 2, except that the plasma density has been increased to 4.28×10^{22} m^{-3}. The z- and r-axes are in m, while E_z is in V/m.

$n_e = 4.270 \times 10^{28}$ m^{-3} and study possible damping of the wake. At $\gamma = 2.2$, the energy loss in liquid hydrogen is $dW/dx = -4.5$ MeV-cm^2-g^{-1}. For a bunch with cross-sectional radii $r = 1$ mm, consisting of 1×10^{12} muons, the density of ionized electrons is

$$n_{ei} = -\frac{\rho_{H_2} N_b dW/dx}{\pi r^2 I}, \qquad (28)$$

where the medium density is $\rho_{H_2} = 0.07099$ g/cm^3 and the ionization energy is $I = 35$ eV. We obtain $n_{ei} = 2.9 \times 10^{23}$ m^{-3}, which is five orders of magnitude less than the density of the bound electrons and can therefore be neglected. The damping of the wake can come from collisions between the neutral polarized hydrogen molecules, since directional changes of the polarized molecules will disturb the plasma oscillations. The mean thermal velocity of the H_2 molecules at 20 K is $v_{H_2} = 235$ m/s (3 degrees of freedom considered). The typical cross section for the hydrogen molecule in the hard-ball model is $\sigma_{H_2} \approx 3 \times 10^{-20}$ m^2. Thus the collision frequency is $\nu_{H_2} \approx 4 n_{H_2} \sigma_{H_2} v_{H_2} \approx 45 \times 10^{11}$ s^{-1}, with n_{H_2} the density of the H_2 molecules. This is still many order smaller than the plasma frequency of $\omega_p = 1.17 \times 10^{16}$ s^{-1}.

Another possibility of damping comes from the damping rates of the bound frequencies of the H_2 molecules. We asserted earlier that the bound frequencies ω_j are an order of magnitude smaller than ω_p. It is reasonable to assume that the damping rates $\frac{1}{2}\Gamma_j$ of the bound frequencies are the same order of magnitude as ω_j. Let us simplify the problem by including only one damping rate $\frac{1}{2}\Gamma$. Then ϵ in Eq. (3) will be replaced by $\frac{1}{2}\Gamma$, and there will be the extra factor of $e^{-\Gamma Z/2v}$ in the wake expressions of Eqs. (6) and (9), where $Z = z - z_1 - vt$. Since $2v/\Gamma \ll \sigma_z$, the bunch length, in the computation of stopping power enhancement in Eq. (20), the longitudinal beam distribution can be replaced by the peak beam density. Taking the tri-Gaussian distribution as an example, instead of Eq. (25), we obtain

$$G_t \ln x_m \approx \frac{N_b}{\sqrt{2\pi}(\sigma_\rho/a_I)^2(\sigma_z/a_I)} \frac{\Gamma}{2\omega_p}. \qquad (29)$$

For a beam with $\sigma_\rho = \sigma_z = 1$ mm and $N_b = 1 \times 10^{12}$, $\Gamma/2\omega_p \sim 0.1$ implies $G_t \sim 3.9 \times 10^{-5}$. In addition, if the transverse distribution is exponential, $(\sigma_\rho/a_I)^2$ in the denominator is replaced by $4(\sigma_\rho/a_I)/\sqrt{2\pi}$ and we have $G_t \sim 190\%$ instead. In short, the enhancement becomes much larger in the presence of some amount of damping.

CONCLUSIONS

The perturbation of stopping power due to collective effect as a charged particle beam traverses a medium is studied in detail. This effect is introduced by the polarization of the medium and depends on a variety of factors such as beam distribution, beam density, and medium density.

The magnitude of the collective perturbation is fundamentally determined by the ratio of the separation of beam particles and the interaction length in the polarized medium, which is also a function of the velocity of incident particles. As this ratio decreases, the collective effect becomes more significant.

The damping of the wake also plays an important role in the wake field. Without any damping consideration, the wake oscillates sinusoidally with period $\lambda_p = 2\pi a_I$. Since the average separation of the incident beam particles is usually much larger than the interaction length, the wake field perturbation on stopping power is negligibly small. Damping comes from two sources: one is the collision rate between absorber molecules, which is slow and insignificant, the other is the damping rates of the bound frequencies of absorber electrons. Under certain circumstances, a shorter damped wake enhances collective perturbation.

The model used in the analysis employs the dielectric constant ε in the form of Eq. (3) where bound frequencies are considered small and neglected. Further analysis should take account of the contribution of bound frequencies to the wake and their effects on the stopping-power enhancement should be fully investigated.

Whether an enhancement of stopping power will increase the ionization-cooling rate has not been proven. In our future analysis, we will study the perturbation on the stopping power for particles traveling at various angles relative to the traveling direction of the bunch center. This will allow us to determine precisely the effect on the cooling rate of the beam due to interaction with the wake generated in the medium.

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