WEDGE ABSORBER SIMULATIONS FOR THE MUON IONISATION COOLING EXPERIMENT

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Abstract

In the Muon Ionisation Cooling Experiment (MICE), muons are cooled by ionisation cooling. Muons are passed through material, reducing the total momentum of the beam. This results in a decrease in transverse emittance and a slight increase in longitudinal emittance, but overall reduction of 6D beam emittance.

In emittance exchange, a dispersive beam is passed through wedge-shaped absorbers. Muons with higher energy pass through more material, resulting in a reduction in longitudinal emittance as well as transverse emittance. Emittance exchange is a vital technology for a Muon Collider and may be of use for a Neutrino Factory.

In this paper, we show that even in the straight solenoidal lattice of MICE, emittance exchange can be demonstrated. In order to achieve this, a muon beam is passed through a wedge shaped absorber; the input beam distribution must be carefully selected to accomodate strong non-linear effects due to chromatic aberrations in the solenoid lattice, which we achieve using a polynomial weighting function to select beam moments.

EMITTANCE EXCHANGE IN THE MUON IONISATION COOLING EXPERIMENT

Ionisation cooling is achieved in the Muon Ionisation Cooling Experiment (MICE) [1] baseline by the placement of absorbing material in the beamline. The absorbing material removes beam momentum, which is replaced only in the longitudinal direction by RF cavities, resulting in a net reduction of emittance. Low-Z materials must be used as absorbers together with carefully designed beam optics. This minimises the effects of multiple Coulomb scattering, which tend to reduce the cooling effect. Overall, transverse emittance is reduced to some equilibrium point while longitudinal emittance stays the same or increases slightly due to stochastic processes in the energy loss.

In this note we consider using MICE to observe a phenomenon known as emittance exchange. In emittance exchange a dispersive beam is passed through a wedge-shaped absorber. Muons with higher energy pass through more material and experience greater momentum loss. In this way the longitudinal emittance of the beam can be reduced either in addition to, or even instead of transverse emittance reduction. Emittance exchange is vital to a Muon Collider and has been considered as an upgrade option to the Neutrino Factory. Ring coolers [2], Helical coolers [3] and Guggenheim coolers [4] have been proposed to perform emittance exchange and longitudinal cooling using a simple wedge or a truncated wedge.

In MICE muons will be passed one-by-one through a short section of ionisation cooling equipment and the six-dimensional phase space vector of each muon will be measured to high precision, enabling the measurement of reduction in 2D, 4D and 6D emittance due to ionisation cooling. The position and momentum of individual muons is determined by high resolution scintillating fibre spectrometers before and after the cooling apparatus. Fast time-of-flight counters measure time-of-flight between the upstream and downstream spectrometers, which together with the spectrometers enables reconstruction of the full 6D phase space vector of individual muons. Rejection of beam impurity before the cooling channel will be achieved using a pair of time-of-flight counters together with a threshold Cherenkov counter. After the cooling channel decay electrons will be identified using the time-of-flight through the experiment together with an Electron-Muon-Ranger.

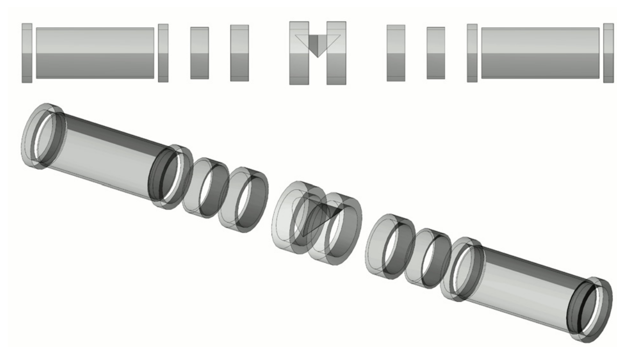

Figure 1: The geometry as simulated in G4MICE code: side and 3D view. The wedge absorber and coils are shown. The total length of the Step IV layout is just over 7.5 m, inner radius of the coils is 258 mm. A steel, cylindrical beam pipe with a 232 mm aperture was also included in simulations but is not shown in these figures.

Wedge Geometry

We study the use of a simple wedge-shaped absorber in a straight solenoid channel. The geometry considered is shown in Figure 1. The case considered here is MICE Step IV, where MICE is operated in a mode without RF cavities. RF cavities will be added to MICE at a later stage, but

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it is expected that the non-linearities discussed below will be worse for a longer cooling section and swamp any cooling measurement. In addition, the thin end of the wedge absorbers will enable dark currents emerging from the RF cavities to enter the detector systems, which may swamp any muon signal.

NON-LINEARITY OF BEAM TRANSPORT

Solenoids have natural chromatic aberrations at 2nd order and spherical aberrations at 3rd order that are significant for the large emittances typical of MICE ($\varepsilon_\perp = 7$ mm, $\varepsilon_\parallel = 0.3$ ns). By carefully matching the input beam β function to the lattice β function and using a cylindrically symmetric beam, emittance growth from these non-linearities is suppressed in transverse phase space, and in longitudinal phase space tightly packed RF cavities and damping from the absorbers prevents emittance growth from getting out of hand.

In this context, introducing a dispersion into the beam might be expected to ruin the cooling performance of the lattice and indeed this can be observed. As expected, in the linear approximation the dispersion D undergoes a Larmor rotation between x and y spaces in addition to the usual evolution of D and D'. Unfortunately this Larmor rotation is momentum-dependent in second order leading to the loss of dispersion from the beam and large emittance growth for energy spreads typical of MICE (20 MeV rms).

The situation is worse when we consider a non-ideal beam. In MICE, we attempt to generate muons from pions generated by a target dipping into the ISIS proton synchrotron. These particles are transported through a system of quadrupoles, dipoles and a single 5 T solenoid to the entrance of the MICE cooling apparatus. While this beamline allows us some flexibility in the muon distribution that is presented to the cooling channel, the control over input dispersion is limited, and the selection of appropriate Twiss parameters may be challenging.

One solution to this issue is to carefully select a set of particles at input to MICE such that the beam is well behaved at the absorber and emittance growth is minimised, and this is the method that we study in this note.

NO MATERIAL

We consider tracking of several cases through the MICE magnetic fields in the absence of material. In the first instance, we take an input beam typical of simulations of the MICE beamline and pass it through the cooling channel. The beam transverse and longitudinal β functions are not well-matched to the cooling channel and the dispersion vector is poorly controlled, as shown by the *Mismatched* curve in Figure 2. This leads to roughly 10% emittance growth in each dimension due to non-linear effects in the beamline, which is enough to mask any cooling signal.

In order to find an ideal beam that does not suffer from these effects, we generate a perfect, Gaussian beam at the

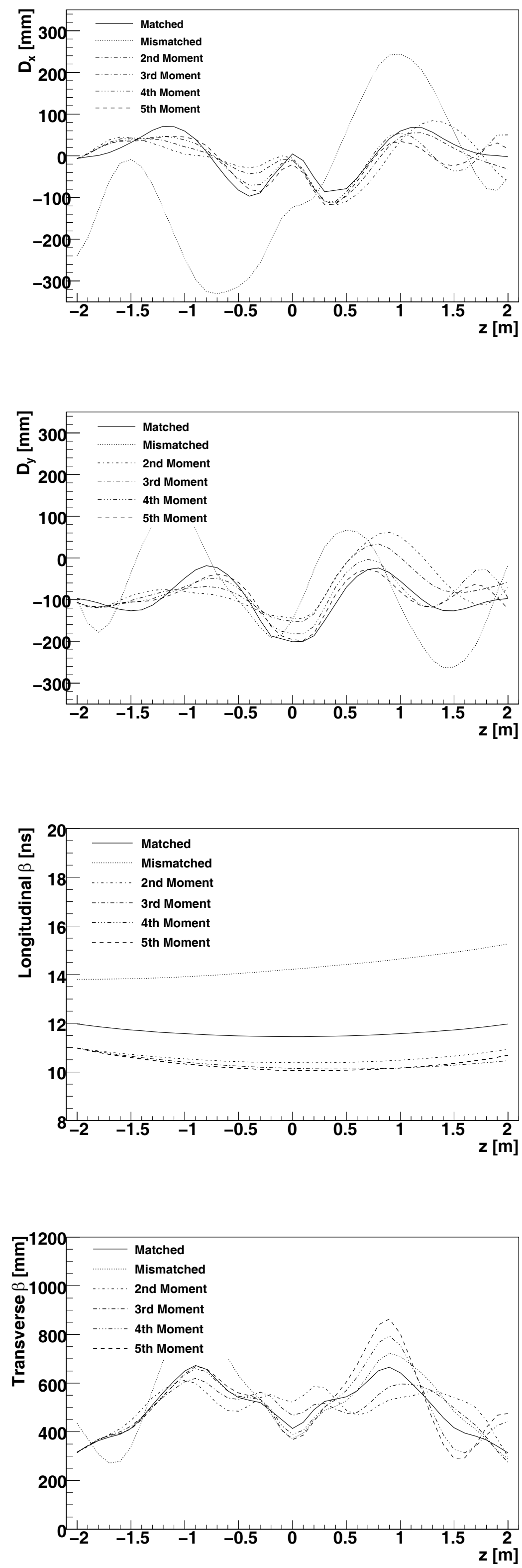

Figure 2: Dispersion in x and y, longitudinal β function and transverse β function vs position in the cooling channel. Plots are made for the ideal beam, a realistic (but mismatched) beam, and the realistic beam with selection of moments up to various orders.

centre of the cooling channel and then pass it to the downstream end. We then apply a z and t parity operation in order to transform the beam to a forward-going beam at input to the cooling channel. By selection of an initial beam with $D' = 0$ and $\beta' = 0$, the evolution of the beam is ensured to be symmetric about the centre of the magnetic lattice. Indeed, it can be seen from the *Matched* curve in Figure 2 that the evolution of linear and non-linear terms is symmetric in this case, such that the optical functions are symmetric about $z = 0$.

Statistical Weighting

In order to demonstrate the cooling channel experimentally, we need to select a beam that matches the parameters outlined above, from experimental data. We achieve this by applying a set of statistical weightings that map the input beam distribution from the beamline to the ideal input beam distribution detailed above. The idea is to count some particles several times to enhance the beam density in certain regions of phase space at the upstream detector, so that the probability distribution of the realistic beam is similar to the probability distribution of the ideal beam at input.

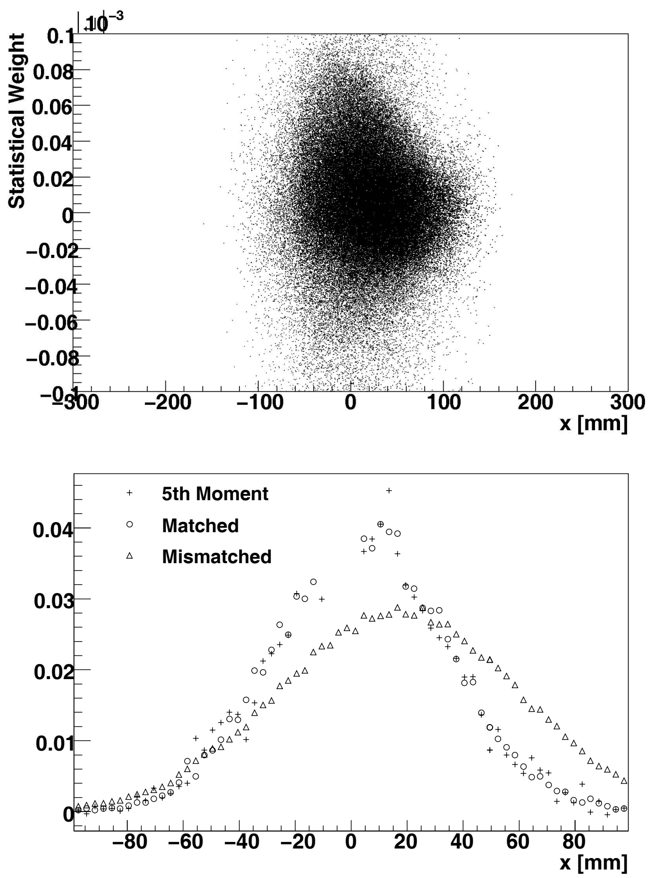

Figure 3: (top) The statistical weights applied as a function of x and (bottom) the ideal distribution in x and the mismatched distribution in x before and after weighting with a 5th order polynomial.

In this case, we attempt to select the moments of the beam using a polynomial weighting function [5]. Statistical weights are found by comparing the moments of the realistic beam with the moments of the ideal beam. We apply a weighting to each particle w^i, where $w(\vec{u})$ is a polynomial in the six-dimensional phase space vector, such that the weighting w^i of the i^{th} particle with phase space vector $\vec{u}^i = (x^i, y^i, t^i, p_x^i, p_y^i, E^i)$ is given by

$$w^i = a_0 + \sum_j a_j u_j^i + \sum_{k_1 k_2} a_{k_1 k_2} u_{k_1}^i u_{k_2}^i + \ldots \quad (1)$$

Applying this weighting to each particle gives us a set of weights that maps the moments of the realistic beam to the moments of the ideal beam, and by extension the distribution of the realistic beam to the distribution of the ideal beam. The projection of the beam distribution to the x axis is shown in Figure 3, together with a scatter plot of the weightings as a function of x. The statistical weights are generally higher in the negative x region, indicating that a statistical enhancement of the beam is required in this region. This is confirmed when we study the x distribution, where we see that the realistic beam is statistically depleted in the negative x region. After weights are applied to select 5th moments, the realistic beam distribution matches the ideal beam distribution rather more closely.

In principle this technique enables us to select moments up to arbitrary order, although in this note we only work up to 5th moments due to computational constraints. The results are satisfactory although not quite ideal - the optical functions evolve in a more controlled manner, and importantly the considerable emittance growth has been avoided.

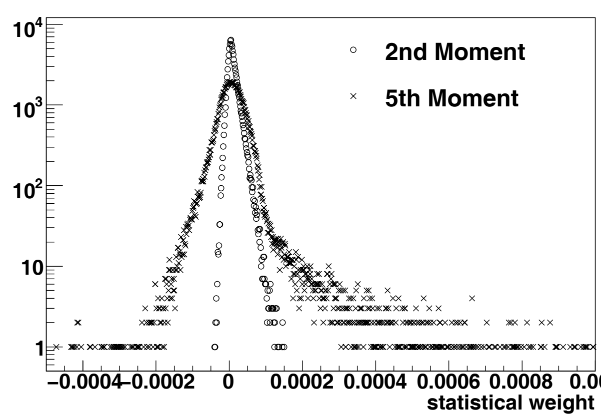

Figure 4: Distribution of statistical weightings required to select a beam for 1st through 2nd moments and for 1st through 5th moments. Note the long tail when higher moments are selected.

The weighting required for selection of 2nd and 5th moments is shown in Figure 4. At higher moments, rather large weightings are required, and it is expected this will have a detrimental effect on the statistical error of the measurement. The magnitude of this error is still to be determined.

WEDGE INCLUDED

Finally the case where a wedge is included is studied. The cooling performance of the various beams discussed above is assessed for a polyethylene wedge with a 90° opening angle. The on-axis length of the wedge has been chosen so that particles lose energy of about 11 MeV/c, which is similar to the energy gain that can be restored by the full MICE experiment.

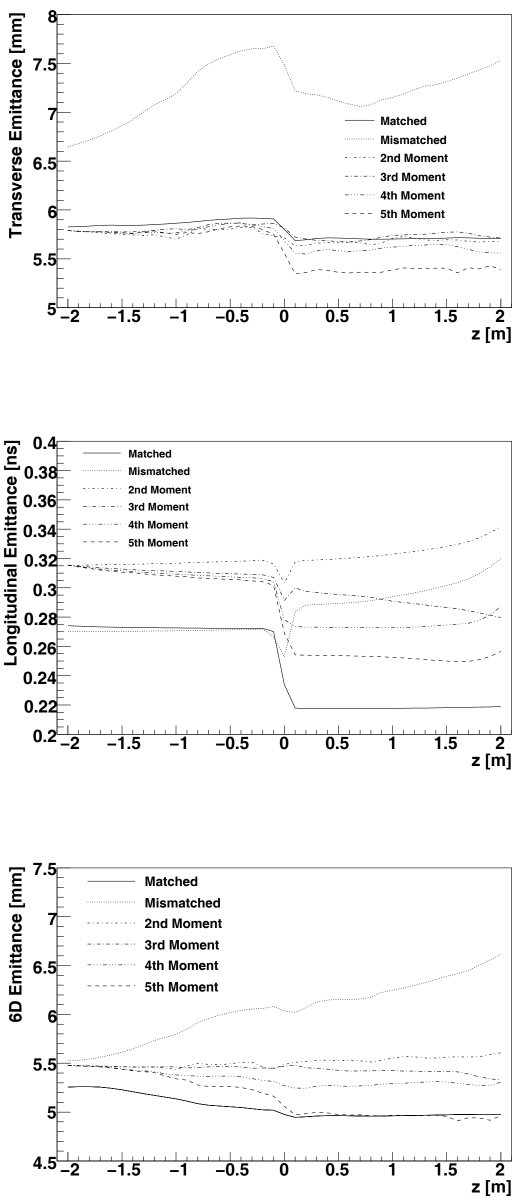

Figure 5: Evolution of transverse, longitudinal and 6d emittance as a function of z. Plots are made for the ideal beam, a realistic (but mismatched) beam, and the realistic beam with selection of moments up to various orders. Note that the realistic beam sees significant optical emittance growth when no statistical weighting is applied.

The emittance evolution is shown in Figure 5. It is clear that for the unweighted beam, the non-linearities ruin the cooling in both transverse and longitudinal phase space. After the weightings are applied there is now a clear cooling signal in both transverse and longitudinal phase spaces, that can be clearly attributed to the wedge. The full six dimensional emittance change is obscured slightly by emittance reduction due to non-linear terms in the optics, although a signal can still be observed in addition to the emittance evolution due to non-linearities. It may be possible to improve the weighting algorithm used here to achieve a closer match between the real beam, after selection, and the ideal case.

CONCLUSION

Emittance exchange is a powerful technique that may be of use for a Neutrino Factory and is vital to a Muon Collider. Observation of this phenomenon in the MICE setup would normally be impossible due to the presence of catastrophic beam blow up from non-linearities in the beam optics. The use of beam selection or statistical weighting presents us with the opportunity to make measurements over a wider range of parameters than can be generated with the MICE beam, and in this case it even enables us to take some control over the non-linear beam optics that is inherent in high emittance beams in solenoidal lattices. This enables us to observe emittance exchange.

REFERENCES

[1] D. Li, Status of the International Muon Ionization Cooling Experiment (MICE), Proc. COOL 2009.

[2] V. Balbekov, Solenoid-Based Ring Coolers, MUCOOL Note 232, 2002.

[3] K. Yonehara, Progress of Helical Cooling Channel Design for Muon Colliders, Proc. COOL 2009.

[4] P. Snopok, G. Hanson, A. Klier, Recent Progress on the 6d Cooling Simulations in the Guggenheim Channel, International Journal of Modern Physics A, Volume 24, Issue 05, 2009.

[5] C. T. Rogers, Statistical Weighting of the MICE Beam, Proc. EPAC 2008.