# SIMULATIONS OF STOCHASTIC COOLING OF ANTIPROTONS IN THE COLLECTOR RING CR

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*Abstract*

The Collector Ring at FAIR will be equipped with pertinent stochastic cooling systems in order to achieve fast cooling of the hot secondary beams, antiprotons and rare isotopes, thus profiting from the repetition rate of the SIS100 synchrotron. Detailed simulations of the system performance are needed for optimization as well as input for the users of the CR pre-coooled beams, e.g. HESR. We presently focus on the antiproton cooling in the band 1-2 GHz. After a comprehensive overview, results from Fokker-Planck simulations with the CERN code of the momentum cooling of antiprotons will be presented. The performance of the betatron cooling of antiprotons, which has to proceed simultaneously with the momentum cooling, was calculated separately by means of an analytical model. First results and their implications will be discussed, including an outlook to future simulation work.

## INTRODUCTION

The main purpose of the Collector Ring (CR) within the FAIR project [1] is the fast reduction of the phase space occupied by the hot secondary beams. The latter are antiprotons at 3 GeV and rare isotope beams (RIBs) at 740 MeV/u, coming from the production targets in a very short (≈ 50 ns) bunch. At injection into the CR, they have the largest momentum spread and fill the transverse aperture. After bunch rotation and adiabatic debunching their momentum spread is reduced, whereas the transverse emittances remain unchanged. The reduced $\delta$p/p is a prerequisite for stochastic cooling. Otherwise, the effect of undesired mixing (see below) would exclude particles at the tails of the momentum distribution from being cooled. In order to meet the requirements of maximum production rate the CR stochastic cooling system has to strongly reduce all 3 phase subspaces, within 9 s for the antiprotons (with the option of 5 s after future upgrade) and 1 s for the highly charged RIBs (Table 1). The recent scenario according to which, in the first phase of the FAIR project, the pre-cooled antiprotons from the CR will be accumulated in the HESR instead of the RESR calls for 20% lower (if possible) final emittances and momentum spread than those in Table 1 in order to match the very small acceptances of the HESR [2].

The CR lattice is governed by the demands from stochastic cooling: (i) flexibility in setting different transition energy values for antiprotons and RIBs to reach an optimal compromise for the mixing parameters of the stochastic cooling, as explained below, (ii) accommodation of pickups and kickers in regions of appropriate dispersion, (iii) control of the horizontal and vertical betatron phase advance between pickups and kickers of the transverse stochastic cooling systems, (iv) reducing chromaticity over the whole momentum range.

Table 1: Requirements for the CR stochastic cooling

| | | |
|---|---|---|
| Antiprotons, 3 GeV, $10^8$ ions | | |
| | $\delta$p/p (rms) | $\epsilon_{h,v}$ (rms) $\pi$ mm mrad |
| Before cooling | 0.35 % | 45 |
| After cooling | 0.05 % | 1.25 |
| Phase space reduction | $9 \times 10^3$ | |
| Cooling down time | ≤9 s | |
| Cycle time | 10 s | |
| Rare isotopes, 740 MeV/u, $10^9$ ions | | |
| | $\delta$p/p (rms) | $\epsilon_{h,v}$ (rms) $\pi$ mm mrad |
| Before cooling | 0.2 % | 45 |
| After cooling | 0.025 % | 0.125 |
| Phase space reduction | $1 \times 10^6$ | |
| Cooling down time | ≤1 s | |
| Cycle time | 1.5 s | |

## OVERVIEW OF THE CR STOCHASTIC COOLING SYSTEM

### *Design criteria*

In a simplified model, one can write the stochastic cooling rate, e.g. for transverse emittance, as

$$\frac{1}{\tau_\perp} = \frac{2W}{N}\left[2gB|\sin(\mu_{pk})| - g^2(M+U)\right] , \quad (1)$$

where $W$ is the system bandwidth, $N$ is the number of particles in the beam, $g$ is the system gain, $U$ is the ratio of power densities of the system thermal noise to the Schottky signal. For transverse cooling the CR lattice satisfies the condition of proper betatron phase advance $\sin(\mu_{pk}) \approx \pm 1$ between pickup and kicker. The undesired mixing parameter B (mixing between pickup and kicker) can be written in the form $B = \cos(m_c\phi_u)$, where $m_c$ is the central harmonic in the band and $\phi_u = -2\pi\chi_{pk}\eta_{pk}\delta p/p$. At the beginning of cooling i.e. for the maximum total ($2\sigma$) momentum spread $m_c\phi_u \leq \pi/2$ , otherwise the cooling force changes sign i.e. heats up the beam. Here, $\chi_{pk} = (s_k - s_p)/C$ is the ratio of the path from pickup to kicker to the closed orbit circumference $C$, $\eta_{pk}$ is the local frequency slip factor between pickup and kicker. The desired mixing parameter $M$ (mixing between kicker and pickup) can be approximated as $M = (m_c|\eta|\delta p/p)^{-1}$, for the total ($2\sigma$) momentum spread, with the frequency slip factor of the ring $\eta = \gamma^{-2} - \gamma_t^{-2}$. This holds if the Schottky bands do not overlap. If they do, $M$=1.

For momentum cooling, the mixing aspects are similar. The Palmer-Hereward technique [3] is a special case of horizontal cooling. It uses a pickup at high dispersion and appropriately located kickers to correct the horizontal orbit displacement due both to betatron oscillation and momentum deviation, the latter correction corresponds to longitudinal cooling. The notch filter method [4] uses the dependence of the particle revolution frequency on its momentum deviation. The transfer function of an ideal filter (with infinitely deep notches), plus a 90° phase-shifter, is

$$H_{nf} = \frac{i}{2}(1 - e^{-i2\pi m\eta\delta p/p}) = -e^{-i\pi f/f_0}\sin(\pi f/f_0).$$

It has no effect at the exact harmonics $mf_0$ of the revolution frequency (notches) and accelerates/decelerates particles with wrong revolution frequency to the nearest harmonic of the correct revolution frequency. Prerequisite for filter cooling are not overlapping Schottky bands ($M$ >1), so that the particles are driven into the notches. For filter cooling the undesired mixing phase becomes [5] $\phi_{u,nf} = -\pi(2\chi_{pk}\eta_{pk} + \eta)\delta p/p$ i.e. the momentum acceptance of the system is further reduced. This dictates very small values of $\eta$ (and $\eta_{pk}$).

Antiproton beams and RIBs set different requirements. Antiproton cooling is limited by the poor ratio of Schottky signal to thermal noise, due to the low charge state. To cope with that it is foreseen in the CR: (i) to keep the pickup electrodes and the pickup signal preamplifiers at cryogenic temperatures (in the present scenario, 20-30 K and 80 K, respectively, yielding an effective temperature of 73 K at the preamplifier input), (ii) to strive for the largest possible electrode sensitivity during cooling by moving (plunging) the pickup electrodes, following the shrinking beam size, (iii) to choose the notch filter technique for momentum cooling, which uses the higher sum signals from the pickups (compared to the low difference signals of the Palmer method) and advantageously filters out the thermal noise at all harmonics of the revolution frequency in the band. In order to have sufficient momentum acceptance (and well-separated Schottky bands) for the notch filter cooling the optimum choice is to operate the CR slightly above transition at $\eta$=-0.011 ($\gamma_t$=3.85). The drawback is that the transverse cooling suffers from the resulting high value of $M \approx 11$. (Eq. 1).

For RIBs, the undesired mixing limits the momentum acceptance of the system. The phase $m_c\phi_u$ must be kept small by minimizing (i) $\eta_{pk}$ by increasing the dispersion in the dipoles and (ii) $\chi_{pk}$ by placing pickup and kicker as close as possible. For the chosen lattice $\eta$=0.186. Even so, initially the Schottky bands overlap, so that only the Palmer method can be applied in the beginning of cooling.

### *System parameters*

Along the above lines, the following concept has been developed (Fig. 1). The CR stochastic cooling system will operate in the frequency band 1-2 GHz. It consists of 2 pickup (PH, PV) and 2 kicker tanks (KH, KV), all in straight sections with zero dispersion, and one Palmer pickup tank (PP) at high dispersion. Antiproton cooling makes use of PH, PV, KH, KV. Longitudinally the notch filter technique is implemented and to improve the signal to noise ratio, signals from both PH and PV are taken in sum mode. For RIBs only the Palmer pickup PP is useful in the first stage. It serves to detect signals in all 3 phase space planes. After the rms $\delta$p/p has decreased below 0.1%, it is possible to switch off the signals from the PP and turn to cooling from PH and PV combined with the notch filter down to the final emittances and momentum spread.

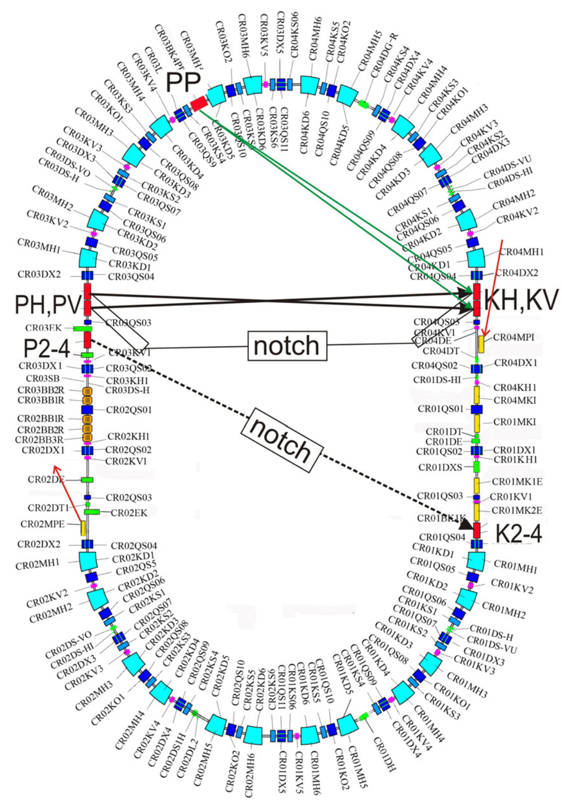


Figure 1: CR layout with stochastic cooling paths, incoming and extracted beams. Solid black line: 1-2 GHz, pbar 3D cooling, RIB 3D cooling final stage; Solid green line: 1-2 GHz, RIB 3D cooling first stage; Dashed black line: 2-4 GHz, pbar longitudinal cooling (future option).

As a future option, for the antiprotons, an additional momentum cooling system in the band 2-4 GHz by means of a notch filter is foreseen. It consists of a pickup tank (probably with plunging electrodes) and a kicker tank, both in dispersion-free straight sections. The design $\eta$=-0.011 guarantees optimum momentum acceptance for both bands. The handshake between the 1-2 and 2-4 GHz momentum cooling systems will have to be investigated by simulations.

Each pickup and kicker tank (PH, PV, KH, KV) consists of two plates (up/down for vertical, left/right for horizontal). Each plate consists of 8 arrays (modules) of 8 identical slotline electrodes, i.e. 64 electrodes. The pickup modules are plungeable. These structures [6] as well as an optical notch filter [1] are being developed at GSI.

In the circuit convention, the longitudinal impedance of the electrodes acting as pickup or kicker is defined in terms of the rms beam current, the rms voltage applied to the beam, time-average pickup signal power and applied power to kicker: $Z_p \equiv P_p/I^2_{b,rms}$ , $Z_k \equiv U^2_{b,rms}/P_k$. Due to reciprocity $Z_k = 4Z_p$. According to HFSS [7] simulations with the present electrode geometry, the longitudinal impedance at midband frequency $f_c$=1.5 GHz, of one electrode pair acting as pickup is 11.25 Ω and 37.75 Ω, for electrode aperture ± 60 mm (unplunged case) and ± 10 mm (fully plunged), respectively. These values are obtained for a beam centered in the pickup without considering its transverse dimensions i.e. they are conservative because in reality particles with high amplitudes "see" a higher sensitivity. Relative measurements on the GSI prototype pickup module [8], indicate that $\sqrt{Z_{p,k}(f,y)} \approx \sqrt{Z_{p,k}(f_c)}S1(y)S2(f)$, where $y$ stands for horizontal or vertical beam coordinate, and yield the functions $S1(y)$ and $S2(f)$. For simplicity, $S1(y)$ is approximated to increase linearly i.e. $\partial\sqrt{Z}/\partial y$=slope=const. The sensitivity $S2(f)$ is about 1 between 1-1.5 GHz and drops down to 0.65 between 1.5-2 GHz, as is characteristic for such slotlines.

The foreseen installed output power at the kickers of the 1-2 GHz system is 4.8 kW, it has to cover both momentum and betatron cooling. It is limited by funding, but could be increased in a future upgrade.

Table 2: Parameters for the stochastic cooling of $E_0$=3 GeV antiprotons with the 1-2 GHz system in the CR.

| | |
|---|---|
| Circumference $C$ | 221.45 m |
| Revolution frequency $f_0$ | 1.315 Tm |
| Slip factor ring $\eta$, PU-K $\eta_{pk}$ | -0.011, -0.033 |
| Distance PU-K/$C$ $\chi_{pk}$ | 0.378 |
| Beam intensity $N$ | $10^8$ |
| Initial rms emittance $\epsilon_{h,v}$ | 45 $\pi$ mm mrad |
| Initial rms momentum spread (Gaussian/parabolic distribution) | 3.5 $\times 10^{-3}$ |
| System bandwidth $W$ | 1-2 GHz |
| PU, K midband impedance $Z_{p,k}(f_c)$ unplunged electrodes at ±60 mm | 11.25 Ω, 45 Ω |
| PU/K sensitivity $S1(y)$=1+slope·$y$ | slope=24.5m$^{-1}$ |
| PU/K sensitivity $S2(f)$ | |
| Number of PU $n_p$, of K $n_k$ (longitudinal) | 128, 128 |
| Number of PU $n_p$, of K $n_k$ (transverse) | 64, 64 |
| Effective temperature (preamp.) $T_{eff}$ | 73 K |
| Total installed power at kickers | 4.8 kW |

Table 2 summarizes the parameters, which are used as input in the following simulations. Plunging of the pickup electrodes i.e. time variation of $Z_p$ is not considered yet. It can be included after complete treatment of the betatron cooling, which provides information on how the beam emittance shrinks with time.

## MOMENTUM COOLING

Momentum cooling is described in terms of the Fokker-Planck (FP) equation for the energy distribution of the particles $\Psi(E,t) \equiv \partial N/\partial E$:

$$\frac{\partial\Psi}{\partial t} = \frac{\partial}{\partial E}\left[-F\Psi + \left(D_s\Psi + D_n\right)\frac{\partial\Psi}{\partial E}\right] . \quad (2)$$

The basic formalism is explained in detail in [9]. In short, the coherent effect F and the incoherent effects (diffusion) due to thermal noise and Schottky noise are:

$$F(E,t) = 2ef_0^2\sqrt{n_pn_k}\sum_{m\in W}\sqrt{Z_p(m)Z_k(m)}\cdot$$
$$\cdot Re\left\{\frac{G(m,E)}{1-S(m,E,t)}\right\},$$
$$D_n(E,t) = \frac{1}{2}f_0^2k_BT_{eff}n_k\sum_{m\in W}Z_k(m)\left|\frac{G(m,E)}{1-S(m,E,t)}\right|^2,$$
$$D_s(E,t) = e^2f_0^3\frac{1}{|\eta|}\frac{\gamma+1}{\gamma}E_0n_pn_k\cdot$$
$$\cdot\sum_{m\in W}Z_p(m)Z_k(m)\frac{1}{m}\left|\frac{G(m,E)}{1-S(m,E,t)}\right|^2, \quad (3)$$

where $m$ are the harmonics of the revolution frequency in the band $W$, $E_0$=3 GeV and $\sqrt{Z_{p,k}(m)} = \sqrt{Z_{p,k}(m_c)}S2(m)$.

The system gain $G(m,E) = G_{||}H_{nf}(m,E)e^{im\phi_u(E)}$ includes the notch filter response and undesired mixing. The electronic gain $G_{||}$, real and constant within the bandwidth, is the variable parameter. The effect of feedback by the beam enters into the open loop gain $S(m,E,t) = \sqrt{n_pn_kZ_p(m)Z_k(m)}G(m,E)BTF(m,E,t)$ via the beam transfer function

$$BTF(m,E,t) = -\frac{ef_0^2}{m}\left[\frac{\pi}{|\kappa|}\frac{d\Psi}{dE}+\right.$$
$$\left.+\frac{i}{\kappa}\cdot PV\left(\int_{-\infty}^{+\infty}\frac{d\Psi/dE^*}{E^*-E}dE^*\right)\right],$$
$$\frac{1}{\kappa} \equiv \frac{1}{2\pi f_0\eta}\frac{\gamma+1}{\gamma}E_0 .$$

The CERN program solves numerically the FP equation and computes the particle density $\Psi(E,t)$. An example is given in Fig. 2. The coefficients $F$ and $D$ are updated in time through their dependence on $d\Psi/dE$ entering into the beam transfer function. Fig. 3 shows the calculated cooling force. Fig. 4 shows that in our case the Schottky noise is higher than the thermal noise before cooling. During notch filter cooling, the Schottky noise dominates over the thermal noise within the limits of the beam distribution. Therefore, one can anticipate that the longitudinal cooling time will be roughly proportional to the beam intensity. As expected, the feedback by the beam suppresses the Schottky noise in the middle of the distribution and deforms accordingly the cooling force F, especially at high gain (see also Eq. 3). The cooling loop was stable since in the Nyquist plot in Fig. 5 the curve lies far on the left from the point $S$=1.

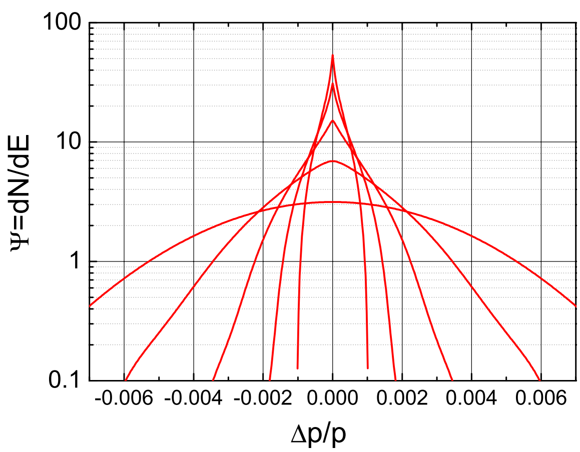


Figure 2: Evolution of the particle density Ψ during cooling with gain=150 dB. Plots at t=0, 2.5 s, 5 s, 7.5 s and 10 s.

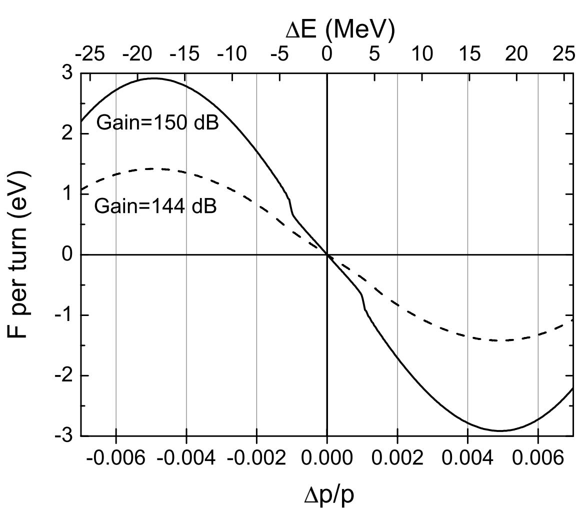


Figure 3: Coherent effect F per revolution at the end of the cooling process (t=10 s for 150 dB gain, t=15 s for 144 dB gain) plotted against the relative momentum spread Δp/p and the deviation ΔE of the beam particles from the nominal kinetic energy of 3 GeV.

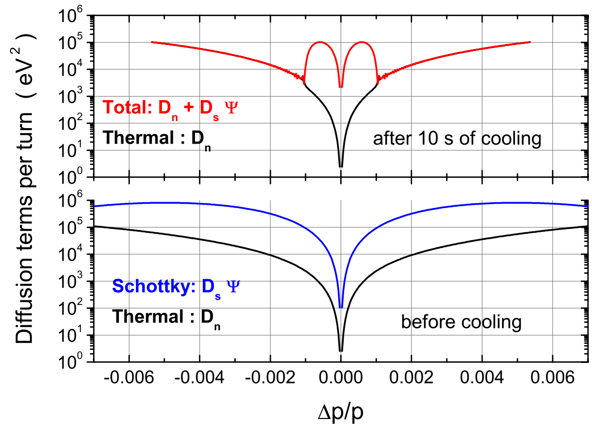


Figure 4: Incoherent effects due to Schottky particle noise $D_s \cdot \Psi$ and (filtered) thermal noise $D_n$ at the beginning and at the end of the cooling process. Gain=150 dB, total cooling time t=10 s.

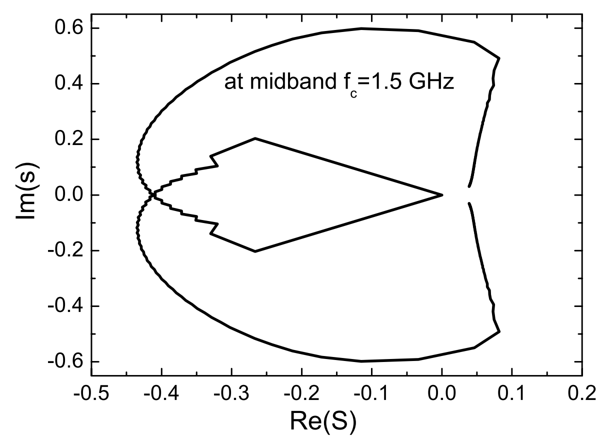


Figure 5: Nyquist plot of the open loop gain $S$ at midband, at the end t=10 s of the cooling process. Gain=150 dB.

The rms energy (momentum) spread is calculated as the second moment of $\Psi(E,t)$. The simulations yield the maximum total cw power in the bandwidth at the kicker. It is the sum of the initial maximum Schottky power and of the constant filtered amplifier power:

$$P_s = 2(ef_0)^2 n_p \sum_{m\in W} \sum_{E} Z_p(m) \left| \frac{G(m,E)}{1 - S(m,E,t)} \right|^2 \Psi(E,t)$$

$$P_n \approx \frac{1}{4} k_B T_{eff} W G_{||}^2 \,.$$

The required installed power is taken by rule of thumb to be 4 times higher than the total cw power, in order to account for statistical fluctuations of the signals. The results are summarized in Fig. 6. The gain of 158 dB was found to be optimum, for higher gain cooling was slower, but the required power is not realistic. As expected [9], for this gain min$[Re(S)]$ is close to -1, i.e. it maximizes the particle flux $\Phi = F\Psi - (D_s\Psi + D_n)\frac{\partial\Psi}{\partial E}$.

To conclude, the requirements of Table 1 can be met with the notch filter cooling at $G_{||}$ =150 dB even with the conservative assumption of no plunging at the pickups. However, there seems to be no safety margin.

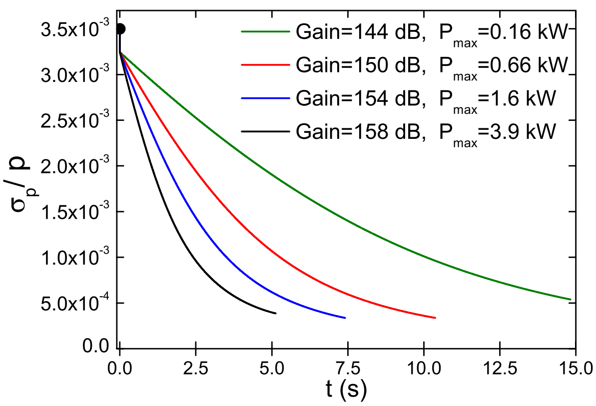


Figure 6: Evolution of the rms momentum spread of the beam during cooling for different gains and corresponding maximum required cw power (up to 20% Schottky, 80% thermal noise).

## BETATRON COOLING

The betatron cooling is studied with an analytical model based on the standard "rms" theory [3, 10, 11] leading to Eq. 1. The instantaneous rate of change of the horizontal or vertical rms emittance is $-(1/\epsilon(t))(d\epsilon(t)/dt) = 1/\tau_\perp(t)$,

$$\frac{1}{\tau_\perp(t)} = \frac{2W}{N}\left[2gB(t)|\sin(\mu_{pk})| - g^2(M(t)+U(t))\right]. \tag{1}$$

Simultaneous momentum cooling takes place for 10 s. The Ansatz for the variation of the momentum spread with time is an exponential fit of the form $\sigma_p(t)/p = 0.0035\ e^{-t/\tau_{||}},\ t \le 10$ s to the results of the FP simulations for gain $G_{||}$=150 dB (see Fig. 6). The parameters entering into Eq. 1 are given by:

$$B(t) \approx \cos[m_c\phi_u(t)],\quad \phi_u(t) = -2\pi\chi_{pk}\eta_{pk}\Delta p(t)/p\,;$$

for an exact expression, the cooling rate term $(2W/N)2gB(t)|\sin(\mu_{pk})|$ in Eq. 1 is replaced by

$$\frac{2f_0}{N}g\sum_{\substack{m=-\infty\\ m\in W}}^{m=+\infty} Re\left\{e^{i(m\phi_u(t)+\mu_{pk}-\pi/2)}\right\},$$

$$M(t) \approx \frac{1}{m_c|\eta|\Delta p(t)/p},$$

$$U(t) = \frac{k_B T_{eff}}{Ne^2 f_0\beta_p \text{slope}^2 n_p}\left(\frac{f_0}{2W}\sum_{\substack{m=-\infty\\ m\in W}}^{m=+\infty}\frac{1}{Z_p(m)}\right)\frac{1}{\epsilon(t)},$$

where $\Delta p(t)/p$ is taken as the $2\sigma$ value, $\beta_p$, $\beta_k$ are the beta functions at the pickup and kicker, respectively.

The system gain $g(t)$, assumed to be constant within the bandwidth, is the variable function in the calculations. It is connected via

$$G_\perp(m) = g\frac{m}{\sqrt{n_p n_k Z_p(m)Z_k(m)}}\frac{4\pi p}{Ne^2\text{slope}^2\sqrt{\beta_p\beta_k}}$$

with the electronic gain $G_\perp(m)$, which ideally should follow the above frequency dependence in the band and also vary in time with shrinking betatron amplitudes.

The gain $g_{opt}(t) = B(t)|\sin(\mu_{pk})|/[M(t)+U(t)]$ maximizes at each t the rate $\tau_\perp^{-1}(t)$, the optimum rate is $\tau^{-1}_{\perp,opt}(t) = (2W/N)\ B^2(t)\sin^2(\mu_{pk})/[M(t)+U(t)]$. During cooling, the heating terms from $M(t)$ and $U(t)$ continuously grow, so that to keep on cooling $g$ should decrease with time, ideally as $g_{opt}(t)$.

Since $G_\perp(m) \sim m$ (a consequence of the Panofsky-Wenzel relation), the power scales with the square of the working frequency in the band. The total cw power in the bandwidth at the kicker is the sum of the initial maximum Schottky power and the constant amplifier power:

$$P_s \sim N\epsilon(t)g^2(t)\sum_{m\in W}\frac{m^2}{S2^2(m)},$$

$$P_n \sim k_B T_{eff} g^2(t)\sum_{m\in W}\frac{m^2}{S2^4(m)}.$$

Depending on how the PU-K pairs in Fig. 1 are assigned to horizontal and vertical cooling, respectively, there are two cases to investigate and choose the best one in terms of cooling performance:

| | | | |
|---|---|---|---|
| Case 1 | | | |
| horizontal | $\beta_p$=20.5 m | $\beta_k$=5.2 m | $\sin(\mu_{pk})$ =-0.92 |
| vertical | $\beta_p$=8.3 m | $\beta_k$=8.3 m | $\sin(\mu_{pk})$ =-1.00 |
| Case 2 | | | |
| horizontal | $\beta_p$=11.0 m | $\beta_k$=11.0 m | $\sin(\mu_{pk})$ =-1.00 |
| vertical | $\beta_p$=5.5 m | $\beta_k$=13.6 m | $\sin(\mu_{pk})$=-0.96 . |

In Figs. 7 and 8 we summarize preliminary results for case 1 and for horizontal cooling only. Similar results are expected for vertical cooling. Also, in this first approach the feedback by the beam is not taken into account. For a proper quantitative treatment it must be included, in particular since in our case with $N = 10^8$ ions the Schottky noise dominates over the thermal one. Nevertheless, these results already allow us to identify the main challenges for the antiproton cooling in the CR. Initially, $M$=11 and $U$=1.2, so that $M$ dominates the heating rate at all times i.e. $M(t) \sim 10\ U(t)$ (Fig.7). The emittance $\epsilon$= 4 $\pi$ mm mrad is reached (Fig. 8), with a maximum required cw power of 950 W (40% Schottky, 60% thermal noise). This goes far beyond the power limitation foreseen for the system (Table 2). In order to reach the required 3 times lower emittance (Table 1), the transverse cooling must proceed at lower gain and take longer than 10 s.

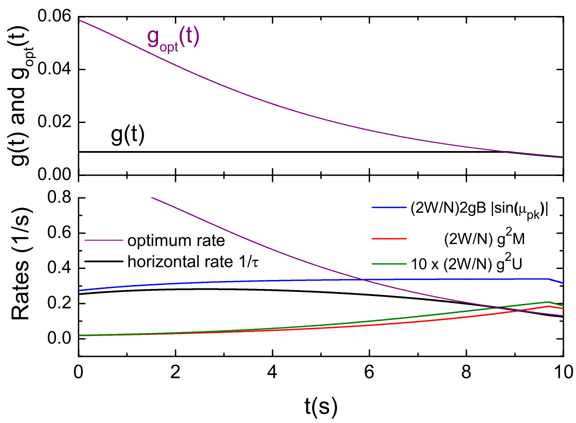


Figure 7: Upper part: Assumed system gain $g(t)$ compared to the optimum gain. The initial $g$ corresponds to an electronic gain $G_{\perp}(m_c)$=141 dB at midband. Lower part: The instantaneous rate $\tau_{\perp}^{-1}(t)$ (black line) and its partial rates according to Eq. 1: cooling rate (blue line), heating rate due to mixing $M$ (red line) and heating rate due to noise/signal ratio $U$ multiplied by the factor 10 (green line). The optimum rate $\tau_{\perp,opt}^{-1}(t)$ is also shown (purple line).

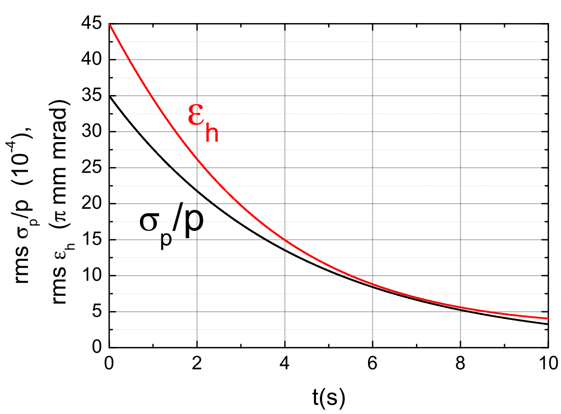


Figure 8: Evolution of the horizontal rms emittance within 10 s of cooling for $g(t)$ as shown in Fig.7. Evolution of the rms momentum spread for simultaneous longitudinal notch filter cooling with $G_{||}$=150 dB.

## CONCLUSIONS AND OUTLOOK

We have seen that, whereas the momentum cooling proceeds optimally, the betatron cooling must proceed very slowly, unless we could somehow restrict $M(t)$. This essentially means to reduce the performance of the momentum cooling at the profit of the betatron cooling until a reasonable compromise is found. A straightforward way is to have in a first stage low-gain notch filter momentum cooling (i.e. slower decrease of $\delta p/p$) with high-gain betatron cooling and inverse the situation at a later stage. An alternative would be to apply momentum cooling in two stages: first with the time-of-flight (TOF) method [12] and then with the notch filter method. The TOF method is not only slower but it also has a larger momentum acceptance through its undesired mixing phase $\phi_u$ (instead of $\phi_{u,nf}$ of filter cooling). Thus, it can be envisaged to increase slightly the $\eta$ (e.g. $\eta \approx -0.02$) before the TOF cooling, thus reducing the initial value of $M$, and bring it down to the required $\eta = -0.011$ when notch filter cooling takes over.

In any case, the interplay between momentum and betatron cooling will have to be investigated in detail. The aim is a simultaneous optimization of both processes, by distributing the available installed power accordingly, so as to fulfill as much as possible the very challenging requirements of the CR. As a next step, plunging of pickup electrodes can be included to study the ultimate performance of the cooling system. It is expected that the plunging will dramatically reduce the diffusion by factors like 4 to 9, especially in the transverse planes.

Cooling of $N = 10^8$ antiprotons is of course the most demanding case, but during the commissioning of the CR intensities $N = 10^6 - 10^7$ are expected. In this case, the filter momentum cooling becomes more relaxed, whereas in the betatron cooling the relative weights of the incoherent effects change ($U$ comparable or greater than $M$), calling for a dedicated optimization procedure.

Concerning the cooling of RIBs, the Palmer pickup has still to be designed. Then, the cooling performance in all 3 planes for the two foreseen stages and in particular the handshake between Palmer and notch filter cooling [5] have to be investigated in extensive simulations.

## ACKNOWLEDGEMENT

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