RADIAL-SECTOR CYCLOTRONS WITH DIFFERENT HILL AND VALLEY FIELD PROFILES

M.K. Craddock#, University of British Columbia and TRIUMF*, Vancouver, B.C., Canada

Abstract

A new class of isochronous cyclotron is described in which more general radial field profiles *B(r)* are allowed than the simple proportionality to total energy found in conventional radial- and spiral-sector cyclotrons. Isochronism is maintained by using differently shaped field profiles in the hills and valleys. Suitably chosen profiles will produce high flutter factors and significant alternating-gradient focusing, enabling vertical focusing to be maintained up to 1 GeV or more using radial rather than spiral sectors.

INTRODUCTION

In an isochronous cyclotron, the constant orbit frequency, independent of ion energy $\gamma m_0 c^2$ and average radius R (circumference/2π), implies that

$$B = \gamma B_c\,, \qquad R = \beta R_c\,, \tag{1}$$

where B denotes the average field around a closed orbit, B_c the "central field" and R_c the "cyclotron radius". Unfortunately the resultant positive field gradient produces a defocusing contribution to the vertical betatron tune ν_z given by $\Delta\nu_z^2 = -\beta^2\gamma^2$. From the beginning, a major problem in cyclotron design has been how to compensate this and ensure vertical focusing. Thomas's [1] suggestion of edge focusing through an azimuthal field variation with N-fold symmetry, and Kerst's [2] of adding alternating focusing by using spiral sectors, have together succeeded in enabling "compact" cyclotrons with spiral sectors to accelerate protons to 230 MeV ($\beta^2\gamma^2 = 0.55$) [3]. *Separate-sector cyclotrons* (SSCs) can achieve higher flutter and so higher energies: the PSI Ring Cyclotron [4] produces 590 MeV protons ($\beta^2\gamma^2$ = 1.65), and designs have been published for energies, up to 15 GeV [5].

Reverse-bend cyclotrons would achieve higher flutter still by making the valley fields negative ($B_v = -B_h$, as in radial-sector FFAGs), rather than zero. Moreover, the alternating-gradient (AG) focusing, minimal in the previous schemes, becomes significant. Thus, if the hills cover a fraction of the orbit $h = 0.6$, the flutter is expected to maintain vertical focusing only up to 3.75 GeV. But a tracking simulation [6] has shown that positive focusing is in fact preserved up to 7.3 GeV.

HILL AND VALLEY FIELD PROFILES

A common feature of the above schemes is that the hill and valley fields, while of different magnitudes, are assumed to have the same radial profiles, *i. e.*:

$$B_v(r)/B_h(r) = \text{constant.} \tag{2}$$

More elaborate designs have also been proposed to achieve isochronism at high energy without using spiral magnets – basically by introducing more free parameters. Thus Rees [7] has designed a non-scaling muon FFAG that remains isochronous over the range 8-20 GeV (5,900 $< \beta^2\gamma^2 <$ 37,000!), relying on AG focusing by "pumplet" cells (OdoFoDoFodO) composed of five straight-sided magnets of three different designs. On a less ambitious scale, Johnstone [8, 9] has designed close-to-isochronous non-scaling proton FFAGs to provide 250-MeV protons and 400-MeV carbon ions for cancer therapy, and 1-GeV protons for ADSR. These all use a 4-cell FDF triplet lattice with straight-sided (though not necessarily radial) edges, and are also remarkable for their low variation in tune, both ν_z and ν_r. In both these authors' studies the *B(r)* profile in each type of magnet is specially determined to produce the desired orbit properties.

Here we propose to explore a simpler possibility for achieving positive vertical focusing at high energy with purely radial sectors – allowing the radial field profiles in hills and valleys to differ. As was found helpful in previous high-energy cyclotron studies [6], we assume hard-edge fields with B_h and B_v each constant along equilibrium orbits. In particular we assume a polynomial variation with energy:

$$B_h(\gamma) = H_0 + H_1\gamma + H_2\gamma^2 + H_3\gamma^3 + \ldots \tag{3}$$

$$B_v(\gamma) = V_0 + V_1\gamma + V_2\gamma^2 + V_3\gamma^3 + \ldots \tag{4}$$

As a first step we consider a "compact" design with no drift spaces and negative valley fields. For an orbit of mean radius R crossing a hill-valley edge at radius R_e, we may write $\ell_h = \rho_h\psi_h$ and $\ell_v = \rho_v\psi_v$ for the arc lengths within a half-cell (Fig. 1), where the radii of curvature $\rho_h = B_cR_c\beta\gamma/B_h(\gamma)$, $\rho_v = B_cR_c\beta\gamma/B_v(\gamma)$, and ψ_h and ψ_v are the bending angles.

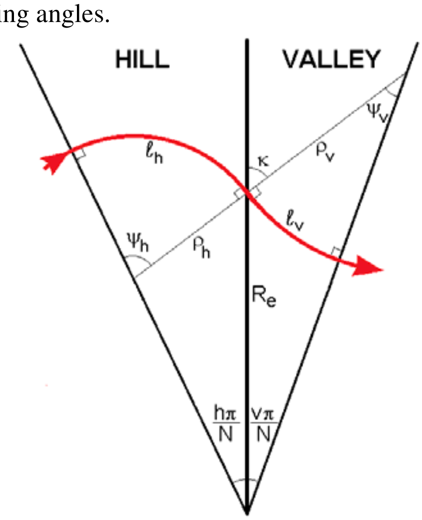

Figure 1: Orbit geometry within a half-cell.

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#craddock@triumf.ca

To maintain isochronism:

$$\ell_h H_1 + \ell_v V_1 = \frac{\pi}{N} B_c R_c \beta \quad \text{and} \quad \ell_h H_n + \ell_v V_n = 0 \ (n \neq 1). \quad (5)$$

Thus if the hill coefficients H_n are specified, the valley coefficients V_n must satisfy:

$$V_1 = \frac{\pi}{N}\frac{B_c R_c}{\ell_v}\beta - \frac{\ell_h}{\ell_v} H_1 \quad \text{and} \quad V_n = -\frac{\ell_h}{\ell_v} H_n \ (n \neq 1). \quad (6)$$

Orbit Geometry

To compute B_v therefore requires knowledge of the bending angles ψ_h and ψ_v and of the radii of curvature – of which ρ_v itself depends on B_v. These parameters may be evaluated by invoking their various geometrical relationships, which after some manipulation yield a transcendental equation for ψ_h, from which the others follow:

$$\psi_h + (\psi_h - \pi/N)\frac{\sin\psi_h \sin[(1-h)\pi/N]}{\sin(\psi_h - \pi/N)\sin(h\pi/N)} = \frac{B_c\beta\gamma}{B_h(\gamma)}. \quad (7)$$

This must be solved numerically, but a good starting point is to make the approximation $R_e = \beta R$, giving:

$$\psi_{h0} = \arcsin\left(\frac{B_h(\gamma)}{\gamma B_c}\sin\left(h\pi/N\right)\right). \quad (8)$$

Betatron Tunes

To calculate the betatron tunes we take a lumped-element approach (validated by tracking with CYCLOPS in previous studies [6]), evaluating the traces of the vertical and horizontal transfer matrices for the full cell:

$$M = M_e M_v M_e M_h. \quad (9)$$

Here M_e is the standard 2×2 matrix for a thin lens, while M_v and M_h are those for focusing and defocusing sector magnets respectively. For M_e we need to evaluate the focal power g of the edge crossing, which depends on the Thomas crossing angle $\kappa = \psi_h - h\pi/N$ and is given by:

$$g = \frac{B_h - B_v}{B_c R_c \beta\gamma}\tan\left(\psi_h - \frac{h\pi}{N}\right). \quad (10)$$

For M_v and M_h we need the field gradients and the phase advances $\phi_{h,v}$. For vertical motion $\phi_h = \ell_h\sqrt{K_h}$ and $\phi_v = \ell_v\sqrt{K_v}$, where the respective coefficients K_h and K_v are:

$$K_h = \frac{dB_h/dr}{B_c R_c \beta\gamma} = \frac{\gamma^2}{B_c R_c^2}\left(H_1 + 2H_2\gamma + 3H_3\gamma^2 + \ldots\right), \quad (11)$$

$$K_v = \frac{dB_v/dr}{B_c R_c \beta\gamma} = \frac{\gamma^2}{B_c R_c^2}\left(V_1 + 2V_2\gamma + 3V_3\gamma^2 + \ldots\right). \quad (12)$$

For the horizontal motion the phase advances are $\phi_h^* = \ell_h\sqrt{K_h^*}$ and $\phi_v^* = \ell_v\sqrt{K_v^*}$, where:

$$K_h^* = \frac{1}{\rho_h^2} + K_h \quad \text{and} \quad K_v^* = \frac{1}{\rho_v^2} - K_v. \quad (13)$$

RESULTS

A number of cases were studied with $H_0 = 0 = H_{n>2}$ to investigate the dependence of the tunes on the size of the H_1 and H_2 components, the hill fraction h and the number of sectors N. Most runs were made for $h = 0.5$ and $N = 8$.

Figure 2 displays an example of such field profiles for one of the cases studied, showing that for $H_2 = 0.2H_1$ the B_h required is 20-40% higher than in an SSC.

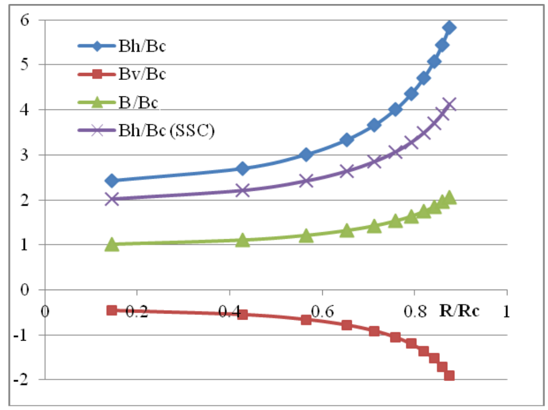

Figure 2: Field profiles for $N = 8$, $h = 0.5$, $H_1 = 2B_c$, $H_2 = 0.4B_c$.

For reference Fig. 3 displays the tunes for the conventional situation where $H_2 = 0$ so that both B_h and B_v are directly proportional to γ. Results are shown for values of H_1/B_c between 2.0 and 2.6, the lower value representing the situation in a separated-sector cyclotron with $h = 0.5$ and near-zero field in the valleys. In that case the flutter $F^2 \approx 1$, and as expected the vertical tune ν_z drops towards zero as $\beta^2\gamma^2$ approaches 1 at around 400 MeV. Higher values of H_1/B_c require more negative valley fields, raising the flutter and the vertical tune values, until with $H_1/B_c = 2.6$ positive focusing is retained up to 1 GeV. The effect of increased H_1/B_c on the horizontal tune is less dramatic: for $H_1/B_c = 2$, $\nu_r \approx \gamma$, and for $H_1/B_c = 2.6$ ν_r still grows linearly but ~40% faster.

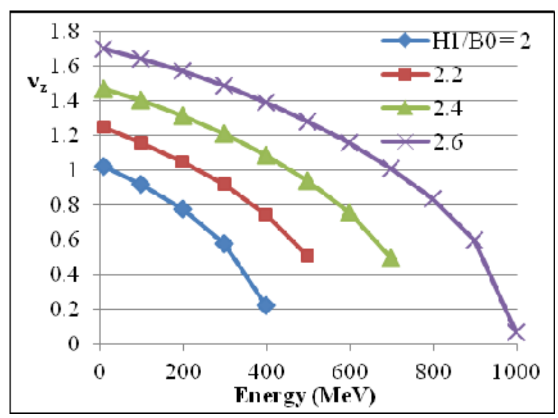

Figure 3: Variation of the vertical tune ν_z with energy for $H_2 = 0$, $h = 0.5$, $N = 8$, and various values of H_1/B_c.

Figure 4 shows the effect of adding a γ^2 component to the field for $H_1/B_c = 2.0$, $h = 0.5$ and $N = 8$. For $H_2 = 0$ we have the same case as before, where the vertical tune drops to 0 near 400 MeV. But increasing the γ^2 component raises ν_z significantly, making it almost constant for $H_2/B_c = 0.4$ and rise with energy for higher values. There is also a significant effect on the horizontal tune, introducing a noticeable quadratic dependence on energy, driving ν_r to the $N/2$ resonance at 900 MeV for $H_2/B_c = 0.6$.

In Fig. 5 we return to the effect of varying H_1/B_c (as in Fig. 2) for $h = 0.5$ and $N = 8$, but now with $H_2/B_c = 0.4$. As before, raising H_1/B_c increases both tunes, but ν_z more

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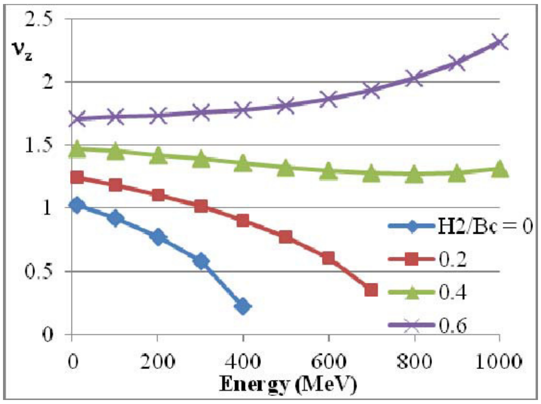

Figure 4: Variation of the vertical tune ν_z with energy for $H_1/B_c = 2$, $h = 0.5$, $N = 8$, and various values of H_2/B_c.

noticeably than ν_r. For this value of the γ^2 component, the least variation in ν_z is obtained for $H_1/B_c = 2.2$. In this case (and in those of varying h and N below) the effect on the horizontal tune is small.

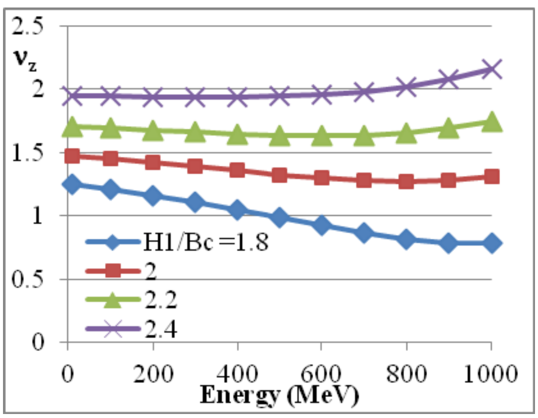

Figure 5: Variation of the vertical tune ν_z with energy for $H_2 = 0.4$, $h = 0.5$, $N = 8$, and various values of H_1/B_c.

The fraction of a sector occupied by the hill also has a powerful influence on the tunes. Figure 6 shows the tunes for hill fractions h between 0.5 and 0.65 for $H_1/B_c = 2$, $H_2/B_c = 0.4$ and $N = 8$. Just as for separated-sector cyclotrons, widening the hills increases the flutter and both ν_z and ν_r.

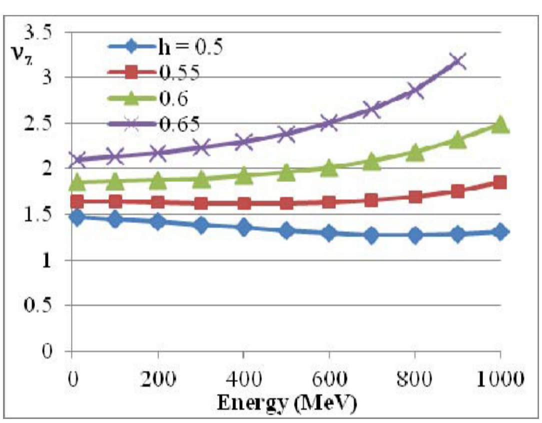

Figure 6: Variation of the vertical tune ν_z with energy for $H_1/B_c = 2$, $H_2/B_c = 0.4$, $N = 8$, and various values of h.

The effect of the number of sectors on the tunes is shown in Fig. 7, where data are plotted for $N = 8$, 10 and 12, with $H_1/B_c = 2$, $H_2/B_c = 0.4$ and $h = 0.5$. Increasing N lowers both ν_z and ν_r, but the effect is a weak one except at the highest energies. Neither tune approaches the $N/2$ resonance in the energy range considered, but ν_r would do so for $N = 8$ not much above 1 GeV, making higher periodicity necessary for higher-energy designs.

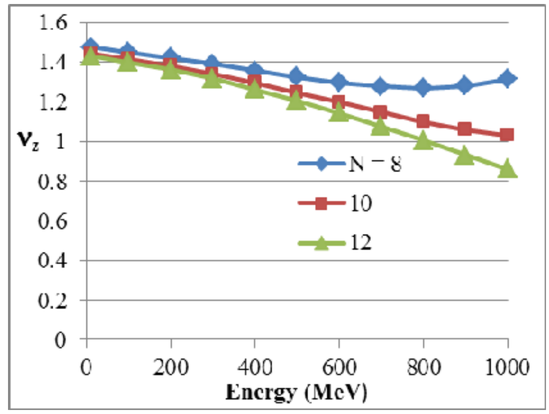

Figure 7: Variation of the vertical tune ν_z with energy for $H_1/B_c = 2$, $H_2/B_c = 0.4$, $h = 0.5$, and various values of N.

CONCLUSIONS

A study has begun of using different radial field profiles in hills and valleys (while maintaining isochronism) to obtain increased flutter and more strongly alternating gradients – and hence increased vertical focusing - in radial-sector cyclotrons. As a first step, adding a γ^2 component to the hill fields in a "compact" design (i.e. no field-free regions) - and subtracting a compensating γ^2 component from the valley fields - has been shown to be a possible way of providing radial-sector cyclotrons with sufficient vertical focusing to reach at least 1 GeV.

The practicality of such a design has not been taken into account, particularly with regard to finding suitable locations for the accelerating cavities and injection and extraction systems. Field-free drift spaces would remove this difficulty and a beam optics study of such an arrangement is under way.

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