# MEDIAN PLANE EFFECTS AND MEASUREMENT METHOD FOR RADIAL COMPONENT OF MAGNETIC FIELD IN AVF CYCLOTRONS 

G. Karamysheva, N. Morozov, P. Shishlyannikov, JINR, Dubna, Russia


#### Abstract

The median plane of the magnetic field in AVF cyclotrons rather often does not coincide with the midplane of their magnetic system. The idea of an effective median plane formulated by J.I.M.Botman and H.L.Hagedorn [1] for the central region of the cyclotron is extended to the entire working region and tolerances for the horizontal components of the magnetic field are estimated. Equipment based on the search coils is proposed and used for measurement of the radial component of the magnetic field and for correction of the magnetic field median plane.


## INTRODUCTION

The vertical deviation of the beam center from the median plane of the vacuum chamber was observed and corrected at some cyclotrons (AVF Cyclotron, Einhoven, Netherlands [1], AGOR, Groningen, the Netherlands [2], U-120M, Rzez, Chech Republic [3], AIC-144, Krakow, Poland [4], JINR Phasotron, Dubna, Russia [5]). Thus it is very important for the cyclotron design to be clear in knowing the tolerances for horizontal components of the magnetic field, their relation to the manufacturing tolerances for magnetic and current elements and the method of magnetic field measurement and correction.

## EFFECTIVE MEDIAN PLANE AND TOLERANCES FOR HORIZONTAL COMPONENTS OF THE MAGNETIC FIELD

When the median plane of the magnetic field is symmetric, the equation of free vertical oscillations of the particle near the median plane of the vacuum chamber is:

$$
\begin{equation*}
z^{\prime \prime}+v_{z}^{2} z=0 \tag{1}
\end{equation*}
$$

where $v_{z}^{2}$ is the total vertical focussing force of the magnetic and electric field. When the symmetry of the magnetic system is lost, the horizontal components $\left(B_{r}\right.$ and $B_{\varphi}$ ) of the magnetic field appear in the vacuum chamber median plane. In this case the magnetic field median plane is not physically the plane where $B_{r}$ and $B_{\varphi}$ are equal to zero at the same time. Stable vertical oscillations exist now near the effective medial plane (EMP) ( $Z_{\text {eff }}$ ):

$$
\begin{equation*}
z^{\prime \prime}+v_{z}^{2}\left(z-Z_{e f f}\right)=0 \tag{2}
\end{equation*}
$$

Physically the EMP is the plane in which the sum of all vertical forces acting on the particles is zero. If in the cyclotron there are some vertical forces with factor $v_{z i}{ }^{2}$ and zero point $\mathrm{Z}_{\mathrm{i}}$, the position of EMP can be evaluated as:

$$
\begin{gather*}
Z_{e f f}=\Sigma_{i}\left(v_{z i}^{2} \cdot Z_{i}\right) / \Sigma_{i} v_{z i}^{2}  \tag{3}\\
Z_{e f f}=\Sigma_{i}\left(v_{z i}^{2} \cdot Z_{i}\right) / v_{z}^{2} \tag{4}
\end{gather*}
$$

The main physical conclusion from (4) is that the influence of each vertical force in the AVF cyclotron on the vertical position of the EMP depends on the relative contribution of this force to the total vertical force $v_{z}{ }^{2}$.
In a classical cyclotron there is only one vertical force generated by the average radial component of the magnetic field, and the vertical beam position follows the position of the plane with $B_{r \text { aver }}=0$. In the AVF cyclotron there are basically two vertical forces, the defocussing force generated by the average radial component of the magnetic field and the focussing force generated by the horizontal components of the magnetic field azimuth variation. When the plane with $B_{r \text { aver }}=0$ shifts, the vertical beam position follows the plane with the overall vertical force equal to 0 (Fig. 1).


Figure 1: AVF cyclotron: vertical beam position follows the plane with zero overall vertical force

If the vertical component of the cyclotron magnetic field is written in the form

$$
\begin{equation*}
B_{z}=B_{z a v}+\Sigma_{i} B_{z i} \cdot \operatorname{Cos}\left[i\left(\varphi-\varphi_{z i}\right)\right] \tag{5}
\end{equation*}
$$

the vertical beam center position, which arises due to the average radial field component ( $\mathrm{B}_{\text {rav }}$ ), and the main ( N ) harmonic of the radial and azimuth component ( $\mathrm{B}_{\mathrm{rN}}, \mathrm{B}_{\varphi \mathrm{N}}$ ), may be found as:

$$
\begin{gather*}
Z_{e f f}=R /\left(B_{z a v} \cdot v_{z}^{2}\right)\left[B_{r a v}+0.5\left(B_{z N} / B_{z a v}\right) B_{\varphi N} / N\right. \\
\cdot \operatorname{Sin}\left\{N\left(\varphi_{\varphi N}-\varphi_{z N}\right)\right\}-\left(B_{z N} / B_{z a v}\right) \cdot B_{r N} \varphi_{z N}^{\prime} R / N \\
\left.\cdot \operatorname{Sin}\left\{N\left(\varphi_{r N}-\varphi_{z N}\right)\right\}\right] \tag{6}
\end{gather*}
$$

If we take into account only the part of $\mathrm{B}_{\mathrm{rN}}$ and $\mathrm{B}_{\varphi \mathrm{N}}$ in the form of their projections on the vector shifted by $\pi / 2 / \mathrm{N}$ from the vector with the phase of the main vertical harmonic designated as $\mathrm{B}_{\mathrm{rNs}}$, $\mathrm{B}_{\varphi \mathrm{Ns}}$ (only these horizontal components of the magnetic field give the vertical force
acting on the accelerated particles), the latter formula may be written as:

$$
\begin{align*}
Z_{e f f}= & R /\left(B_{z a v} \cdot v_{z}^{2}\right)\left[B_{r a v} \pm 0.5\left(B_{z N} / B_{z a v}\right) B_{\varphi N s} / N \pm\right. \\
& \left. \pm\left(\left(B_{z N} / B_{z a v}\right) \cdot B_{r N s} \varphi_{z N}^{\prime} \cdot R / N\right)\right] \tag{7}
\end{align*}
$$

To find separately the allowable values (tolerances) for the horizontal field components from the allowable vertical beam offset the latter formula can be transformed into three ones:

$$
\begin{align*}
& B_{r a v}=Z_{e f f} B_{z a v} \cdot v_{z}^{2} / R \\
& B_{q N s}=Z_{e f f} B_{z a v} \cdot v_{z}^{2} / R \cdot 2 N\left(B_{z a v} / B_{z N}\right)  \tag{8}\\
& B_{r N s}=Z_{e f f} B_{z a v} \cdot v_{z}^{2} / R \cdot N / R\left(B_{z a v} / B_{z N}\right) / \varphi_{z N}^{\prime}
\end{align*}
$$

## TOLERANCES FOR THE HORIZONTAL COMPONENTS OF THE MAGNETIC FIELD OF THE C400 CYCLOTRON

For hadron therapy project, the C400 superconducting cyclotron [6] is proposed and designed. With the simulated magnetic field map and computed vertical betatron frequency, we can use formulas $(8)(\mathrm{N}=4)$ to obtain the radial dependence of tolerance for the horizontal field components of this cyclotron per 1 mm of acceptable vertical offset of the beam (Fig. 2). By increasing the frequency $v_{z}$ at the extraction region of C400 close to 0.45 , it was possible to increase the tolerances for the horizontal magnetic field components. For the extraction region those tolerances are $\mathrm{B}_{\text {rav }}<3-4 \mathrm{G}$, $\mathrm{B}_{\varphi 4}<70-90 \mathrm{G}$ and $\mathrm{B}_{\mathrm{r} 4}<15-20 \mathrm{G}$. The region most sensitive to $B_{\text {rav }}$ is the central one ( $\mathrm{R}<40 \mathrm{~cm}$ ). To check the analytic formulas, the beam dynamic was simulated for separate C 400 working regions. The result of the simulation for $B_{r 4}=50 \mathrm{G}$ is presented in Fig. 3 .

## MEASUREMENT AND CORRECTION OF THE AVERAGE RADIAL COMPONENT

The average radial component of the AVF cyclotron's magnetic field is the first among the most dangerous causes for the vertical beam offset and the most difficult part of the magnetic field measurement procedure. The difficulties of the measurement are so great that the accelerator physicists very often prefer not to measure this component of the field but rather to correct it by the accelerated beam effect during the cyclotron commissioning. So, the accelerator physicists have to go along three ways:

- to use as high a vertical force $\left(v_{z}\right)$ as possible during the cyclotron design;
- to get and implement an accurate $\mathrm{B}_{\mathrm{rav}}$ measurement method;
- to be prepared for correcting the vertical offset of the accelerated beam.


Figure 2: $\mathrm{B}_{\mathrm{rav}}, \mathrm{B}_{\mathrm{r} 4}, \mathrm{~B}_{\varphi 4}$ for the possible vertical beam 1 mm offset


Figure 3: Vertical beam offset due to $B_{\mathrm{r} 4}=50 \mathrm{G}$ according to the analytic formula and the beam dynamics simulation

The best solution for the $\mathrm{B}_{\text {rav }}$ problem is to have an accurate measurement method. Considering the available cyclotron design experience it is possible to distinguish two ways in the $\mathrm{B}_{\text {rav }}$ measurement:

- direct measurement of the radial component of the magnetic field;
- integration of the difference between the vertical components in two planes at the distances $\pm \mathrm{a}$ from the median plane:

$$
\begin{equation*}
B_{r a v}(R)=-\frac{1}{2 a R} \cdot \int_{0}^{R} \Delta \bar{B} z(r) r d r \tag{9}
\end{equation*}
$$

## Measurements by the Hall Probe

Direct $B_{r}$ measurements by the Hall probe reveal a very strong influence of the large vertical component of the magnetic field: $\mathrm{B}_{\mathrm{r} \text { error }}=\mathrm{B}_{\mathrm{z}} \bullet \delta$, where $\delta$ is the accuracy of the Hall probe orientation in the radial direction. It is impossible to have $\delta$ at the level better than $10^{-3}$. To overcome the above difficulties the Hall probe was used in the way of a pendulum [8] or with laser alignment [9]. The accurate measurement by the Hall probe may be additionally complicated by the planar effect.
If the Hall probe is used to measure $\Delta \mathrm{B}_{z}$, the influence of the vertical component is less, it is $\sim 1 / 2 \mathrm{~B}_{\mathrm{z}} \bullet \delta^{2}$ but additional problems arise:

- keeping the angle $\delta$ constant when the Hall probe shifts from one measurement plane to another;
- small measurement error at the inner radius gives a large overall effect at the outer radius after integration by (9).


## Measurements by Search Coils

The use of search coils for the measurement of $\Delta B_{z}$ makes the measurement of $B_{r \text { av. }}$ more accurate due to high sensitivity of the coils and lower influence of the vertical component of the magnetic field than in the case of the Hall probes.
The first type of search coil use is the one of a small diameter. The measurements are performed in the polar grid, and for signal generation the coil is shifted in the vertical direction in positions $\pm$ a from the median plane of the cyclotron. The second possibility is the use of two similar coils (at the distance $\pm$ a) with opposite connection. The signal is generated as the probe moves radially from the center of the cyclotron to the final radius R . The signal can be processed in the floating mode to get the radial $\Delta B_{z}$ dependence. For this type of measurements the same hardware and software can be used as for the measurement of the vertical component of the magnetic field [10].
The AVF magnetic field was first measured by a small search coil (with vertical shift) at the JINR Phasotron [11]. It was demonstrated that for this type of measurements the systematic error is proportional to the gradient of the vertical component of the magnetic field:

$$
\begin{equation*}
\Delta_{s} \cdot(\Delta \overline{B z})=2 a \frac{d \overline{B z(r)}}{d r} \delta \tag{10}
\end{equation*}
$$

and accumulation of the systematic error occures under integration of $\Delta \overline{B z}$ by formula (9).
The second type of search coils is the one with radius R which is needed for measurement of the component $B_{r \text { av. }}$. of the magnetic field. The signal is generated during the vertical shift of the coil at the positions $\pm$ a from the median plane of the cyclotron. To get a full magnetic field map, it is required to use a set of coils with the designed set of radii. This method was tested and used at the JINR Phasotron [5]. It was shown that the influence of the vertical component of the magnetic field on the radial component is:

$$
\begin{equation*}
\Delta_{e r r}\left(B_{r a v}\right)=1 / 2 B_{z a v} \cdot \delta^{2} \tag{11}
\end{equation*}
$$

The practical accuracy of the measurement of $B_{r \text { av. }}$ is about $0.2-0.3 \mathrm{G}$. This measurement method was used for correcting the radial component of the magnetic field and the magnetic field responses of iron elements in the central part of the Phasotron. The value of the radial component was corrected from 15 G to 1-2 G.
To measure the radial component of the magnetic field at the IBA C235 cyclotron, a measurement equipment on the basis of a set of search coils was designed and tested. The system consists of the measurement disk with 35 search coils, alignment and pneumatic system for the
vertical shift of the disk. The system at the test stand is shown in Fig.4. The search coil system is planned to be used for measuring of the radial component of the magnetic field at version V3 of the C235 cyclotron.


Figure 4: Measurement disk with search coils for the C235 cyclotron.

## REFERENCES

[1] J.I.M.Botman, H.L.Hagedorn, "Median Plane Effects in the Eindhoven AVF Cyclotron", IEEE Trans. On Nucl. Science, Vol. NS-28, No.3, p. 2128.
[2] S.Brandenburg et al., "Vertical Beam Motion in the AGOR Cyclotron", Proc. of EPAC 2004, Lucerne, Switzerland, p. 1384-1386.
[3] J.Stursa et al., "The Axial Injection System of the Isochronous Cyclotron", Proc. of EPAC 2002, Berlin, Germany, p. 1513-1515.
[4] E. Bakevicz et al., "Extraction of Protons with the Energy of 35 MeV from the Upgraded AIC-144 Cyclotron", NUKLEONIKA 2001; 46(2), p. 51-57.
[5] N. Zaplatin et al., "Magnetic Field Median Plane Determination and Shaping for the JINR Phasotron", JINR, P9-82-925, Dubna, 1982.
[6] Y. Jongen et al., "IBA C400 Cyclotron Project for Hadron Therapy", Proc. of 18-th Cyclotrons and Their Application International Conference, 2007, Giardini Naxos, Italy, p. 151-156.
[7] Y. Jongen et al., "Computer Modeling of Magnetic System for C400 Superconducting Cyclotron", Proc. of EPAC 2006, Edinburgh, UK, p. 2589-2591.
[8] D.P. Vasilevskaya, Yu.N. Denisov, Instruments and Experimental Techniques, No.3, 1959.
[9] D. Vandeplassche, Y. Paradis, "Radial Field Mapping in C235", IBA internal report, January 6, 2006.
[10] A.Roy et al., "Median Plane Magnetic Field Mapping for Superconducting Cyclotron (SCC) in VECC", Proc. of APAC 2007, Indore, India, p. 652654.
[11] N. Morozov, P. Shishlyannikov, "The Method of Experimental Determination of the Mean Radial Magnetic Field Component for the JINR Phasotron", JINR, P9-82-926, Dubna, 1982.

