OPTIMIZATION OF SECTOR GEOMETRY OF A COMPACT CYCLOTRON BY RANDOM SEARCH METHOD

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Abstract

This paper describes the procedure of optimizing the sector geometry of the magnet to obtain the desired isochronous field. The hill shape of the magnet is described in terms of a small number of parameters which are iteratively determined by random search technique geared to minimize the frequency error. 3D magnetic field data and results of equilibrium orbit code are used as input for the iterative optimization process.

INTRODUCTION

A 10 MeV, 5 mA four sector compact cyclotron for proton is under development at VECC Kolkata. Proton beam at 80 keV from 2.45 GHz microwave ion source (under testing) will be first collimated and bunched [1]. It will be injected axially in the central region where a spiral inflector will place the beam on the proper orbit. Two delta type resonators located in the opposite valleys will accelerate the beam and an electrostatic deflector will be used for the extraction. In general the magnet pole shimming is an iterative process [2,3]. Analytical formulas [4,5,6] are available for calculating the average magnetic field and betatron frequencies for a given configuration of the magnet geometry. But these formulas are not valid for high flutter field and particularly at the lower radii in the case of a compact cyclotron. Hence an equilibrium orbit code becomes necessary to obtain the frequency error. An acceptable phase shift of the particles with respect to rf determines the tolerance of the magnetic field isochronism. In this paper we present a shimming method, which gives smooth sector geometry of the hill. We have approximated the shape of the sector by a polynomial function of the radius, and minimized the frequency error by optimizing the coefficients of the polynomial by using random search technique.

METHOD OF OPTIMIZATION

We have used a 3D magnetic field code MagNet [7] to calculate the field in the median plane and obtained the frequency errors as a function of energy using equilibrium orbit code GENSPEO [8]. These frequency errors are then minimized by modifying the sector geometry.

For an N sector cyclotron, using hard edge approximation, we can write the following relations:

$$\theta_h(r)+\theta_v(r)=\frac{2\pi}{N} \tag{1}$$

$$\bar{B}(r)=\frac{\theta_h(r)\cdot B_h+\theta_v(r)\cdot B_v}{\theta_h(r)+\theta_v(r)} \tag{2}$$

$$\bar{B}(r)=\frac{B_0}{\sqrt{1-ar^2}} \tag{3}$$

where B_h and B_v are the hill and valley field respectively. B_0 is the isochronous field at the center of the cyclotron and $a=\left(\frac{qB_0}{m\cdot c}\right)^2$. Using equations (1-3) we can write

$$\theta_h(r)=\frac{2\pi}{N\cdot[B_h-B_v]}\left[\frac{B_0}{\sqrt{1-ar^2}}-B_v\right] \tag{4}$$

Expanding the above equation we get

$$\theta_h(r)=a_0+a_1r^2+a_2r^4+a_3r^6+...+a_mr^{2m} \tag{5}$$

The polynomial coefficients a_0, a_1 etc. depend on B_h, B_v and average central field B_0. For the optimisation one has to start with an initial set of a_n values, and iteratively correct these to obtain the final optimized hill angle. At first the z-component of the magnetic field at the median plane is calculated for the initial sector geometry and frequency errors are obtained at n different discrete energies. The frequency error is defined as

$$\Omega(k)=\frac{\omega_0}{\omega(E_k)}-1 \tag{6}$$

where, ω_0 is the constant rotation frequency of the particle and $\omega(E_k)$ is the rotation frequency for the calculated magnetic field at energy E_k.

The second step involves the calculation of the elements of the $n\times(m+1)$ correlation matrix. For this it is required to calculate the magnetic field by slightly changing the coefficients say $a_i=a_i+\Delta a_i$ of the polynomial one at a time keeping all other coefficients and geometry constant. The same procedure is repeated for all other coefficients one by one. For the small change in polynomial coefficients Δa_i, the deviation in frequency errors $\Delta\Omega(1), \Delta\Omega(2),\ldots \Delta\Omega(n)$ can be related linearly as

$$\Delta\Omega(k)=\frac{\partial\Omega(k)}{\partial a_0}\Delta a_0+\frac{\partial\Omega(k)}{\partial a_1}\Delta a_1+\cdots\frac{\partial\Omega(k)}{\partial a_m}\Delta a_m \tag{7}$$

and $$\frac{\partial\Omega(k)}{\partial a_i}=\frac{\Omega(k)\big|_{a_i+\Delta a_i}-\Omega(k)\big|_{a_i}}{\Delta a_i}. \tag{8}$$

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While calculating the correlation matrix we have found that the values of the matrix elements at lower radii are very small. So for the optimisation of sector shape at lower radii we have added another term to the equation of hill angle $\theta_h(r)$. Now the modified hill angle becomes

$$\theta_h(r) = \sum_{i=0}^{m} a_i r^{2i} + g \cdot e^{-\alpha(r-r_0)^2} \qquad (9)$$

Here, the parameter g controls the pole shimming in the central region, r_0 and α are chosen constant. Further, we have included the coil current as a parameter for the optimization so the total number of parameters becomes m+3 and the size of the correlation matrix is $n\times(m+3)$. The linear set of equations in the matrix form

$$\begin{bmatrix} \Delta\Omega(1) \\ \Delta\Omega(2) \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \Delta\Omega(n) \end{bmatrix} = \begin{bmatrix} \frac{\partial\Omega(1)}{\partial a_0} & \frac{\partial\Omega(1)}{\partial a_1} & \cdots & \frac{\partial\Omega(1)}{\partial a_{m+2}} \\ \frac{\partial\Omega(2)}{\partial a_0} & \frac{\partial\Omega(2)}{\partial a_1} & \cdots & \frac{\partial\Omega(2)}{\partial a_{m+2}} \\ & \vdots & & \\ & \vdots & & \\ \frac{\partial\Omega(n)}{\partial a_0} & \frac{\partial\Omega(n)}{\partial a_1} & \cdots & \frac{\partial\Omega(n)}{\partial a_{m+2}} \end{bmatrix} \begin{bmatrix} \Delta a_0 \\ \Delta a_1 \\ \vdots \\ \vdots \\ \Delta a_{m+2} \end{bmatrix} \qquad (10)$$

For the optimization of the parameters a_0, a_1,…a_{m+2}, we have used random search method. Here parameter a_{m+1} represents the coefficient g in equation (9) and a_{m+2} is the coil current parameter.

Random Search Method

In this method we have minimized the frequency error of the particle by a suitable combination of all the unknown parameters. The error term R is defined as

$$R = \sum_{k=1}^{n} \left(\Omega(k) - \sum_{i=0}^{m+2} \left(a_i \frac{\partial\Omega(k)}{\partial a_i} \right) \right)^2 \qquad (11)$$

Here $\frac{\partial\Omega(k)}{\partial a_i}$ are the elements of the correlation matrix. Now the quantity R is minimized by varying the parameters a_i randomly. A set of parameters is chosen randomly within a small range for a_i, and the quantity R is calculated. This is done repeatedly until a small value of R is obtained. This gives an intermediate set of a_i. The process is repeated with random values chosen around the intermediate set of values. At each step the range for each parameter is decreased so that the search becomes faster. After a number of cycles of the process one obtains the minimum value of R and new values of parameters a_i. With this set of new parameters a_i, a new hill shape is obtained, and the magnetic field is calculated once again. The iterations are continued until the frequency error falls below the required tolerance. In random search technique one can include constraints in the problem such as maximum coil current, maximum and minimum hill angle etc. One can reject a solution if the constraints are not fulfilled.

RESULTS

The above iterative method has been applied to optimize the sector geometry of our 10MeV, 5mA four sector compact cyclotron for proton. The preliminary design of the magnet was obtained using hard edge approximation method [1,2]. We have chosen maximum hill field of 1.5T and valley field of 0.25T so that we can get high flutter. A high flutter provides strong focusing in the vertical direction. The main idea was to provide the vertical betatron tune > 0.5 at all radii for handling the space charge defocusing force of 5mA beam. The hill gap is 5 cm and the valley gap is 50 cm. We have used 3D MagNet code for the field calculation. Proper symmetry considerations allowed us to use only 1/16 portion of the magnet, as shown in Figure 1 for field calculation.

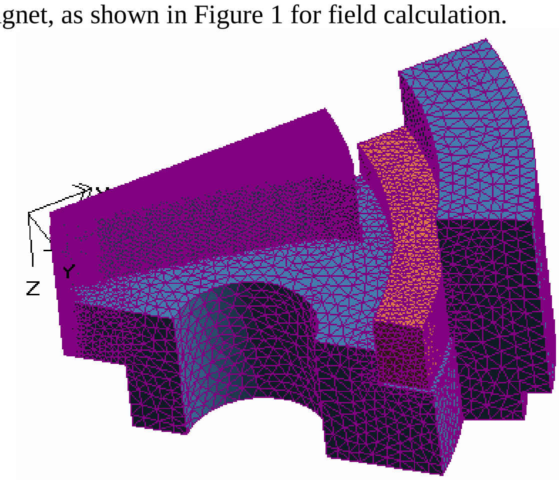

Figure 1: Model of the magnet with mesh.

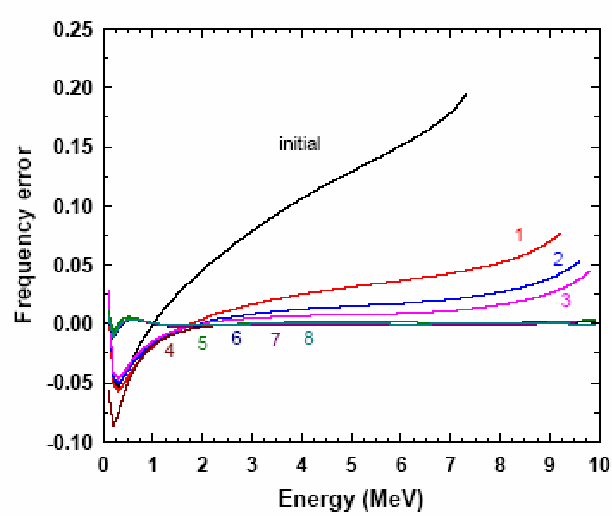

Figure 2: Decrease in frequency error as a function of energy at different iterations 1,2,3… .

In the optimization we have used six parameters a_0, a_1, a_2, a_3, a_4, a_9 from the polynomial and other two as coil current and g. We have started with constant angular width of the hill shape equal to 20 deg. and current in the coil is set equal to 500 A (total no. of turns ~200). The values of frequency error as a function of energy are shown in Figure 2 for successive iterations. After 8 iterations we achieved the magnitude of frequency error < 10^{-4} at all energies.

Figure 3 shows the variation of the mean square frequency error $\sigma_\Omega{}^2$ as a function of the iteration number.

$$\sigma_\Omega{}^2 = \frac{1}{n}\sum_{k=1}^{n}\Omega^2(k) \qquad (12)$$

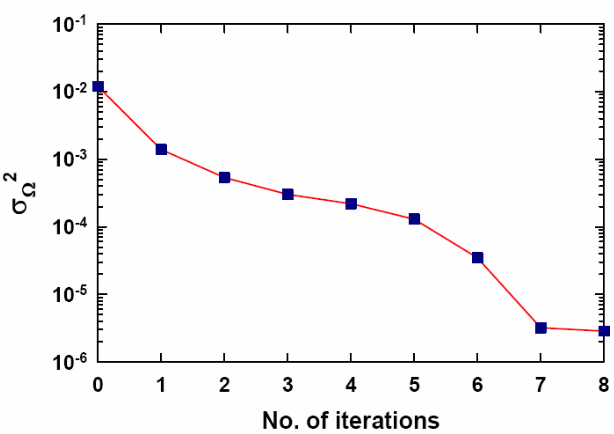

Figure 3: Mean square frequency error as a function of the number of iterations.

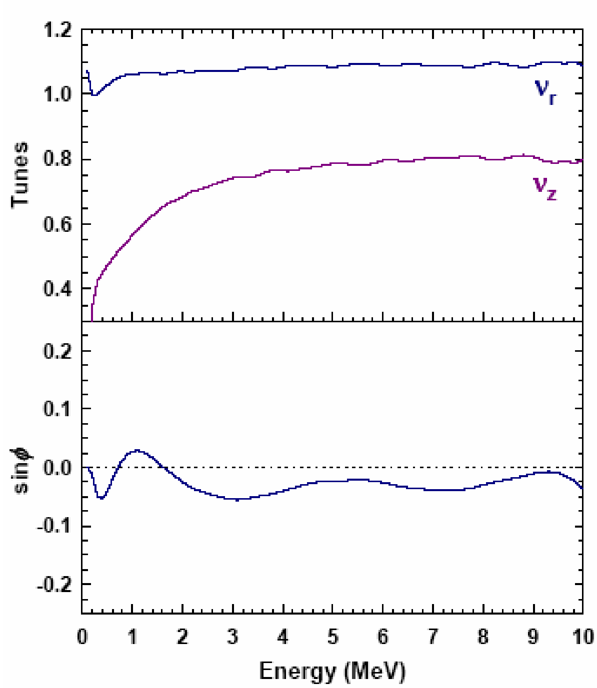

Figure 4: Variation of betatron tunes and phase slip sinϕ as a function of energy.

The radial and axial tunes, integrated phase shift etc. were found out for the optimized sector shape using the equilibrium orbit program as shown in Figure 4. The phase excursion in the entire region is limited within ± 2^0.

We have also checked the centering of the accelerated orbits using the optimized magnetic field data. The electric field E in the accelerating gaps of the two resonators has been approximated by a Gaussian function. Figure 5 shows the position of the accelerating gaps (G-1 to G-4) in the median plane and accelerated orbits of protons up to the extraction radius.

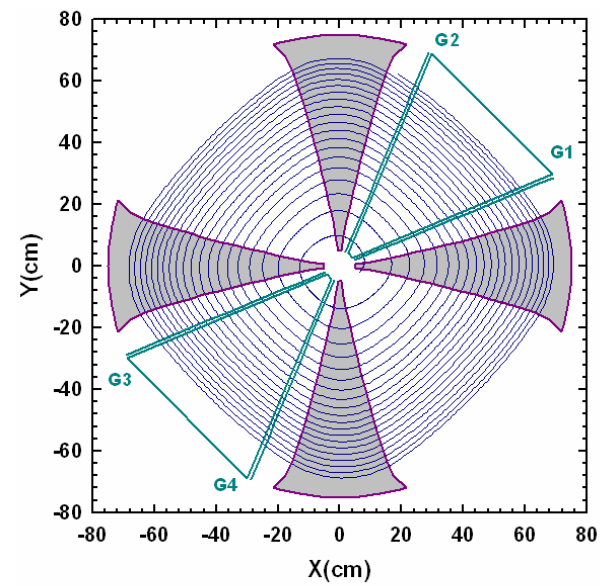

Figure 5: Location of the accelerating gaps and optimised accelerated orbits for proton from 80 keV to 10 MeV.

CONCLUSION

This paper describes a technique based on random search for obtaining the isochronous magnetic field for a 10MeV, 5mA compact cyclotron by optimization of pole profile of the hill. This method uses less number of parameters and convergence is fast.

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