# **BEAM BASED GAIN CALIBRATION OF BEAM POSITION MONITORS AT J-PARC MR**

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#### Abstract

The output data from a beam position monitor (BPM) system was usually calibrated at the test bench and so on.

The relative gain of the output data may drift due to unpredictable imbalance among output signals from the pickup electrodes, because the output signals must travel through separate paths, such as cables, connectors, attenuators, switches, and then are measured by detectors.

The gain calibration process has been tried to apply for the BPM system in J-PARC Main Ring. However, we were not able to apply a same method as KEKB to analyze a gain of BPMs in J-PARC. We noticed linear relations among 4 outputs voltage and analyzed the imbalance by the total least-squares method. This paper proposes this new method to estimate the related gains from four output data of a BPM head.

#### **INTRODUCTION**

The J-PARC accelerator comprises an H<sup>-</sup> linac, a Rapid-Cycling Synchrotron, a slow-cycling Main Ring Synchrotron (MR) and related experimental facilities.

For closed orbit distortion (COD) measurement, there are 186 BPMs with 4-diagonal cut electrodes installed in the MR [1]. Stability of the closed orbit is one of very important points for stable operations to keep a small beam loss in MR. In the installation of the BPM head, its mechanical reference axis was calibrated against the electric center of the output signals from 4 electrodes. In other hand, we have the electric errors such as signal transmission cable and the signal detecting circuit etc. The calibration include the correction of these errors, too.

In KEKB, we found noticeable errors larger than 0.1mm in the almost all BPM readings. These errors come from the imbalance among 4 output voltage of a BPM. By this reason, the gains of every BPMs of KEKB have been calibrated by a non-linear chi-square method [2]. The same process of gain calibration as in KEKB has been tried to apply for the BPM system in J-PARC Main Ring, however the fitting result gave indefinite solutions.

Since the relations among 4 outputs voltage from the pickups with a diagonal-cut is linear, we found the relative gains of J-PARC BPM heads with calibration. We used the conventional least square fitting method, but this method was unstable when output has larger errors. To obtain stable calibration, we introduced the total least square fitting method [3]. It is described that the total least square fitting method gives better result in the calibration.

#### GAIN CALIBRATION IN KEKB BPM

The beam based calibration (BBGC) was usually used to guarantee accuracy of BPM. The BBGC is based on a nonlinear chi-square method.

#### Modelling of Output Data

The BPM has four electrodes to pick up and four output signals V<sub>i</sub>'s . the configuration of the 4 electrodes are shown in Fig. 1. The output signal from each electrode is given by,

$$V_i = g_i \cdot q \cdot F_i(x, y)$$
  $i = 1, 2, 3, 4,$ 

where q denotes the beam charge, and x,y denote horizontal and vertical displacements of the beam against the geometrical center. Function  $F_i(x,y)$ , denotes response of four electrodes, and normalized to  $F_i(0,0)=1$ . Quantities, gi's, show overall gains of each electrode. The response function depends only on the geometrical structure of the pick-up head.



Figure 1: BPM model in KEKB.

#### Gain Calibration

The beam positions are measured m times with a pickup head, by changing the orbit at the monitor in each time, the signal from the i-th electrode at the j-th measurement is given by,

$$V_{i,j} = g_i \cdot q_j \cdot F_i(x_j, y_j)$$
  $i = 1, 2, 3, 4, j = 1, 2, m.$ 

Since we can set  $g_1$  to 1 with a proper scaling factor for the beam charge, there exist only 3 unknown gains, g2,g3 and  $g_4$ . We measure  $V_{1,j}$ ,  $V_{2,j}$ ,  $V_{3,j}$  and  $V_{4,j}$  at each measurement. Since gi will not change at each measurement,  $q_i$ ,  $x_i$  and  $y_i$  are unknown parameters. After the m-th measurement the number of the unknown

parameters is 3+3m. The known parameters is 4m. When m is larger than 4, the 4m exceeds the 3+3m, then the unknown parameters, including the gains, can be calibrated using a nonlinear chi-square,

$$\chi^{2}(\mathbf{a}) = \sum_{i}^{4} \sum_{j}^{m} \frac{\left[V_{i,j} - g_{i}q_{j}F_{i}(x_{j,}y_{j})\right]^{2}}{\sigma_{i,j}^{2}},$$
  
$$\mathbf{a} = \left(g_{2}, g_{3}, g_{4}, q_{1}, x_{1}, y_{1}, \cdots, q_{m}, x_{m}, y_{m}\right)$$

where **a** denotes the array of fitting parameters.  $\sigma_{ij}$  denotes the data error in the i-th electrode at the j-th measurement. This pick-up model has such a pleasant symmetry that all of the response functions can be expressed with only one function,

$$\begin{split} F_1(x,y) &= 1 + a_1 x + b_1 y \\ &+ a_2 \Big( x^2 - y^2 \Big) + b_2 \Big( 2 x y \Big) \\ &+ a_3 \Big( x^3 - 3 x^2 y \Big) + b_3 \Big( 3 x y^2 - y^3 \Big) \\ &+ a_4 \Big( x^4 - 6 x^2 y^2 + y^4 \Big) + b_4 \Big( x^3 y - x y^3 \Big) \end{split}$$

$$F_2(x,y) &= F_1(-x,y), F_3(x,y) = F_1(-x,-y), F_4(x,y) = F_1(x,-y).$$

The BBGC was performed every month. The result of gain calibration is shown in Fig. 2.



Figure 2: Gain analysis in KEKB HER.

Here, horizontal scale is BPM No. and vertical scale is relative gain to  $g_{1}$ 

#### **NEW METHOD FOR J-PARC MR BPM**

In J-PARC MR, we adopted an electrostatic pickup with a diagonal-cut cylinder or racetrack duct to obtain good linearity over full aperture. The simulation was performed by using the method in previous section to estimate gains of MR BPMs. The simulation result showed that the gains were changed depending on the given initial values for  $q_j$ ,  $x_j$  and  $y_j$  in the fitting process. The non-linear fitting method was not able to apply for the gain analysis of such electrode as diagonal cut as shown in Fig. 3. We propose a new method for the BBGC for a diagonal-cut type BPM.



Figure 3: BPM with diagonal cut in J-PARC.

#### Model for a Diagonal Cut Pickup

Four outputs  $(V_L, V_R, V_U, V_D)$  of a diagonal cut electrode are defined as follow,

$$\begin{cases} V_{L} = \lambda \cdot \left(1 + \frac{x}{a}\right), & V_{R} = g_{R} \cdot \lambda \cdot \left(1 - \frac{x}{a}\right) \\ V_{U} = g_{U} \lambda \cdot \left(1 + \frac{y}{a}\right), & V_{D} = g_{D} \cdot \lambda \cdot \left(1 - \frac{y}{a}\right) \end{cases}$$
(1)

where a denotes cylinder radius. As same as in previous section, we set  $g_L=1$ . Two horizontal electrodes (Left, Right) and two vertical electrodes (Upper, Down) are attached independently each other as shown in Fig. 3.

Eliminating  $\lambda$ , x, y and a from four output Eq. 1 gives,

$$V_{\rm L} = -\frac{1}{g_{\rm R}} V_{\rm R} + \frac{1}{g_{\rm U}} V_{\rm U} + \frac{1}{g_{\rm D}} V_{\rm D}$$
 (2)

This linear equation express three gains in terms of four outputs. When beam positions are measured m times, the j-th measurement can be given by,

$$V_{Lj} = -\frac{1}{g_R} V_{Rj} + \frac{1}{g_U} V_{Uj} + \frac{1}{g_D} V_{Dj}, \quad j = 1, \cdots, m$$
(3)

The simultaneous linear equations are expressed in a matrix representation as follows,

# Total Least Squares Fitting

Calibration of gains is able to carry out from the simulated data, with the Least Square method (LS). The LS approximation is obtained optimum  $\hat{\mathbf{X}}$  from  $AX = B + \Delta B$ , then  $\left\| \Delta B \right\|_{E}$  are optimized as little as possible in the Frobenius norm sense. In this case of Eq. 4, **A** has an error  $\Delta \mathbf{A}$  too, the subject is  $(A + \Delta A)X = B + \Delta B$ . Because the LS minimizes the only one-dimensional sum of the squared vertical distance from the data points to the fitting surface, we must use the total least squares methods (TLS) for such 2-dimensional problem. TLS is very effective for the minimal correction such  $\left\| \left[ \Delta \mathbf{A} \ \Delta \mathbf{B} \right] \right\|_{\mathbf{F}}$  on given data A and B.

# Simulation Result

We compare the TLS method with LS method by using a simulations. In this simulations, the mapping data were generated from model outputs with the defined Eq. 1. 12500 points at 25 displaced positions with 0.2% Gaussian noise, as shown in Fig. 4. The gains were given reasonable values, set  $g_{1=1}$ ,  $g_{2=1.01}$ ,  $g_{3=1.005}$ , g4=0.975.

The results of values given to relative gains and variation from true gains in both TLS and LS simulations are summarized in Table 1. The TLS gives a smaller variations than LS. Corrected positions by obtained gains shows black points of Fig. 4.

Table 1: Simulation Result				
	<b>g</b> <sub>2</sub>	<b>g</b> 3	<b>g</b> 4	
TLS	1.0138	1.0066	0.9771	
Variation	0.0038	0.0016	0.0021	
LS	1.0381	1.0183	0.9889	
Variation	0.0281	0.0133	0.0139	



Figure 4: Reconstructed mapping data. Red: (x, y)without correction, Black: (x, y) with TLS.

## Test with Real Beam

We tested both of the TLS and the LS method for diagonal cut BPM in J-PARC MR. The position measurements was done in nine displacements of beam positions at the BPM. The results of gain calibrations are summarized in Table 2. The results of beam position corrected by obtained gain are shown in Fig 5. We can see differences in the relative gains depend on the fitting method.

Table 2: Test of Beam Based Gain Calibrations

BPM001	g <sub>2</sub>	g 3	g 4
TLS	1.0062	1.0024	0.9873
LS	1.0103	1.0045	0.9892
BPM002	$\mathbf{g}_2$	<b>g</b> <sub>3</sub>	$g_4$
TLS	0.9568	0.9811	0.9463
15	0.9617	0.9838	0 9487



Figure 5: Reconstructed mapping data. Red: (x, y) without correction, Black: (x, y) with TLS.

## SUMMARY

We developed a new analyzing method using TLS for the beam based gain calibration about diagonal cut BPM. The simulation result shows that this new method will give a more reliable results for the gain calibration.

## ACKNOWLEDGMENT

We would like to thank to Dr. J.Takano for his helpful support.

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