# COMBINED APPROACH USING CLOSED-ORBIT AND MULTITURN DATA FOR MODEL-INDEPENDENT AND FAST BEAM OPTICS DETERMINATION IN STORAGE RINGS 

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## Abstract

Multiturn-capable Beam Position Monitors (BPMs) have been used successfully for characterization of storage ring beam optics. While their use eases determination of optical parameters (e.g. $\beta$ function and phase) by observation of non ring-periodic beam centroid oscillation, the installation of multiturn electronics in all storage ring BPMs causes a high monetary effort.

The presented method aims at combining multiturn and closed-orbit measurement methods in a cost-effective way. This is done using a single drift section in the ring, being equipped with two multiturn BPMs at its ends. Measuring the centroid motion in the full transverse phase space, one can completely determine all local beam optics parameters inside the drift space. Then, four additional dipole correctors inside this drift are used to create closed-orbit perturbations along the ring. Because of the known drift optics, it is then possible to extract all data that would be available if all storage ring BPMs were multiturn-capable, by using only closed-orbit BPM data of the mentioned four perturbations (incl. betatron coupling).

This fast and model-independent approach may be increased in accuracy by two kinds of feedback system.

## THEORY

The perturbations of a closed orbit under influence of a dipole perturbation are dependent on the Twiss parameters that describe betatron oscillations along the ring. If no further information is obtainable, phase and amplitude of the oscillation can not be distinguished in the closed orbit data.

By using additional multiturn information from two longitudinal positions in the ring and two dipole correctors between them, this degree of freedom can be removed and $\beta, \phi$ parameters can be obtained along all other BPM positions (which are not multiturn capable) in the ring. Thus the method presented here consists of three steps:

- Determination of the optical functions within a drift section via multiturn BPMs [1],
- application of small closed-orbit perturbations using two correctors inside the forementioned drift section and
- calculation of optics parameters at all BPM positions, using the response to the closed-orbit perturbations, and known optics of the drift section.

[^0]
## Measure Drift Space Lattice Parameters Using Multiturn Data

If the transfer matrix between two multiturn BPM positions is known (and invertible), the full Poincaré section of the beam centroids motion can be obtained. After a short transverse kick, the beam centroid follows the betatron oscillation trajectory (neglecting damping) and thus betatron functions and phases can be derived. Although this technique is demonstrated in the $x-s$ (machine) plane with uncoupled betatron motion for simplicity, it is possible to include vertical motion and betatron coupling [1].

The transverse phase space coordinates $\left(x, x^{\prime}\right)^{T}$ of the undamped beam centroid at BPM $j$ after a short transverse kick and $n$ turns around the ring can be written as [3]

$$
\binom{x}{x^{\prime}}_{n j}=\sqrt{\Upsilon_{x}}\left(\begin{array}{cc}
\sqrt{\beta} & 0  \tag{1}\\
-\alpha / \sqrt{\beta} & -1 / \sqrt{\beta}
\end{array}\right)_{j}\binom{\cos \left[\phi_{j}+\mu_{x} n\right]}{\sin \left[\phi_{j}+\mu_{x} n\right]}
$$

with the horizontal Courant-Snyder invariant $\Upsilon_{x}$, betatron phase $\phi$, horizontal betatron phase advance per turn $\mu_{x}=$ $2 \pi Q_{x}$ and Twiss parameters $\alpha, \beta$. Since these sequences have sinusoidal form, each sequence $x_{n j}, x_{n j}^{\prime}$ can be described using only one phasor

$$
\begin{equation*}
X_{j}=\sqrt{\Upsilon_{x} \beta_{j}} e^{\mathrm{i} \phi_{j}}, \quad X_{j}^{\prime}=-\sqrt{\frac{\Upsilon_{x}}{\beta_{j}}}\left[\alpha_{j}-\mathrm{i}\right] e^{\mathrm{i} \phi_{j}} \tag{2}
\end{equation*}
$$

In a drift space of length $l$ with BPMs $j=1,2$ at its ends, the intercept theorem relates the phasors to each other via

$$
\begin{equation*}
X_{1}^{\prime}=X_{2}^{\prime}=\frac{X_{2}-X_{1}}{l} \tag{3}
\end{equation*}
$$

Since $X_{1 / 2}^{\prime}$ can be obtained from experimental data in a drift space, it is possible to determine the ring-global invariant $\Upsilon_{x}$ because it equals the area spanned up by the phasors $X_{1}, X_{1}^{\prime}$ in the complex plane (see Fig. 1). With known $\Upsilon_{x}$, one can determine $\beta_{j}$ and $\phi_{j}$ using the absolute value and complex angle of $X_{j}$ at all multiturn BPMs. The Twiss parameter $\alpha$ can also be obtained inside the drift section and is proportional to the slope of $\beta(s)$.

By further calculations [2], $\beta$ and $\phi$ functions inside the drift space can be expressed completely using measurable values from its edges and are thus known $(\beta(s)$ is a parabola inside the drift).

This method also works when decoherence phenomena are considered, because of its calculations in frequency domain [2].


Figure 1: Multiturn phasors $X_{1}, X_{1}^{\prime}$ span up an area in the Gauss plane that equals the Courant-Snyder invariant $\Upsilon_{x}$.

## Application of Closed-orbit Perturbations

A closed-orbit perturbation $x$ at the position of BPM $j$ by an additional corrector $k$ kick $\theta_{k}$ is described by [5]

$$
\begin{equation*}
x_{j k}=C_{Q} \sqrt{\beta_{j} \beta_{\tilde{k}}} \cos \left(\phi_{j}-\phi_{\tilde{k}}\right) \theta_{k} \quad \text { with } \phi_{j}>\phi_{\tilde{k}} . \tag{4}
\end{equation*}
$$

We propose a setup using two dipole correctors inside a drift section. Since the Twiss parameters at the corrector positions, including $\beta_{\tilde{k}}$ and $\phi_{\tilde{k}}$, are known from multiturn data, the closed-orbit perturbations only depend on the unknown optical functions $\beta_{j}$ and $\phi_{j}$ at the position of the BPMs $j$ where the perturbation is measured. The additional factor $C_{Q}$ relates to the betatron tune and is assumed constant during the measurement.

## Calculating Twiss Parameters

Thus, a system of two equations (4) for each BPM $j$ with the two unknowns $\beta_{j}$ and $\phi_{j}$ can be derived. This system is solvable with two exceptions that can be avoided in practice. The first exception is linear dependence of both corrector perturbations, equivalent to a difference in betatron phase $\Delta \phi_{12}=n \pi$ for $n \in \mathbb{Z}$ between both correctors. The second exception is the same condition for the multiturn BPM positions. A new variable

$$
\begin{align*}
u_{j}=\arctan \left(\frac{1}{\tan \Delta \phi_{\tilde{1} 2}}\right. & \left.-\frac{1}{\sin \Delta \phi_{\tilde{1} 2}} \sqrt{\frac{\beta_{\tilde{2}}}{\beta_{\tilde{1}}}} \frac{x_{j 1}}{x_{j 2}}\right) \\
& + \begin{cases}\pi & \text { for } x_{j 2}<0 \\
0 & \text { for } x_{j 2}>0\end{cases} \tag{5}
\end{align*}
$$

is introduced. Using this variable, one obtains

$$
\begin{gather*}
\beta_{j}=\frac{x_{j 2}^{2}}{\beta_{\tilde{2}} C_{Q}^{2} \theta^{2}}\left(1+\tan ^{2} u_{j}\right)  \tag{6}\\
\phi_{j}= \begin{cases}u_{j}+\phi_{\tilde{2}}+\mu_{x} / 2 & \text { for } \phi_{j}>\phi_{\tilde{2}}, \\
u_{j}+\phi_{\tilde{2}}-\mu_{x} / 2 & \text { for } \phi_{j}<\phi_{\tilde{2}} .\end{cases} \tag{7}
\end{gather*}
$$

as solution of the equation system.
Because it is possible to measure closed-orbit perturbations using conventional low-frequency BPM systems, $\beta$
and $\phi$ can be calculated at the positions of all BPMs in the ring. For unknown dipole kick strengths $\theta_{k}$ of the corrector electromagnets, one can scale the perturbation values using the multiturn drift measurement results.

## EXPERIMENTAL SETUP

The Dortmund Electron Accelerator (Delta) shown in Fig. 2 is a synchrotron light source at TU Dortmund University, comprising a 70 MeV linac, a full-energy synchrotron and a 1.5 GeV electron storage ring of 115.2 m length. Three insertion devices and 54 BPMs are installed in the storage ring, of which 44 BPMs work with ana$\log$ electronics $\left(f_{s}=10 \mathrm{~Hz}\right)$ [6]. 10 BPMs are multiturncapable ( $f_{s}=2.6 \mathrm{MHz}$ ) [7]. Up to now, BPM 39-41 are not capable of correct multiturn measurements.

For obtaining the results presented here, the insertion space ( $l=5.137 \mathrm{~m}$ ) between BPM 14 and BPM 15 is used as diagnostic section. The undulator between these BPMs was turned off, but exhibits small correction coils that were not in use before. We applied [8] a sinusoidal current with a frequency $<0.2 \mathrm{~Hz}$ and an amplitude $I_{\text {corr }}=3 \mathrm{~A}$ subsequently on the first and the last correction coil of the undulator and recorded all BPM readings. For the multiturn measurements, we utilized a diagonal "slotted-pipe" [9] kicker.

## RESULTS

Exemplary results of the combined method are shown in Figure 3. The theoretical $\beta$-function (red) and phase (green) from a lattice model, corresponding values from 7 multiturn BPMs (black crosses) and results from the combined method (blue circles) are shown. In the diagnostic section at $s \approx 30 m$, one can determine $\beta_{x}$ and $\phi_{x}$ continuously (indigo line).

The combined method results show good agreement with the multiturn method results which were derived solely from multiturn BPMs (also model-independent) and are acceptable in comparison with the lattice model.


Figure 2: Sketch of the Dortmund Electron Accelerator.


Figure 3: Top: Clipping of 2048 turns of position data from BPM14. Middle: Fourier transform of multiturn data, frequency axis normalized to circulation frequency. The maximum component (tune: 0.16) is taken as phasor for multiturn calculations. Bottom: Results for horizontal $\beta$ function and phase for different beam currents $I_{\text {beam }}$.

## Comparison to Other Diagnostic Methods

One of the presented methods advantages is a fast measurement procedure which is slower than in multiturn, but faster than in closed-orbit methods. Since only two multiturn BPMs have to be synchronized, also less synchronization issues are occuring. The combined method is modelindependent, an advantage to fitting procedures like LOCO. Another important advantage is the compatibility to multi-
turn data analysis, since all combined method results can be converted to full multiturn data at all BPMs [2].

Possible error sources are imperfect beam position measurements, e.g. by pincushion distortion, and timing difference errors of the two used multiturn BPMs. Timing errors are common to multiturn measurements, but are reduced by the comparably low synchronization issues.

## Planned Bbв/Fof Experimental Setup

A bunch-by-bunch feedback system (Ввв) [10] is about to be installed at the Delta storage ring. It enables to drive transverse modes by an arbitrary excitation kicker signal. This system may be used to obtain multiturn data, thus replacing the short pulses of the installed kicker system. Since the oscillation is driven, no decoherence damping should occur, enabling data retrieval from $\approx 6 \cdot 10^{4}$ turns [7] of oscillation information, thus multiplying present resolution of the combined method by $\approx 30$.

A fast closed-orbit feedback (Fof) [4] is also planned for Delta, allowing application of closed-orbit perturbations with higher frequencies ( $f_{s}=300 \mathrm{~Hz}$ ), thus increasing signal-to-noise level of closed orbit measurements.

## ACKNOWLEDGMENTS

The authors thank G. Schünemann, P. Towalski, H. Rast, P. Hartmann, D. Schirmer and S. Khan for valuable discussions. This work is partly funded by the BMBF under contract 05K10PEB.

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