Production of Thermal Positrons at ERL

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Abstract. For generation of slow positrons at ERL, we are suggesting the usage of helical ~10T wiggler installed at 5 GeV route for generation of hard gammas. Despite the critical photon energy of radiation is about 170 keV only, the flux in hard part of spectra with photon energy $\hbar\omega > 2mc^2$ generated by 100 *mA* 5 *GeV* beam is big enough for generation of polarized thermal positrons with the rate ~10^{11} e^+ /sec.

OVERVIEW

ERL oriented generally for generation of electromagnetic radiation (SR/X-rays) for further usage this radiation for investigation in different sciences. One more application of ERL might be in creation of positrons by these radiations and usage of positrons instead of photons may open new possibilities in sciences.

Slow (thermal) positrons are a powerful instrument for investigation of properties of materials [1]-[7] due to their negative affinity to the media. Typically slow positron energy lies within <1 *keV*. Broad usage of positrons for this business slowed down by absence of intense source of low energy positrons with appropriate flux.

What for slow positrons could be used is described well in the references mentioned above. Among them are:

- Transmission and scanning microscopy; mostly promising emerges the possibility to switch between electrons/positrons for better resolution.
- Probing the surface by measuring the energy loss, diffraction and re-emission.
- Defects searching. As positrons could be trapped easily in volume defects even by single missed atom defect, theirs annihilation could be identified by measuring to point of creation of gammas created by annihilation process.
- Probing the Fermi-surface. Pair annihilation and following two-photon emission rate is proportional to the local electron density. The point of creation of two (or rarely three) photons could be resolved with adequate resolution ~nm^3 by measuring Doppler shifts in each photon and deflection from straight line.
- Positron holography.
- Some others, see [1] and references in there.

One way in use for positron creation is a beta decay of isotopes ^{22}Na(2.6 year half life time) or ^{58}Co (71day). The isotopes of ^{64}Cu (12.7 hour), ^{18}F (110 min), ^{11}C(20 min) are in use for these purposes also.

There is basically other practical way for getting the positrons in vast amounts: via electron-positron pair creation by gamma quanta (photon) of appropriate energy and flux in a field of nuclei. The photons in its turn could be generated either by beamstrahlung of electrons in the field of nuclei or by synchrotron or undulator radiation (SR or UR). SR or UR radiation to be effective must create the quantas of appropriate energy $E_\gamma \geq 2mc^2 \cong 1.22\,MeV$. Typically to be effective, the energy of the primary beam must be high if *K*-factor is low, $K = eH\lambda_u / 2\pi mc^2$, (which corresponds to operation at low, even single harmonic) or the magnetic field value in wiggler must be high (operation at high harmonics) so the critical energy of SR $\hbar\omega_c = \frac{3}{2}\hbar c\gamma^3 / \rho \approx mc^2$ (where ρ stands for the local bending radius). Typically smaller the aperture of magnet-easier the required value of magnetic field could be achieved.

Typically insertion devices in damping ring have wiggler either with movable poles or SC wigglers with relatively large aperture. The last circumstance is a limiting factor for maximal field achievable in a wiggler. ERL (as well as any FEL) has one undoubted advantage over traditional storage ring: the insertion devices could have very small aperture, as there is no necessity for any kind of damping of betatron amplitude of injected beam. Although the same result could be achieved with usage of booster (pre-damping) rings, or injectors with very small emittance, ERL solves the problem with insertion devices mostly natural way.

THE CONCEPT

The concept we are suggesting is in line with our proposal [9]. The positrons generated by photons created in helical wiggler by energetic electron beam. In [10] the idea was developed to use strong planar wiggler installed in a damping ring for generation of hard photons, inspired by [9].

So basically we review this old idea for possible implementation at ERL. As the energy of beam is not as high as originally suggested in [9], the only way to get hard energy photons is to operate at high harmonics of UR. Spectrum of radiation becomes pretty much the same as for planar magnet, however. Although we are not interesting much in polarization of positrons, the overall polarization of the positron beam could reach ~30% if no special measures applied. As we are suggesting using collimator for the photons, polarization of gammas might reach 100% theoretically, but as we are not selecting these positrons in narrow energy margins, further enhancement of polarization is possible only by reduction of intensity.

SPECTRUM AND ENERGY SPREAD

Any wiggler installed to the beam line yields to the growth of energy spread and emittance. However if it is installed and the end of high energy branch, before entering the recuperation linac, the only important issue remains is the energy spread. We will see that emittance growth remains within acceptance of transporting optics. Let we calculate all this in more detail.

Cyclotron frequency for reference field is

$$\omega_0 = \frac{c}{\rho} = \frac{eB}{m\gamma} \tag{1}$$

Which for 10T comes to $\omega_0 = \dfrac{eB}{m\gamma} \cong 1.758 \cdot 10^{11} rad \cdot s^{-1} \cdot T^{-1} \cdot 10\,T \cdot 10^{-4} \cong 1.76 \cdot 10^{8} rad \cdot s^{-1}$.

Local radius of curvature comes to $\rho \cong \dfrac{mc\gamma}{eB}$. Energy of quanta at first harmonic is $\hbar\omega_0 \cong 6.58 \cdot 10^{-22} \cdot 1.76 \cdot 10^{8} \cong 1.16 \cdot 10^{-13} MeV$. Intensity of radiation could be expressed as

$$I = \tfrac{2}{3} mc^2 \frac{r_0 c\gamma^4}{\rho^2} = \tfrac{2}{3}\frac{e^2 r_0 c\gamma^2 B^2}{m}, \tag{2}$$

while the spectrum is [6]

$$\frac{dI}{d\omega} = \tfrac{2}{3} mc^2 \frac{r_0 c\gamma^4}{\rho^2}\frac{9\sqrt{3}}{8\pi}\frac{\omega}{\omega_c^2}\int\limits_{\omega/\omega_c}^{\infty} K_{5/3}(x)dx = I\frac{9\sqrt{3}}{8\pi}\frac{\omega}{\omega_c^2}\int\limits_{\omega/\omega_c}^{\infty} K_{5/3}(x)dx \tag{3}$$

As integral of spectral density over all frequency interval must be equal to full intensity, i.e. $\int\limits_0^{\infty}\frac{dI}{d\omega}d\omega = I$, the $P(\omega) = \frac{9\sqrt{3}}{8\pi}\frac{\omega}{\omega_c^2}\int\limits_{\omega/\omega_c}^{\infty} K_{5/3}(x)dx$ could be treated as the probability of radiation of photon with frequency ω, i.e. with energy $E_\gamma = \hbar\omega$. Expanding Bessel function for the argument values of our interest $x \equiv \omega/\omega_c >> 1$ as $K_{5/3}(x) \cong \sqrt{\frac{\pi}{2x}}e^{-x}$ spectral distribution of intensity (3) could be transformed to

$$\frac{dI}{d\omega} = I\frac{9\sqrt{3}}{8\sqrt{\pi}}\frac{\omega}{\omega_c^2}\int\limits_{\omega/\omega_c}^{\infty}\frac{e^{-x}}{\sqrt{x}}dx \cong I\frac{9\sqrt{3}}{8\sqrt{2\pi}}\frac{1}{\omega_c}\sqrt{\frac{\omega}{\omega_c}}\exp(-\frac{\omega}{\omega_c}) \tag{4}$$

Photon SR spectrum represented in Fig.1.

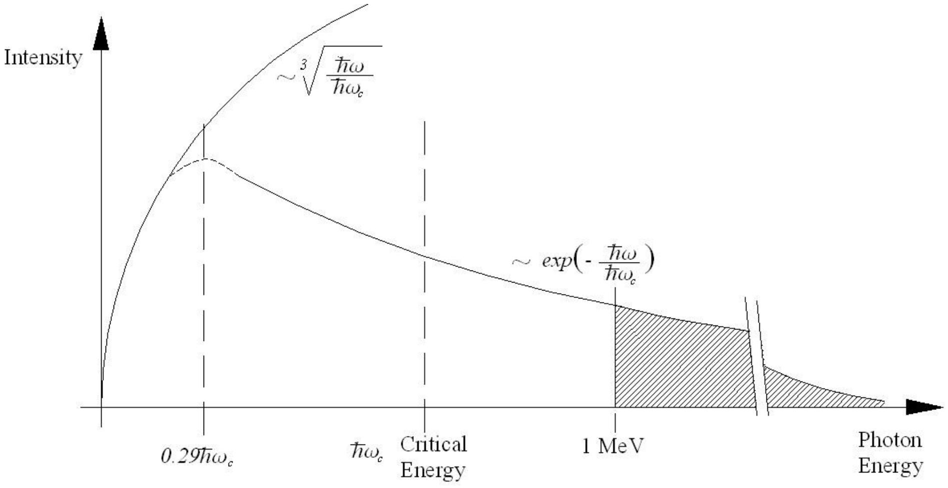

Figure 1: SR spectrum. Hatched area corresponds to the photon flux able to create positrons.

Spectral distribution of the photon flux could be obtained from (4) by dividing it by the energy of the photon ($\hbar = 6.58\cdot10^{-22} MeV\cdot s$)

$$\frac{d\dot{N}_\gamma}{d\omega} = \frac{dI}{\hbar\omega\cdot d\omega} \cong I\frac{9\sqrt{3}}{8\sqrt{2\pi}}\frac{1}{\hbar\omega}\frac{1}{\omega_c}\sqrt{\frac{\omega}{\omega_c}}\exp(-\frac{\omega}{\omega_c}). \tag{6}$$

By introducing a variable $y = \omega/\omega_c$ this expression could be rewritten as (y>>1)

$$\frac{d\dot{N}_\gamma}{dy} \cong I\frac{9\sqrt{3}}{8\sqrt{2\pi}}\frac{1}{\hbar\omega_c}\frac{\exp(-y)}{\sqrt{y}} . \tag{7}$$

Threshold value of variable y comes to $y \geq y_c = 2mc^2 / \hbar\omega_c$ (in our case $y_c \cong 5.8$) . So the total number of photons radiated per one second, which energy is enough to create a pair coming to

$$\dot{N}_\gamma \cong I \frac{9\sqrt{3}}{8\sqrt{2\pi}} \frac{1}{\hbar\omega_c} \int_{y_c}^{\infty} \frac{\exp(-y)}{\sqrt{y}} dy \cong I \frac{9\sqrt{3}}{8\sqrt{2}} \frac{1}{\hbar\omega_c} Erfc[\sqrt{y_c}] \tag{8}$$

Graph of function *Erfc*[y_c] is represented in Fig. 2.

Let us evaluate the photon flux. First the local bending radius in (2) for 10*T* (=100*kG*) field comes to $\rho = mc^2\gamma / eB \cong 167\ cm.$ As $r_0 = e^2 / mc^2 \cong 2.8 \cdot 10^{-13} cm$, $\gamma \cong 10^4$, total energy carried out by all photons while the particle passes the wiggler having length *L*=100 *cm* comes to

$$I \cdot \frac{L_w}{c} \cong \tfrac{2}{3} mc^2 \frac{r_0 c \gamma^4}{\rho^2} \frac{L_w}{c} \cong 1.022 MeV \frac{2.8 \cdot 10^{-13} \cdot 3 \cdot 10^{10} \cdot 10^{16}}{3 \cdot 2.8 \cdot 10^4} \frac{100}{3 \cdot 10^{10}} \cong 3.66[MeV] \tag{9}$$

So the number of photons with their energy >1.022 MeV, radiated by each electron per one pass according to (8) comes to

$$\Delta N_\gamma \cong 3.66[MeV] \cdot 1.37 \cdot 10^{-3} \cdot 10^{-3} / 0.17[MeV \cdot \sec] \cong 3.2 \cdot 10^{-5} [Photons / pass]$$

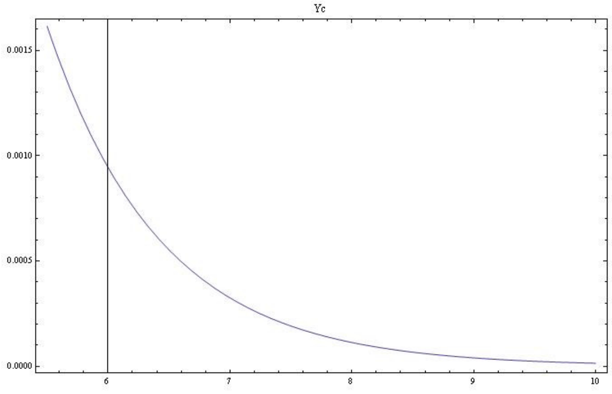

Figure 2. Function *Erfc*[y_c].

As the number of electrons in ERL corresponds to average current $I = eN_b f \cong 0.1[A]$, where *f*=1.3*GHz*, then the number of “useful” photons (which energy >1.022 MeV) per second comes to

$$\dot{N}_\gamma \cong \Delta N_\gamma N_b f = \Delta N_\gamma I / e \cong 3.2 \cdot 10^{-5} \cdot 0.1 / 1.6 \cdot 10^{-19} \cong 2 \cdot 10^{13} [Photons / \sec] \tag{10}$$

Of cause not all of these photons are equally effective for positron production, as the threshold cross section of pair production is rather sensitive to the energy, $\sigma_{\gamma \to e^- e^+} \cong \frac{\pi}{12} Z^2 \alpha\, r_0^2 (\frac{\hbar\omega - 2mc^2}{mc^2})^3$ [12].

For calculation of ***energy spread*** in the beam generated after passage the wiggler field, the number of photons with critical energy is important. The number of photons with critical energy radiated by high energy beam with $\gamma = E / mc^2$ bending with angle φ

$$N_{\gamma} \cong \alpha\gamma\varphi\,,$$

where $\alpha = e^2/\hbar c$. So for 5 GeV beam, $\varphi \cong \lambda_w/\rho \cong \frac{3}{2}\cdot 30/167 \cong 0.27$ (per $1m$) each electron radiates $N_{\gamma} \cong 19.7$ per one pass. We expect that these will be a ~0.17-MeV gammas (magnetic field chosen so it is), so the energy loosen by each particle per one pass is

$$\hbar\omega N_{\gamma} \cong 0.17MeV\cdot 19.7 \cong 3.35MeV\,,$$

which is in good agreement with (9). The energy spread comes to

$$\Delta E \cong \hbar\omega\sqrt{N_{\gamma}} \cong 0.17MeV\cdot 4.4 \cong 0.75MeV\,, \tag{5}$$

i.e. relative spread about $\delta E \equiv \Delta E/E \cong 1.6\cdot 10^{-4}$. We would like to underline here that this absolute energy spread remains constant down to the collector. So it will be necessary slightly increase of dump energy (which is ~15 *MeV* now) by this value (5).

Looks, that method [9] is feasible for ERL. Slight difference between [10] and present proposal: helical wiggler instead of planar with period 30 *cm* and with axial field up to 8-10*T*.

As the harmonics number for critical energy is $n_c = \frac{3}{2}\gamma^3 \cong 1.5\cdot 10^{12}$, the critical energy value comes to $\hbar\omega_c \cong n_c\cdot\hbar\omega_0 \cong 0.174$·MeV. Radiation with energy ~ 1MeV corresponds to the number of harmonic

$$n_{c\gamma} \cong n_c\cdot\frac{2mc^2}{\hbar\omega_c} = \frac{2mc^2}{\hbar\omega_0} = \frac{2m^2c^2\gamma}{\hbar eB} \cong 10^{13}$$

Angle of radiation is a conic one with opening $\vartheta \cong K/\gamma$ which is $\vartheta \cong 10^{-2}$ in our case, so at the distance of *L*=5 m the radius of gamma beam will be $r = L\vartheta = KL/\gamma \cong 5cm$. The gamma beam will be hollow, however, so the trace of the gamma beam on the target will be a ring-like line with characteristic thickness ~0.5*mm*.

Emittance growth could be calculated on the basis

$$\frac{d\varepsilon_x}{ds} \cong \left\langle\left(H_x + \frac{\beta_x}{\gamma^2}\right)\frac{d(\Delta E/E)^2_{tot}}{ds}\right\rangle - 2\alpha_x\varepsilon_x\,, \tag{6}$$

with similar equation for vertical motion, where defined

$$H_{x,y} = \frac{1}{\beta_{x,y}}\left(\eta^2_{x,y} + (\beta_{x,y}\eta'_{x,y} - \frac{1}{2}\beta'_{x,y}\eta_{x,y})^2\right), \tag{7}$$

$\eta_{x,y}$ –are dispersion functions in wiggler. As outside values of dispersion could be chosen so they are to be about zero, the dispersion in (7) is the one generated bu wiggler itself. Partial decrements $\alpha_{x,y,s}$ are defined as $\alpha_i = J_i/2l_s$, where $J_x \cong 1, J_y = 1, J_s \cong 2, J_x + J_s = 3$. Partial decrement for energy spread is the same as the one for emittance. If dispersion generated by wiggler itself, then

$$\eta_x = \frac{K_x\bar{\lambda}}{\gamma}Sin\frac{s}{\bar{\lambda}} = \frac{\bar{\lambda}^2}{\rho_x}Sin\frac{s}{\bar{\lambda}}\,, \tag{8}$$

where $\lambdabar \equiv \lambda_w / 2\pi$, $\rho_x = \lambdabar\gamma / K_x$, λ_w stands for the wiggler period (and the same for other coordinate). So change of emittance comes to

$$\Delta\varepsilon_x \cong \frac{1}{2}\frac{K^2\bar{\beta}_x}{\gamma^4}\Delta(\frac{\Delta E}{E})^2_{tot} - 2\alpha_x\varepsilon_x L \tag{9}$$

Estimating the first term (source of heating) one can obtain

$$\Delta\varepsilon_x \cong \frac{1}{2}\frac{K^2\bar{\beta}_x}{\gamma^4}\Delta(\frac{\Delta E}{E})^2_{tot} \cong \frac{1}{2}\frac{250^2\cdot 10}{10^{16}}10^{-6} \sim 3\cdot 10^{-17}\, m\cdot rad \,,$$

i.e. a negligible value.

HELICAL WIGGLER

Difference from [10] is in a wiggler: we suggesting *helical* wiggler with period 30 cm and with axial field up to 8-10T. Basically design of helical wiggler is similar to high field dipole magnet. The necessary twist with period 30 is big compared with aperture, which is 2*a*~30*mm*. Field distribution inside aperture is shown in Figs. 3-4 calculated with numerical code MERMAID.

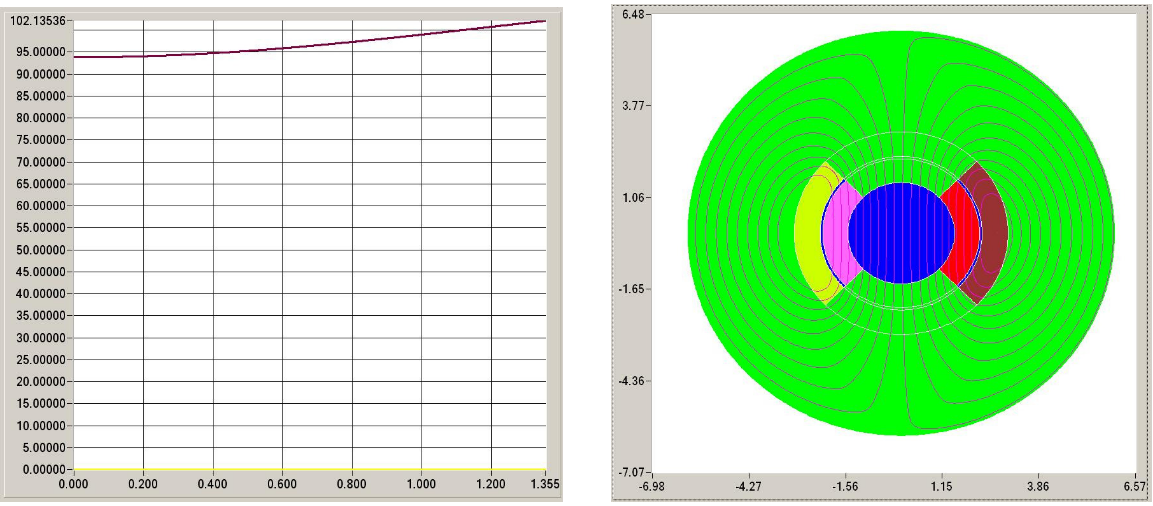

Figure 3: Field across aperture, at the left. Lines of magnetic field, at the right.

Coil is sectioned in two separate ones. The inner one carries total current ~80kA having area ~2cm^2. Outer coil carries has ~450 kA with area~3cm^2. So the current density runs 39.3 and 149.07 kA/cm^2 respectively.

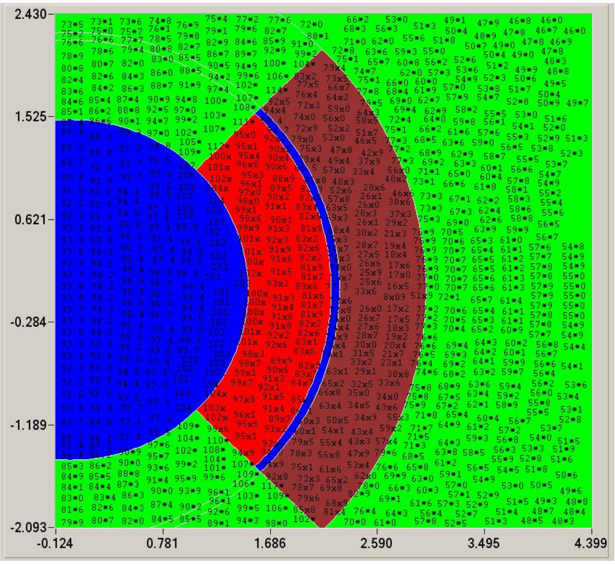

Figure 4: Numerical values of the field in helical magnet.

Trajectory of particle in a wiggler looks like a helix having radius $r = \bar{\lambda} K / \gamma \cong 1mm$. Some set of end correction coils required to keep the first and the second field integrals around zero values.

TARGET AND MODERATOR

Positrons have negative affinity to the solid state media, see Fig.5 . So finally all positrons must came out if not trapped into defects and if not annihilated with electrons riding between atoms. That is why the purity of moderator plays important role here.

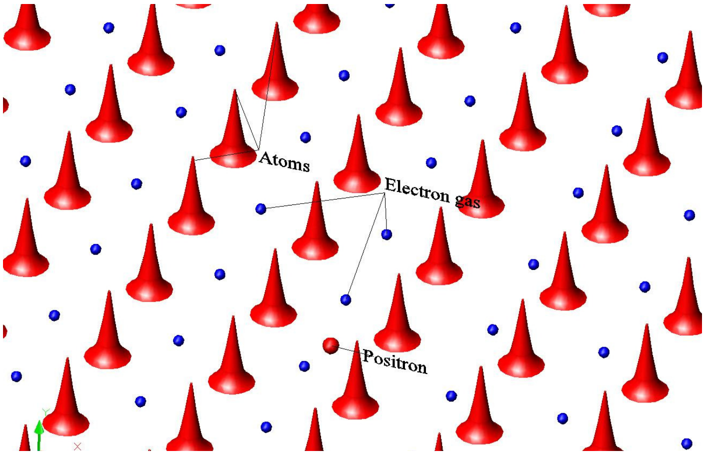

Figure 5: Schematics of potential well for positrons.

Positrons are under repulsive force from nucleus, Fig.5, the only valence electrons acting with positrons, yield annihilation finally.

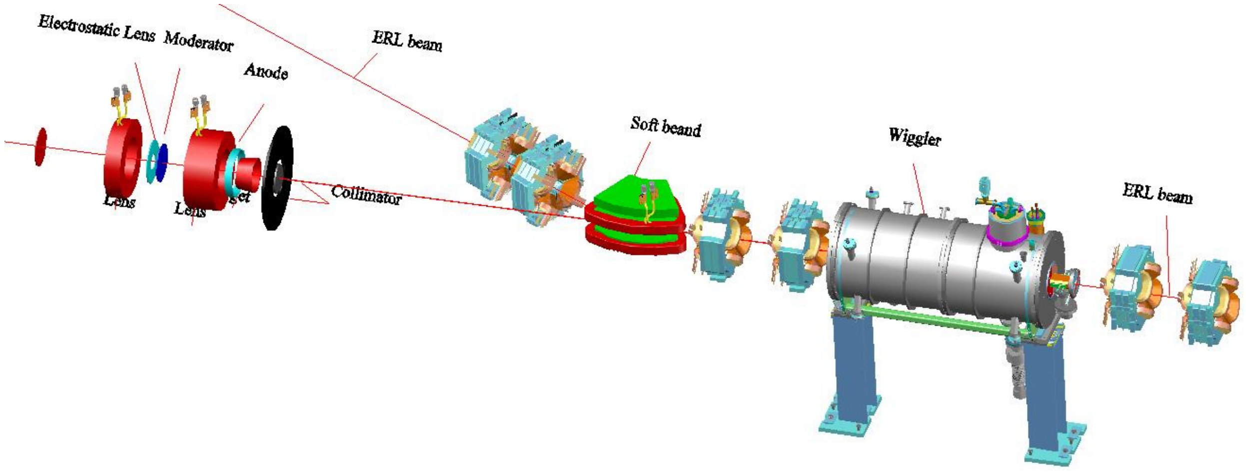

Figure 6: 3D view on the installation. ERL beam coming from the right.

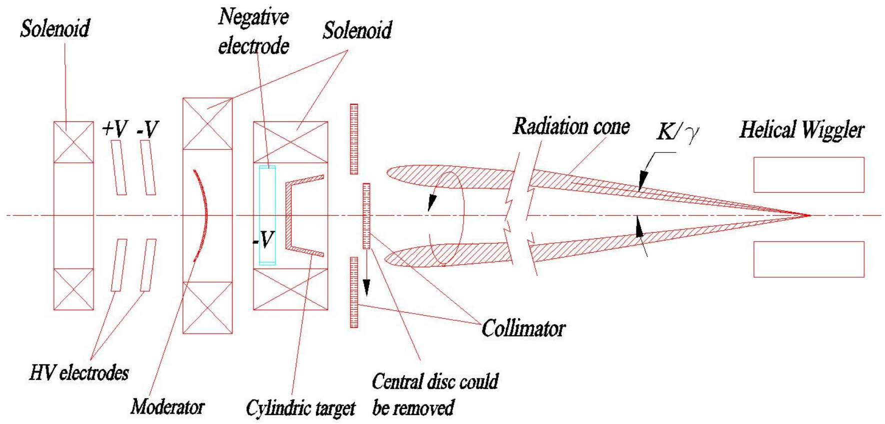

Figure 7: Positron conversion system schematics. Radiation cone has opening ~ K/γ .

Positron target in Fig.6 Fig.7 is a pot-like one. The side conical wall has some small angle and thickness, but effective thickness of the target in longitudinal direction is ~few X_0, where $X_0^{-1} \cong 4r_0^2\alpha\frac{N_0}{A}(Z+1)Z\cdot\ln(\frac{183}{Z^{1/3}})[cm^2/g]$ stands for the Radiation length ($\alpha \cong 1/137$ –fine structure constant, N_0 –Avogadro number, A-is the atomic weight, Z–is the atomic number). Positrons created in a target can escape easily in transverse direction. The positrons from outer side only could be collected for further usage. So basically this positron target resembles a type of magnetron electron gun serving for generation of hollow beam.

So the probability of positron creation in this target ~100%, but we estimate that only 15% could be transformed into positrons which could be collected.

Moderator made from single crystal foil (sheet) with spherical profile. Together with electrostatic electrode kept at negative potential, it serves for better collection and focusing of positrons. Presence of electric field helps positrons to leave the surface of moderator. Tungsten, Copper could serve as materials for moderator; what is important-absence of defects in crystal structure. Yield between 10^{-2} -10^{-4} could be expected here.

As the energy loss per one pass is ~3.66 *MeV*, then for current 100 *mA* the total power radiated comes to ~366 *kW*, distributed on a ring of 5cm in diameter. We estimate that at least 60% of this power will be accepted by collimator. Absorption of ~100kW of power illuminating the target itself will be not a problem as the power density remains low.

SUMMARY

For generation of thermal positrons we revisited old idea about positron production from gammas. The gammas generated by the high-energy electron beam by Wiggler/Synchrotron Radiation in magnetic field. This might be a planar of helical wiggler installed in ERL. One important peculiarity of ERL is that the beam is small at all times, so the aperture of such wiggler might be small. In its turn this allows high magnetic field value. Recuperation of energy is also very important item here as the primary electron current must be high to support the positron yield of interest. Perturbation of emittance and introduced energy spread remains within acceptable for further recuperation.

More detailed calculation could be done if necessary, as all phenomena is well developed topics.

By itself, the method of generation of hard radiation might be interesting for research carrying with high energy photons (above 1 MeV).

Table 1. Basic parameters of system

Parameter	**Value**
Energy of the beam	5 GeV
Current	100 mA
Magnetic field	~9T
Period	~0.3m
Number of periods	3
K factor	~250
Emittance grows	negligible
Energy spread grows	~0.75 *MeV**
Positron flux	10^{8}-10^{11}/sec
Polarization	>30%

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* depends on positron flux required.

REFERENCES

[1] K.G.Lynn, M.H.Weber," *Positrons for Applied Science*", a talk at JPOS09-International Workshop on Positrons at Jefferson Lab, Newport News, VA, March 25-27, 2009.

[2] A.Uedonot et al, "*Positron Annihilation in Silicon in Thermal Equilibrium at High Temperature*", Phys.: Condensed Matter **12** (2000) 719-728.

[3] P.X.Gan, et al., "*Introduction to Beam-Based Positron Spectroscopy Techniques*", Physics 152A: Condensed Matter Physics, March 2, 2004.

[4] A.G.Zaluzhnyi et al., "*Thermalization of Positronium Atoms in Ionic Crystals*", ISSN 1027-4510, Journal of Surface Investigation, X-ray, Synchrotron and Neutron Techniques, 2008, Vol.2 pp.92-97.

[5] F.Komori, " *Thermalized-Positron Induced Ion Deposition from Surfaces*", Current Option in Solid State and Material Science 6(2002) 163-167.

[6] A.Dupasuier, A.P.Millsjr, editors.” *Positron Spectroscopy of Solids*”, Amsterdam, Oxford, Tokyo, Washington DC:IOS Press; 1995.

[7] Terry L. Dull, "*Probing Low-k Dielectric Films with Depth-Profiled Positronium Annihilation Lifetime Spectroscopy*." Workshop on Positron Studies of Semiconductor Defects (PSSD-99), McMaster University, ON, Canada, July 21 - 23, 1999. http://www.obrien-dull.com/profpages/talks.html .

[8] G.A.Schott, “*Electromagnetic Radiation and the Mechanical Reactions Arising from It*”, Cambridge: at the University Press (1912), 327pp.

[9] V.Balakin, A.Mikhailichenko,”*The Conversion System for Obtaining High Polarized Electrons and Positrons*”, Budker Institute of Nuclear Physics, BINP 79-85 (1979).

[10] A.Ando, V.Belyaev, S.Daté, H.Kamitsubo, A.Kiselev, G.Kulipanov, N.Kumagai, N.Mezentsev, Y.Miyahara, T.Nakamura, A.Skrinsky, K.Soutome, M.Takao, I.Voronchev, ”*Production of Intense Low-Energy Positron Beams with Synchrotron Radiation*”, J. Synchrotron Rad. (1996), **3**, 201-206.

[11] P.Stefazzo, “*Slow positrons and Thermal Ps Production Using Thin Foils*”, Appl. Phys. A 36, 93-95 (1985).

[12] V.Berestetsky, E.Lifshits, L.Pitaevsky,”*Quantum Electrodynamics*”, §94.