INVESTIGATION OF THE EFFECT OF SPACE CHARGE IN THE COMPACT-ENERGY RECOVERY LINAC

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Abstract

Compact energy recover linear(ERL) accelerator is a prototype of the 5 GeV ERL accelerator at KEK. The injector system has two SRF cavities which have the frequency of 1.3 GHz. It accelerates the bunches to the energy of 5 MeV. This beam was injected to the main ring and then it was accelerated to energy of 35 MeV at the main superconducting RF linac. Due to the low beam energy on the main ring, the investigation of the effect of space charge (SC) which causes the growth of the energy spread is important to produce the low emittance beam. For the production of the low emittance beam, the optimization of the merger was performed. To obtain smaller emittance at the exit of merger, the effect of the energy spread was also investigated by changing of the k_d which is defined by the ratio of energy spread to length of the bunch. In this calculation, we got the noralized transverse emittance of 0.735 mm·mrad at the exit of merger section.

INTRODUCTION

The Energy Recovery Linear accelerator (ERL) is one of the candidates for the fourth generation light sources that can meet these requirements. The main feature of the ERL is production of low-emittance(pm) beam with energy recovery in the main linac. The ERL requires sophisticated technology of superconducting accelerator. The generation of ultra-low emittance beams is need to demonstrate before constructing Multi-GeV ERL. The compact-ERL at KEK, in the final stage, will provide a beam energy of around 125 MeV and a bunch charge of 77 pC, which is a prototype for the future 5 GeV ERL at KEK. The layout of the compact-ERL is shown in Fig 1. The c-ERL consists of an injector system, a merger section, a superconducting RF (SRF) section, two return loops and two straight sections[3]. In the early comissioning phase of the compact-ERL, the energy is 35 MeV with a bunch charge of 7.7 pC. The electron injector system consists of a 500 kV photo cathode DC gun, two solenoid magnets, a buncher cavity, three superconducting RF cavities, seven quadrupole magnets and a merger section. In the second comissioning phase, the injector produces electron beams with a bunch charge of 77 pC, beam energy of 5 MeV and bunch length of 0.6 mm rms. The beam energy is increased by 30 MeV with two 9 cell SRF cavities. Since the beam energy in c-ERL is a low with high charge, we need to consider the several effects, e.g., the space charge effect, the coherent synchrotron radiation (CSR) effect, the wake function, ion effects and beam break up[4]. In the case of low energy, the electric force which caused the growth of the energy spread is more stronger than the magnetic force. It called SC effect. The emittance growth due to the space charge (SC) effect is dominated for the case of low-energy, around 5 MeV [5], and causes growth of the energy spread. The energy spread induced in an achromatic cell results in the growth of projection emittance at the exit of the achromatic cell. It is known that this emittance growth can be compensated by setting the cell-to-cell betatron phase advance at an appropriate value[6].

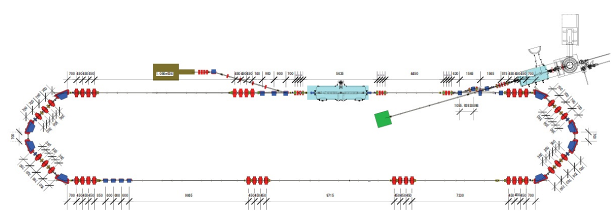

Figure 1: Layout of a compact-ERL.

ENERGY SPREAD GROWTH DUE TO THE SC EFFECT

The low energy beam injected from the injector system merges with the circulating high energy beams. For the beam mergence, after passing the merger section, the ratio of circulating energy to injected energy should be large because the circulating beam is also kicked and needs to be bumped at the merger section. A merger section with 3-dipole was adopted for the flexible beam transport of the high energy circulating beam. The layout of the 3-dipoles merger is shown in Fig. 2.

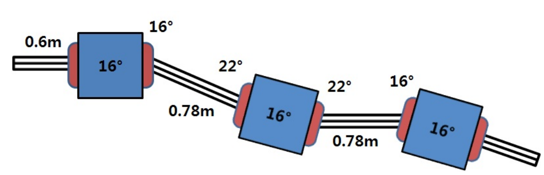

Figure 2: Layout of a merger section.

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As shown in Fig. 2, the bending magnet at the center of the merger is sector type and bending magnets at the entrance and exit of the merger are rectangular type. The center bending magnet have an edge angle to achieve zero dispersion at the exit of the merger section. The space charge is dominated in the low-energy injector system. The characteristic field, which produced by the cylinderical unifom distribution, is given by Eq. 1.

$$E_z(z) = \frac{Q}{2\pi\epsilon_0 a^2}\left[-\left|\frac{1}{2}-\frac{z}{L}\right|+\left|\frac{1}{2}+\frac{z}{L}\right| + \sqrt{\left(\frac{z}{L}-\frac{1}{2}\right)^2+A^2}-\sqrt{\left(\frac{z}{L}+\frac{1}{2}\right)^2+A^2}\right] \tag{1}$$

, where a is radius, ϵ_0 is permittivity, $A = a/\gamma L$, Q is bunch charge and $L = \sqrt{12}\sigma_z$ is full length of bunch. Based on the characteristic field, the growth of the energy spread due to longitudinal space charge force(LSCF) can be calculated that is shown in Fig. 3.

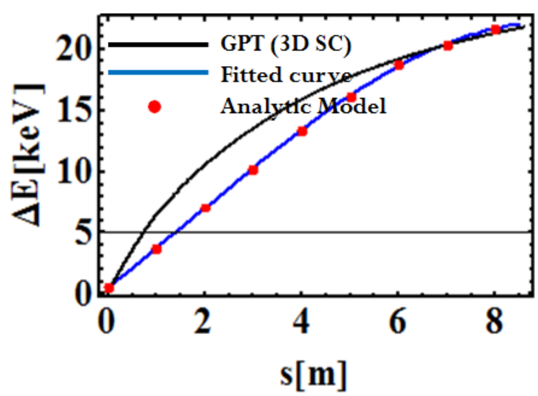

Figure 3: Growth of the energy spread due to the LSCF.

The growth of the energy spread due to the LSCF is fitted by $\Delta E = k_0 + k_1 s + k_2 s^2 + k_3 s^3$ formula. The fitted paramter is given by $k_0 = 0.45 \pm 0.09$, $k_1 = 3.20 \pm 0.11$, $k_2 = 0.092 \pm 0.033$ and $k_3 = -0.020 \pm 0.0027$ when the charge of bunch is 77 pC, energy of bunch is 5 MeV, radius of bunch is 0.2 mm and the length of bunch is 3 ps with uniform distribution on longitudinal phase space. Based on the this calculation, the analytical calculation of the emittance growth in the merger section was performed by using the first-order theory[7]. In the constant LSCF regime, the Eq. 2 can be solved analytically, and electron dynamics through an achromatic cell is calculated using the extended R-matrix [4].

$$x'' = -\frac{x}{\rho^2} + \frac{1}{\rho}\left(\delta_0 + \delta_{SC} + \frac{k_1}{E_0}(s-s_0) + \frac{k_2}{E_0}(s-s_0)^2 + \frac{k_3}{E_0}(s-s_0)^3\right) \tag{2}$$

A vector was defined to express electron motion: $(x, x', \delta_0, \delta_{SC}, k_1/E_0, k_2/E_0, k_3/E_0)^T$. And an R-matrix for a sector magnet is given by

$$\begin{pmatrix} \cos\theta & \rho\sin\theta & \rho(1-\cos\theta) & \rho(1-\cos\theta) & \rho^2(\theta-\sin\theta) & \rho^3(\theta^2+\cos\theta-2) & \rho^4(\theta^4+6\sin\theta-6\theta) \\ \frac{-\sin\theta}{\rho} & \cos\theta & \sin\theta & \sin\theta & \rho(1-\cos\theta) & 2\rho^2(\theta-\sin\theta) & 3\rho^3(\theta^2+2\cos\theta-2) \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \rho\theta & (\rho\theta)^2 & (\rho\theta)^3 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{3}$$

The LSCF is the main source of the emittance growth in the low-energy beam. In the analytical calculation, we assume that the longitudinal and transverse bunch lengths and sizes are not largely changed in the merger section. The energy spread of the beam due to the SC force was induced in the upstream part of the merger section. The slice of the beam in horizontal phase space has a different position in the downstream part of the merger section, due to the energy spread. The projected emittance grew due to the SC effect, shown in Fig. 4.

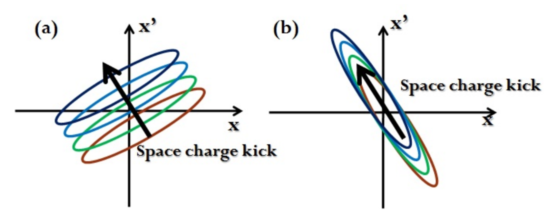

Figure 4: Growth of projected emittance due to the SC effect in merger section. (a) Maximum case of projected emittance growth (b) Mimimum case of projected emittance growth.

The emittance growth due to the displacement of the bunch slices in phase space can be minimized by matching the displacement to the orientation of the phase ellipse at the exit of the merger. The first-order theory was used to calculate the space charge kick angle, because the displacement of the bunch slice is laid on the $\zeta_x x' - \zeta'_x x = 0$, where ζ is the space charge dispersion, ζ' is its derivative, x is the horizontal position and x' is its derivative. Therefore, the angle of the displacements due to the SC effect is given by

$$\phi_\zeta = \tan^{-1}(\zeta'_x/\zeta_x) \tag{4}$$

The analytical calculation of the space charge dispersion requires the space charge kick angle. The result of the space charge dispersion in the merger section is shown in Fig. 5.

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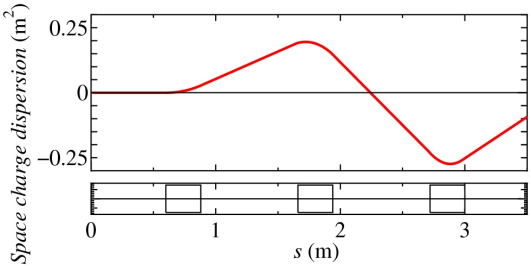

Figure 5: Space charge dispersion at the merger section.

In the analytical calculation, by using the first-order theory, the transfer matrix for each element is derived by Green's function method[8]. At the exit of the merger section, the space charge dispersion, ζ_x, and derivative of the space charge dispersion, $\zeta_{x'}$, are -0.067 m^2 and 0.134 m, respectively. From the Eq. 4, the space charge kick angle, ϕ_s, in horizontal phase space was calculated to be -1.11 rad which is based on the first-order theory. Using the above calculation parameters, the LSCF wake potential was given by ΔW_s= 13.75 [keV/m]. Also, κ_s is the normalized space charge wake potential in the bending path and it is dependent on the space charge wake potential W[eV/m], and the reference energy E_0[eV] at $s = s_0$, which is the entrance of the bending magnet. It is given by $\Delta\kappa_s$=$\Delta W_s/E_0$=2.75$\times$10^{-3}[1/m]. When the merger section is optimized by matching the envelope between the LSCF-induced dispersion function and the betatron function at the exit of the merger section, all the bunch slices align along the orientation of the phase ellipse and have a distribution of the displacement $(\Delta\kappa_s\zeta, \Delta\kappa_s\zeta')$=(-0.184 mm, 0.368 mrad). Based on the first-order theory, the transverse emittance growth in the merger section is given by

$$\varepsilon^2 = (\varepsilon_0\beta_x + D^2)(\varepsilon_0\gamma_x + D'^2) - (\varepsilon_0\alpha_x - DD')^2 \quad (5)$$

, where ε_0 and ε are the initial and final emittance as un-normalized values, respectively, and (D, D')=$(\Delta\kappa_s\zeta, \Delta\kappa_s\zeta')$ is the rms spread of the bunch slice displacement in (x, x') phase space. From Eq. 5, the result of analytical calculation of the emittance growth in the merger section is shown in Fig. 6.

To study the space charge effect, the particle tracking was carried out using General Particle Tracer (GPT) with mesh based method to calculate space charge effect, which includes the calculation of 3-dimensional SC force with actual electric and magnetic fields [9]. The particle tracking simulation gives the particle distribution in six dimensional phase space, $(x, x', y, y', z, \dot{z})$, where $'$ and $\dot{}$ denote d/dz and d/dt, respectively. Here, x, y and z are the particle coordinates for the horizontal, vertical and longitudinal directions, respectively. When the beam consists of the N macro particles, the particle coordinate of the i-th particle is $(x_i, x'_i, y_i, y'_i, z_i, \dot{z}_i)$. The betatron function is defined by

$$\beta_x = \frac{\langle x_c^2\rangle}{\epsilon_x}, \quad (6)$$

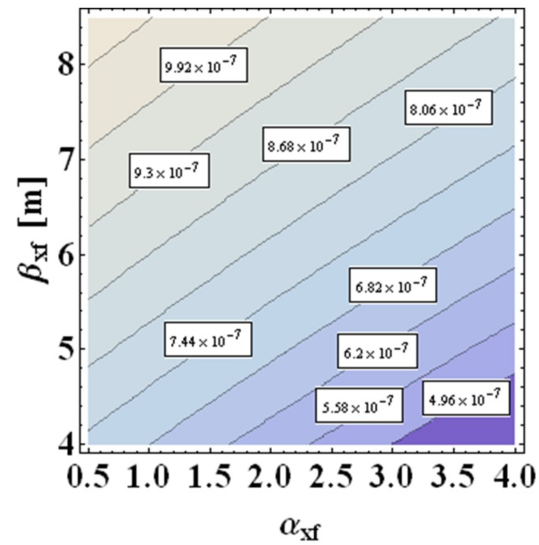

Figure 6: Analytical calculation results of emittance as function of twiss parameter at the exit of merger.

where $x_{c,i} = x_i - \langle x\rangle$, $x'_{c,i} = x'_i - \langle x'\rangle$, and $\epsilon_x = \sqrt{\langle x_c^2\rangle\langle x_c'^2\rangle - \langle x_c x'_c\rangle^2}$. Here, $\langle\rangle$ denotes an average, e.g, $\langle x\rangle = \sum x_i/N$. In this paper, the betatron function is calculated by Eq. (6) from the calculated particle distribution. In the particle tracking simulation by using GPT code, the initial normalized transverse emittance is 0.1 mm·mrad, the bunch length is 3 ps (rms), the beam energy is 5 MeV, radius of the bunch is 1.5 mm and the the bunch included the particle distribution of 10000 macroparticles. The bunch distribuition was assumed to be beer-can shape, which has same vertical and horizontal emittances. The transverse emittance growth was scanned by the initial and final CS parameter at the entrance and exit of merger section. In the tracking simulation, the β_{xi} and α_{xi} were varied from 0.5 m to 10 m by 0.5 m step and from -4 to 4 by 0.2 step, respectively. Fig. 7 (a) and (b) show transeverse emittances as a function of CS parameter at the entrance and the exit of the merger section.

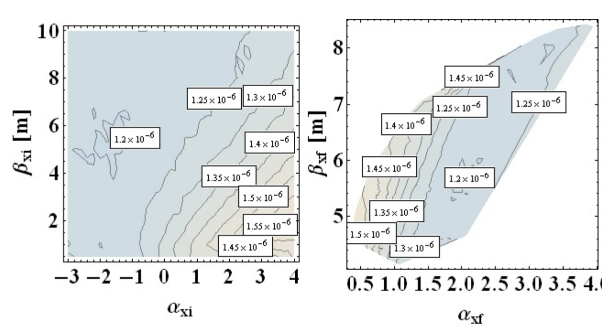

Figure 7: Transverse emittance at the exit of the merger. (a) as function of the initial α_{xi} and β_{xi}. (b) as function of the final α_{xf} and β_{xf}.

As shown by Fig. 6 and 7, amount of the growth of the transverse emittance due to the SC effect is around 1.09

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mm-mrad. Also, the growth of the transverse emittance has minmum value at the minus value of the α_{xi} with small value of β_{xi}. But result of the growth of the emittance calculated by GPT different with result of the growth of the emittance calculated by first-order theory. These difference was caused by the 3-dimensional SC effect. In the numerical simulation by using GPT, the transverse emittance growth due to the 3-dimensional SC force in the merger section was observed. The results show that the emittance was minimized by changing the initial CS parameter. The initial CS parameter is α_x = -1.6, β_x = 5.5 m when growth of the emittance is minimized. The growth of the transverse emittance as function of the orientation of the phase ellipse in the (x, x') phase space, which calculated by using the results of the numerical calculation as shown by Fig. 7, is shown in Fig. 8.

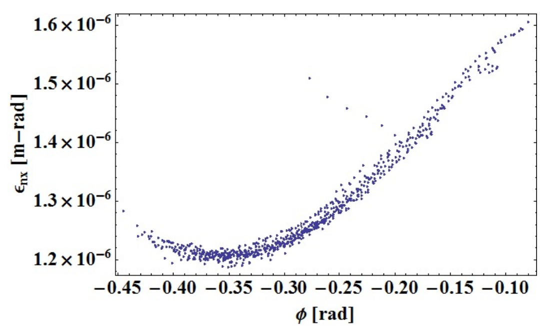

Figure 8: Transverse emittance at the exit of merger section as a function the orientation of the phase ellipse.

From the numerical calculation result, the emittance growth induced by the LSCF in the merger section was compensated by changing the orientation angle of the ellipse in horizontal phase space. We found a minimum transverse emittance growth of 1.09 mm·mrad in the merger section when the vertical CS parameter was fixed to β_y = 9 m, with α_y = 0. The CS parameters and dispersion function in merger section after compensation of the growth of the emittance due to the SC effect are shwon by Fig. 9.

Also, the effect of coupled motion of the beam was investigated to minimize the emittance growth in the merger section. The vertical CS parameter was changed to investigate the effect of coupled motion. In the calculation, the horizontal CS parameter was fixed to β_x = 5.5 m with $\alpha_x = -1.6$ and shows a minimum emittance growth. The result is shown in Fig. 10.

As shown in Fig. 10, the minimum horizontal emittance growth at the exit of the merger becomes 0.684 mm·mrad at the β_y = 0.5 m and α_y=-10. From this result, the orientation of the phase ellipse was calculated to -0.62 rad. It is shows that the smaller angle of the phase ellipse which is calculated by the analytical model, -1.09 rad, gives the smaller growth of the emittance. To achieve the small angle of the phase ellipse, which is around -1.09 rad, it needs the small β_y with large α_y. It causes the growth of the vertical betatron function after passing the merger section. Therefore, the optimumu of βx, α_x, βy and αy were determined to 5.5 m, -1.6, 0.5 and -10 m, respectively.

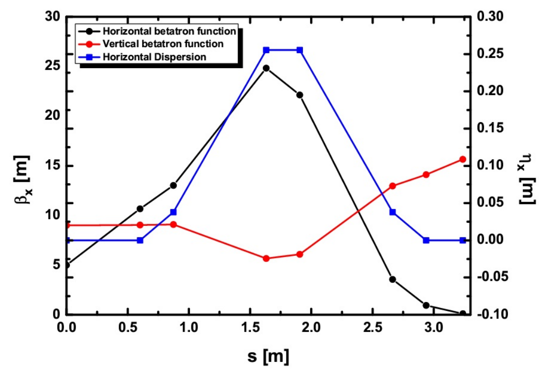

Figure 9: The envelope of the CS parameters and dispersion function in merger section after compensation of the growth of the emittance due to the SC effect.

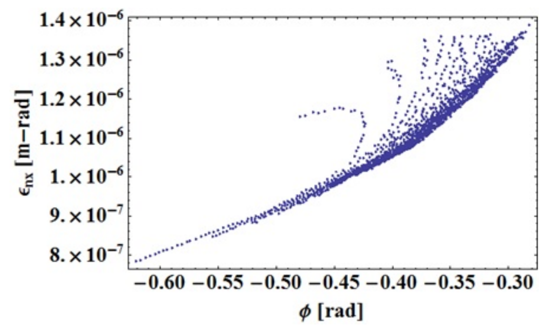

Figure 10: The change of transverse emittance at the exit of merger section as function the orientation of the phase ellipse due to the change of vertical CS parameters.

We also investigate the dependency of the energy spread in the merger section. The SC effect was reduced by the energy spread interact with the dispersion which caused by the dipole magnet. For the study of the dependency of the energy spread, the d_z was defined by ratio of the energy spread to bunch length. Also the sign of the d_z was defined by sign of gradient of the slope. The d_z was changed from the -30 MeV/m to the 50 MeV/m by 20 MeV/m step. In this tracking simulation, the length and charge of the bunch with 10k macro-particles was assumed to 0.9 mm and -80 pC, respectively. The vertical CS parameters also was fixed to β_y=0.5 m and α_y=-10. The calculation result of the energy spread dependency of the SC effect is shown in Fig. 11

As shwon by Fig. 11, the d_z caused the compression of the bunch length at the exit of merger. For longer bunch length with $d_z > 0$, charge density decreases, and SC effect becomes weaker. It causes the decreasing of the growth of the emitance when the beam has $d_z > 0$. From the calculation result, the emittance at the exit of the merger was acheved to 0.735 mm·mrad when the bunch has 27.78 MeV/m of energy spread with 0.9 mm of bunch length,β_x=5.5 m, α_x=-1.6, β_y=0.5 m and α_y=-10 of CS pa-

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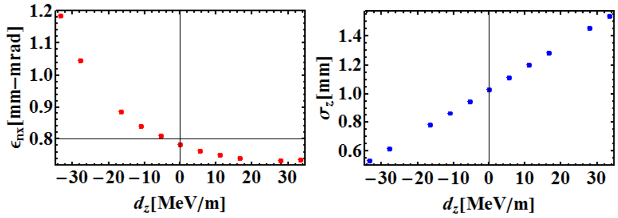

Figure 11: The energy spread dependency of the emittance(left) and bunch length (right) in the merger section.

rameters. The longitudinal phase space beam distribution at the exit of the merger section is shown in Fig. 12.

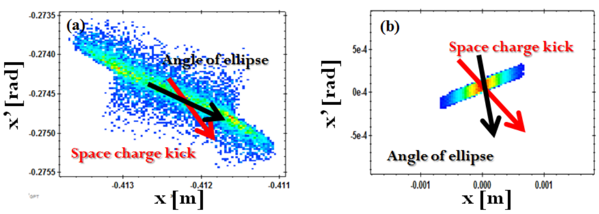

Figure 12: The longitudinal phase space beam distribution at the exit of merger section.(a)The emittance minimized by the CS paramter scan (b)The emittance minmized by changing of energy spread.

This merger system has the large emittance growth due to the SC effect. The increasing of the beam energy at the injector system was needed to achieve the small emittance growth. Because the goal of the emittacne of 5 GeV ERL is 0.1 mm·mrad at the arc-section. So, the energy dependacny of the transverse emittance growth is shown in Fig. 13.

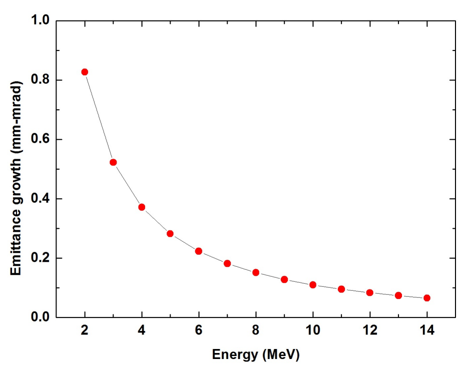

Figure 13: Energy dependency of the horizontal emittance growth in merger section.

From the analytical calculation result, the transverse emittance growth in merger section depends on the energy of beam in injector system. The both method are good to reduce the emittance growth in merger section. But, the increasing of the beam energy is more effective to achieve the smaller emittance. Therefore, the beam energy at the injector system of the 5 GeV ERL is larger than 14 MeV to achieve the 0.1 mm·mrad of transverse emittance after passing the merger section.

SUMMARY

The study for the compensation of emittance growth on the low energy around 5 MeV in the merger section wasperformed to minimize the growth of the emittance in a beam with 77 pC of bunch charge, a bunch length of 3 ps and a bunch energy of 5 MeV. Based on the first-order theory, the emittance growth due to the displacement of bunch slices in phase space was minimized by matching the orientation of the phase ellipse to the kick angle induced by the SC force at the exit of the merger. The dependency of the energy spread at the merger section was investigated. From the calculation results, we minimize the growth of the emittance. From the results, the emittance growth in the merger section was showed to be around 0.635 mm·mrad.

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