# MONITORING BEAM POSITION IN THE MULTIBEAM ACCELERATORS* 

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## Abstract

In this paper we present the concept of a beam position monitors for the accelerating structures with multiple beams. Both cases with common and separated nominal orbits are considered. For the first case we utilize the phase information, when for the second case additional pick-up electrodes can be utilized.

## INTRODUCTION

Most commonly used method for the beam position monitors (BPMs) is based on the evaluation of the signals induced on the pick-up electrodes (PUEs) by a circulating beam. Beam location is calculated from the signal amplitudes using delta over sum [1]. For the vertical plane BPM with two PUEs the equation can be written as

$$
\begin{equation*}
y=k \frac{U_{u p}-U_{\text {dow } n}}{U_{\text {up }}+U_{\text {dow } n}} \tag{1}
\end{equation*}
$$

where $U_{\text {up }}$ and $U_{\text {down }}$ are the amplitudes, $k$ is a scaling factor, which is determined by the geometry of a vacuum vessel and the electrodes. For a symmetrical system and beam in the center both signals have equal amplitudes and the corresponding position readback is zero.

With two (or more) beams circulating inside the vacuum chamber we need to separate the signals and process them individually. For the colliders with beams moving in the opposite directions this task is solved by utilizing the striplines, which have directional properties. The signals from the different beams appear on the different ports and conventional processing units can be utilized.

This technique is not suitable for energy recovery linacs (ERL) and fixed-field alternating gradient accelerators (FFAG) where two or more beams co-propagate through a vacuum system in the same direction and each beam has its own trajectory.

## PROPOSED METHOD

In the energy recovery linacs the beams pass either though an arc where only accelerated and decelerated beams of the same energy are present (on the last pass the only beam is present) or through an accelerating/decelerating section common for the all beams.

For ERL the time delay between accelerated and decelerated bunches is fixed by design and it is possible to employ the phase of the PUE signal to extract information on the position of each bunch. First we consider an arc

[^0]where only two beams are present. If bunches, separated by a flyby time $\Delta t_{12}$, have different positions then each PUE sees different longitudinal "center of gravity" of the two bunches (see Fig. 1) and there is a phase shift between two signals. For a processing unit, utilizing signal processing at frequency $\omega$, and small displacements of the first and the second bunches $\delta_{1}$ and $\delta_{2}\left(S \delta_{1}, S \delta_{2} \ll\right.$ 1 , where $S=1 / k$ is a sensitivity coefficient) we can write the linearized equations:


Figure 1: Signals induced on the pick-up electrodes by two bunches with different coordinates. Because of bunches' displacements the amplitudes of the induced voltages differ.

$$
\begin{gather*}
U_{u p}=U_{1}\left(1+S \delta_{1}\right) \sin \omega\left(t+\Delta t_{12} / 2\right)+ \\
U_{2}\left(1+S \delta_{2}\right) \sin \omega\left(t-\Delta t_{12} / 2\right) \\
U_{\text {down }}=U_{1}\left(1-S \delta_{1}\right) \sin \omega\left(t+\Delta t_{12} / 2\right)+  \tag{2}\\
U_{2}\left(1-S \delta_{2}\right) \sin \omega\left(t-\Delta t_{12} / 2\right)
\end{gather*}
$$

When both bunches have equal charges (a valid assumption for ERL) then we re-write Eq. 2 as

$$
\begin{align*}
U_{u p}= & U_{0} \cos \frac{\omega \Delta t_{12}}{2}\left(2+S\left(\delta_{1}+\delta_{2}\right)\right) \sin \omega t+ \\
& U_{0} S \sin \frac{\omega \Delta t_{12}}{2}\left(\delta_{1}-\delta_{2}\right) \cos \omega t \\
U_{\text {down }}= & U_{0} \cos \frac{\omega \Delta t_{12}}{2}\left(2-S\left(\delta_{1}+\delta_{2}\right)\right) \sin \omega t-  \tag{3}\\
& U_{0} S \sin \frac{\omega \Delta t_{12}}{2}\left(\delta_{1}-\delta_{2}\right) \cos \omega t
\end{align*}
$$

Neglecting second order terms we can estimate amplitudes $\left(A=\sqrt{U_{s i n}^{2}+U_{\text {cos }}^{2}}\right)$ of the signals induced on PUE

$$
\begin{gather*}
A_{u p} \approx 2 U_{0}\left(1+S \frac{\delta_{1}+\delta_{2}}{2}\right) \cos \frac{\omega \Delta t_{12}}{2} \\
A_{\text {down }} \approx 2 U_{0}\left(1-S \frac{\delta_{1}+\delta_{2}}{2}\right) \cos \frac{\omega \Delta t_{12}}{2} \tag{4}
\end{gather*}
$$

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Substitution of the found amplitudes into the Eq. 1 gives

$$
\begin{equation*}
\hat{y}=k \frac{S\left(\delta_{1}+\delta_{2}\right)}{2}=\frac{\left(\delta_{1}+\delta_{2}\right)}{2} \tag{5}
\end{equation*}
$$

That means that using information on the amplitude we measuring the average position of the beam. Now we will consider the phases of the signals. Using the same assumptions we will find

$$
\begin{gather*}
\varphi \approx \frac{U_{\text {cos }}}{U_{s i n}} \\
\varphi_{u p} \approx \frac{s\left(\delta_{1}-\delta_{2}\right)}{2} \tan \frac{\omega \Delta t_{12}}{2}  \tag{6}\\
\varphi_{\text {down }} \approx-\frac{S\left(\delta_{1}-\delta_{2}\right)}{2} \tan \frac{\omega \Delta t_{12}}{2}
\end{gather*}
$$

and difference of the two phases gives us the difference between two positions

$$
\begin{equation*}
\delta_{1}-\delta_{2}=k \frac{\varphi_{u p}-\varphi_{\text {down }}}{\tan \left(\omega \Delta t_{12} / 2\right)} \tag{7}
\end{equation*}
$$

The processing frequency is usually equal to the RF frequency. Such a choice allows processing of any fill pattern because all bunches are separated by a multiple of the RF period. In the ERL distance between bunches can be different. It may be as small as half of the RF wavelength because the decelerated bunch should be in the opposite phase vs. accelerated one. Moreover, this distance can differ from the multiple of the half periods due to the shifts in the merge lines. Also, there is no revolution period in the ERL but only a round-trip time. Hence, choice of the processing frequency is somewhat arbitrary and can be used for optimization of the system. There are frequencies that should be avoided: if $\cos \left(\omega \Delta t_{12}\right)=0$ then there is no signal for the average position calculation and with $\sin \left(\omega \Delta t_{12}\right)=0$ phases will be constant and difference in orbits can not be found. Therefore, the processing frequency should be in between

## SIGNAL-TO-NOISE RATIO

Modern digital beam position electronics have the required capability to process both amplitude and phase of the PUE signals [2].

To estimate the accuracy of the measurement let us consider vector diagram shown in Fig. 2. When the measurement error is not correlated with the signal the probable error will draw a circle around the end of the vector and r.m.s. phase error will be equal to the signal to noise ratio of the amplitude measurement

$$
\begin{equation*}
\sigma_{\varphi}=\sigma_{A} /_{A} \tag{8}
\end{equation*}
$$

That means that achievable error in the difference of position is the same as for the average position. For the system described in [2] r.m.s. position error is 3 microns
for $\mathrm{k}=10 \mathrm{~mm}$, which means that noise is $3 \times 10^{-4}$ of the signal level and the ratio is close to the r.m.s. fluctuations of the phase measurements $0.01^{\circ}\left(1.75 \times 10^{-4}\right.$ radians $)$.


Figure 2: Vector diagram for the PUE signals. Two longer vectors indicate phases and amplitudes of the signal when two smaller ones show the noise amplitude.

So far we have neglected phase shifts in the system associated with propagation of the signals the cables and in the processing electronics. The system transfer function will not be identical for all channels and a phase shift corresponding to the identical beam positions should be established. Otherwise these phase shifts will include a systematic error in the measurement of the difference of the two positions. Similarly, the unequal gain and/or losses in the channels generate systematic offset in the average position readback.

Both phase and amplitudes can be calibrated using a RF generator with a splitter. In this case the whole chain from the cable to electronics can be characterized. For the phase calibration exclusively it is also possible to use exclusively the accelerated bunches (dumping the beam at high energy). In this case the BPM processing unit sees signal from a single bunch, and there are no ambiguities associated with the second bunch.

## FOUR PUES SYSTEM

In the system with four pick-up electrodes, as shown in Fig. 3, the beam position is calculated with the modified equations.


Figure 3: Geometry of the four PUE beam position monitor. Such configuration is used to avoid exposure of the PUEs to the synchrotron light.

$$
\begin{align*}
\overline{\mathrm{x}} & =k_{x} \frac{U_{B}-U_{A}+U_{C}-U_{D}}{U_{A}+U_{B}+U_{C}+U_{D}}  \tag{9}\\
\bar{y} & =k_{y} \frac{U_{B}-U_{C}+U_{A}-U_{D}}{U_{A}+U_{B}+U_{C}+U_{D}}
\end{align*}
$$

As before, for the small beam displacements we can write

$$
\begin{gather*}
U_{A}=U_{0}\left(1-S_{x} \delta_{x 1}+S_{y} \delta_{y 1}\right) \sin \omega\left(t+\Delta t_{12} / 2\right)+ \\
U_{0}\left(1-S_{x} \delta_{x 2}+S_{y} \delta_{y 2}\right) \sin \omega\left(t-\Delta t_{12} / 2\right) \\
U_{B}=U_{0}\left(1+S_{x} \delta_{x 1}+S_{y} \delta_{y 1}\right) \sin \omega\left(t+\Delta t_{12} / 2\right)+ \\
U_{0}\left(1+S_{x} \delta_{x 2}+S_{y} \delta_{y 2}\right) \sin \omega\left(t-\Delta t_{12} / 2\right) \\
U_{C}=U_{0}\left(1+S_{x} \delta_{x 1}-S_{y} \delta_{y 1}\right) \sin \omega\left(t+\Delta t_{12} / 2\right)+  \tag{10}\\
U_{0}\left(1+S_{x} \delta_{x 2}-S_{y} \delta_{y 2}\right) \sin \omega\left(t-\Delta t_{12} / 2\right) \\
U_{D}=U_{0}\left(1-S_{x} \delta_{x 1}-S_{y} \delta_{y 1}\right) \sin \omega\left(t+\Delta t_{12} / 2\right)+ \\
U_{0}\left(1-S_{x} \delta_{x 2}-S_{y} \delta_{y 2}\right) \sin \omega\left(t-\Delta t_{12} / 2\right)
\end{gather*}
$$

With the similar to the above calculations we can find that Eq. 9 is applicable for the calculation of the average beam position when the difference can be found from

$$
\begin{align*}
& x_{\text {diff }}=\frac{k_{x}}{\tan \left(\omega \Delta t_{12} / 2\right)}\left[\left(\varphi_{B}-\varphi_{D}\right)-\left(\varphi_{A}-\varphi_{C}\right)\right] \\
& y_{d i f f}=\frac{k_{x}}{\tan \left(\omega \Delta t_{12} / 2\right)}\left[\left(\varphi_{B}-\varphi_{D}\right)+\left(\varphi_{A}-\varphi_{C}\right)\right] \tag{11}
\end{align*}
$$

## SUPPRESSION OF THE CABLE DRIFTS

The temperature changes and other slow processes can change physical length of the cable and/or phase velocity of the signal in the cable. To suppress the drifts one can lower the processing frequency to the range where the sensitivity to the variations of delays is less. Unfortunately, it also lowers the sensitivity to the position difference due to the $\tan \left(\omega \Delta t_{12} / 2\right)$ term. To have both high sensitivity to the differential beam position (and low noise) and to suppress influence of drifts we can convert down the PUE signals. In this case the sensitivity is defined by sum of the local oscillator frequency and processing frequency, while drifts are defined by processing frequency only. Local oscillator signal should be brought to the PUE with a single cable and split to required number of mixers with shortest connection.

## COMMON SECTION

In common section there more then two beams and we do not have sufficient data to resolve the position of each beam. To overcome this obstacle we can process at different frequencies. The set of the frequencies depends on the machine configuration and general rules are subject for development. For the system with all buckets filled (total number of independent bunches is equal to double of number of passes $\mathrm{N}_{\text {pass }}$ ) all frequencies from the $\mathrm{F}_{\text {rep }}$ to $2 \mathrm{xN}_{\text {pass }} \mathrm{xF}_{\text {rep }}$ can be utilized.

## FFAG

In the FFAG the beams are separated in space as shown in Fig. 4. Nominal levels of the signals and linear
coefficients can be easily found with numerical simulations (one need to account for the phase shifts for each bunch).


Figure 4: Arrangement of the circulating beams (red) and pick-up electrodes (green) for FFAG.
The difference with conventional accelerator will be that ratio will be not scalar but vector:

$$
\begin{align*}
v_{i} & =\frac{U_{u p i}+U_{u p i+1}-U_{\text {downi }}-U_{\text {downi }+1}}{U_{u p i}+U_{u p i+1}+U_{\text {downi }}+U_{\text {down } i+1}} \\
h_{i} & =\frac{U_{u p i}-U_{u p i+1}+U_{\text {downi }}-U_{\text {downi}+1}}{U_{u p i}+U_{u p i+1}+U_{\text {downi }}+U_{\text {downi }+1}} \tag{12}
\end{align*}
$$

Matrices of the linear coefficients can be inverted and used for position calculations:

$$
\begin{align*}
& \vec{x}=M_{H} \vec{h} \\
& \vec{y}=M_{V} \vec{v} \tag{13}
\end{align*}
$$

## REFERENCES

[1] R.E. Shafer. "Beam Position Monitoring", BIW'89, AIP Conf. Proc. 212 (NY, 1990), p. 26.
[2] M. Znidarcic et al., "Testing of New Hadron Beam Phase and Position Monitor at CIEMAT Laboratory," DIPAC'11, Hamburg, May 2011, MOPD26, p. 104; http://www.JACoW.org


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