TRANSVERSE-EMITTANCE PRESERVING ARC COMPRESSOR: SENSITIVITY STUDY TO BEAM OPTICS, CHARGE AND ENERGY*

S. Di Mitri#, Elettra Sincrotrone Trieste, 34149 Basovizza, Trieste, Italy

Abstract

Magnetic compression of ultra-relativistic electron bunches in a 180 deg periodic arc, made of identical double bend achromats, has recently been proposed [1]; its performance surpasses, to the author's knowledge, that of any past studies on the subject, by reaching higher compression factors at larger bunch charges, and by upper-limiting the normalized transverse emittance growth to 0.1 µm level. Chromatic aberrations and nonlinearity in the particle longitudinal dynamics are reduced with sextupole magnets. Coherent synchrotron radiation-induced emittance growth is minimized with a proper choice of beam optics and compression factor per cell. The design is supported by analytical considerations that found confirmation in particle tracking results. In this article, we study the dependence of beam final emittance on optics, charge and mean energy. The range of validity of the linear optics approach is discussed, together with limits of applicability of the lattice under consideration.

BACKGROUND

Bunch length magnetic compression is routinely used in high brightness electron linacs driving free electron lasers (FELs) and particle colliders in order to shorten the bunch, that is increase the peak current of the injected beam from few tens to kilo-Amperes (see, e.g., [2] and references therein). To date, magnetic compression is performed in dedicated insertions made of few degrees bending magnets inserted on the accelerator straight path; the compression factor is limited by the degradation of the beam transverse emittance owing to emission and absorption of coherent synchrotron radiation (CSR). In a recent paper [1], we show through analytical and numerical results, that a 500 pC charge beam can be time-compressed by a factor of up to 45, reaching peak currents of up to 2 kA, in a periodic 180 deg arc at 2.4 GeV beam energy, while the normalized emittance is rather immune to CSR, namely its total growth does not exceed the 0.1 µm rad level. To achieve such emittance control, we reformulated the known concept of CSR-driven optics balance [3] for the more general case of varying bunch length, and found that it works for bending angles larger than previously thought advisable and practical. The proposed solution applies adiabatic compression throughout the arc. Optical aberrations and longitudinal nonlinearities are controlled with sextupole magnets.

In the past, several energy recovery linac (ERL) designs [4–6] have attempted to use recirculating arcs for bunch length compression in the energy range 0.2–7 GeV, while keeping the normalized emittance growth below approximately 0.1 µm rad. Double and triple bend achromatic cells were tested for compression factors $C < 30$. The highest beam charge compatible with the target emittance control was set at 150 pC at the highest energy, and at 77 pC at 3 GeV. While in [5, 6] some degree of optics control was exercised in order to minimize the CSR effect following the optics prescriptions given in [7], in all those designs CSR was mainly neutralized by a low beam charge.

In comparison with the existing literature, our solution allows larger compression factors at high charges, simplifies ERL lattice designs (since, in principle, no dedicated chicane is anymore needed for compression as the arc acts both as final stage of recirculation *and* compressor) and paves the way for repeated compressions at different stages of acceleration, i.e., at different energies. Although it finds an immediate application to ERLs, the proposed CSR-immune arc compressor promises to be applicable to more general accelerator design, thus offering the possibility of new and more effective layout geometries of single-pass accelerators and of new schemes for beam longitudinal gymnastic, both in electron-driven light sources and colliders.

ARC COMPRESSOR DESIGN

The 180 deg arc compressor is made of 6 modified Chasman-Green achromats (one cell shown in Fig.1) separated by drift sections that allow optics matching from one DBA to the next one. The arc is 125 m long (40 m long radius) and functional up to 2.4 GeV. The bending angle per sector dipole magnet is $\theta = 0.2618$ rad and the dipole arc length $l_b = 1.4489$ m. R_{56} of one dipole is 17.2 mm, while that of the entire arc is 207.1 mm. If, for example, a total compression factor $C = 45$ were required at the end of the arc (1.9 per DBA cell), an energy chirp $h = \left(\frac{1}{C} - 1\right)\frac{1}{R_{56}} \approx 4.7 m^{-1}$ would be needed at its entrance, which corresponds roughly to a fractional rms energy spread of $\sigma_{\delta,0} \approx h\sigma_{z,0} = \frac{1}{E}\frac{dE}{dz}\sigma_{z,0} \approx 0.3\%$ for a 3 ps rms long bunch. Figure 2 shows the nominal optics functions along the arc.

According to the analysis in [1], the normalized emittance growth in the bending plane and in the presence of CSR for a *single* DBA cell can be estimated by (using the same notation than in [1]):

$$\Delta\varepsilon_{nf} = \varepsilon_{nf} - \varepsilon_{n0} \cong \varepsilon_{n0}\left(\sqrt{1 + \frac{\gamma J_3}{\varepsilon_{n0}}} - 1\right), \qquad (1)$$

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#simone.dimitri@elettra.eu

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where the single particle Courant-Snyder invariant *2J₃* is:

$$2J_3 = \beta_2 x_3'^2 + 2\alpha_2 x_3 x_3' + \left(\frac{1+\alpha_2^2}{\beta_2}\right)x_3^2 = \tag{2}$$

$$\cong \left(\frac{k_1\rho^{1/3}\theta^2}{2}\right)^2 \left\{\left[\sqrt{\beta_2}\left(C^{4/3}+1\right) - \frac{\alpha_2}{\sqrt{\beta_2}}\left(\frac{l_b}{6}\right)\left(C^{4/3}-1\right)\right]^2 + \left[\frac{1}{\sqrt{\beta_2}}\left(\frac{l_b}{6}\right)\left(C^{4/3}-1\right)\right]^2\right\}$$

It is worth noticing that $(k_1\rho^{1/3}\theta)$ is the rms value of the fractional energy spread induced by CSR in the first dipole magnet of a DBA cell, and that its evolution along the cell (thus the arc), as well as that of the bunch length, is taken into account by the cell compression factor C. The invariant reaches a minimum for $\beta_2 \cong \beta_{2,\min} = \frac{l_b}{6}\left(\frac{C^{4/3}-1}{C^{4/3}+1}\right)$; β_2 is the betatron function inside the dipole magnet, where the beam size is forced to a waist. In the lattice under consideration, $l_b \geq \beta_2$, thereby we should consider average values of β_2 and α_2 inside the magnet; we expect to have minimal CSR effect on the emittance for $\beta_{2,\min} \sim 0.2$ m.

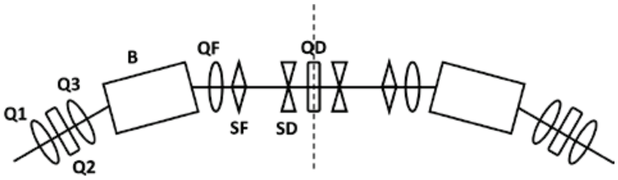

Figure 1: Sketch (not to scale) of the arc compressor DBA cell. Dipole magnets (B), focusing (QF, Q1 and Q3) and defocusing (QD, Q2) quadrupole magnets, focusing (SF) and defocusing sextupole magnets (SD) are labelled. The geometry and the magnets' arrangement is symmetric with respect to the middle axis (dashed line) [1]. Copyright by Europhysics Letters.

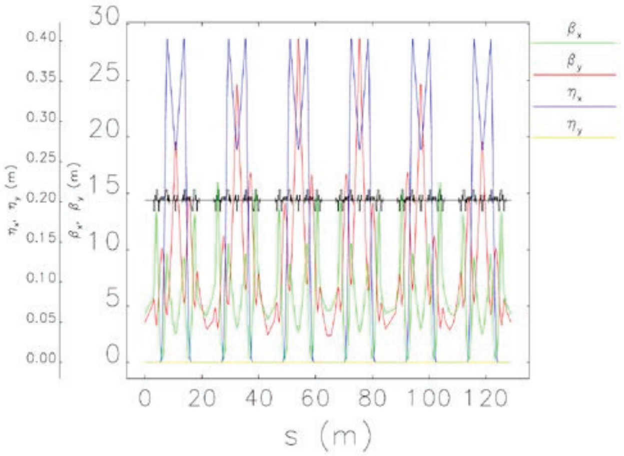

Figure 2: (Colour on-line) Linear optics functions along the 180 deg. arc compressor. Optics functions are quasi-symmetric in each DBA cell of the arc compressor, and totally symmetric with respect to the middle axis of the arc. The minimum β_x is in the dipole magnets, and it ranges from 0.14 m to 0.26 m over the six cells.

In summary, in order to minimize the CSR-induced emittance growth in a periodic 180 deg. arc compressor, we prescribe the use of several symmetric DBA cells with linear compression factor not far from unity in most of the cells. We also impose a beam waist in the dipoles of all the DBAs and find that, unlike in a magnetic chicane [8], there is an optimum value for β_2 inside the dipoles that minimizes the chromatic emittance growth due to CSR.

SENSITIVITY STUDY

The dependence of the final horizontal emittance on the optics, charge and mean energy is investigated through Elegant [9] simulations and compared with analytical predictions based on linear optics analysis. The nominal values of beam and arc parameters are listed in Tab.1. When beam charge and energy is varied, all the other parameters are meant to remain the same. For each charge and energy value, the beam Twiss parameters at the entrance of the arc are varied in order to be matched to the optics periodic solution. This way, the minimum value of β_x in the dipoles is scanned. Being the optics fully symmetric w.r.t. the arc central axis, but only quasi-periodic inside each DBA cell, the betatron phase advance between the DBA dipoles slightly deviates from π. This, together with dipole magnets finite length, CSR transient field at the dipoles' edges and in drift sections, is expected to generate some deviations from the analytical predictions based on Eqs.1 and 2.

Table 1: Beam and Arc Compressor Parameters

Parameter	Value	Units
Charge	0.1 – 1.0	nC
Mean Energy	0.5 – 2.4	GeV
Initial Bunch Duration, FWHM	10	ps
Linear Energy Chirp	-4.7	m^{-1}
Initial Norm. Emittance, RMS	0.8	μm
Number of Arc DBA Cells	6	
Total Compression Factor	45	

Varying the Optics Parameters

Figure 3 shows, for each charge value, the normalized emittance at the arc end at different simulation steps, each step corresponding to a different optics and thereby to a different value of β_2 (see Eq.2). Since the initial bunch length and the total compression factor are fixed, the final peak current is different for the three charge values (final peak current is 1.5 kA for 0.5 nC; it scales almost linearly with the charge for all the other cases). The initial emittance is also kept the same, for direct comparison of the behaviour of the final emittance in the presence of CSR.

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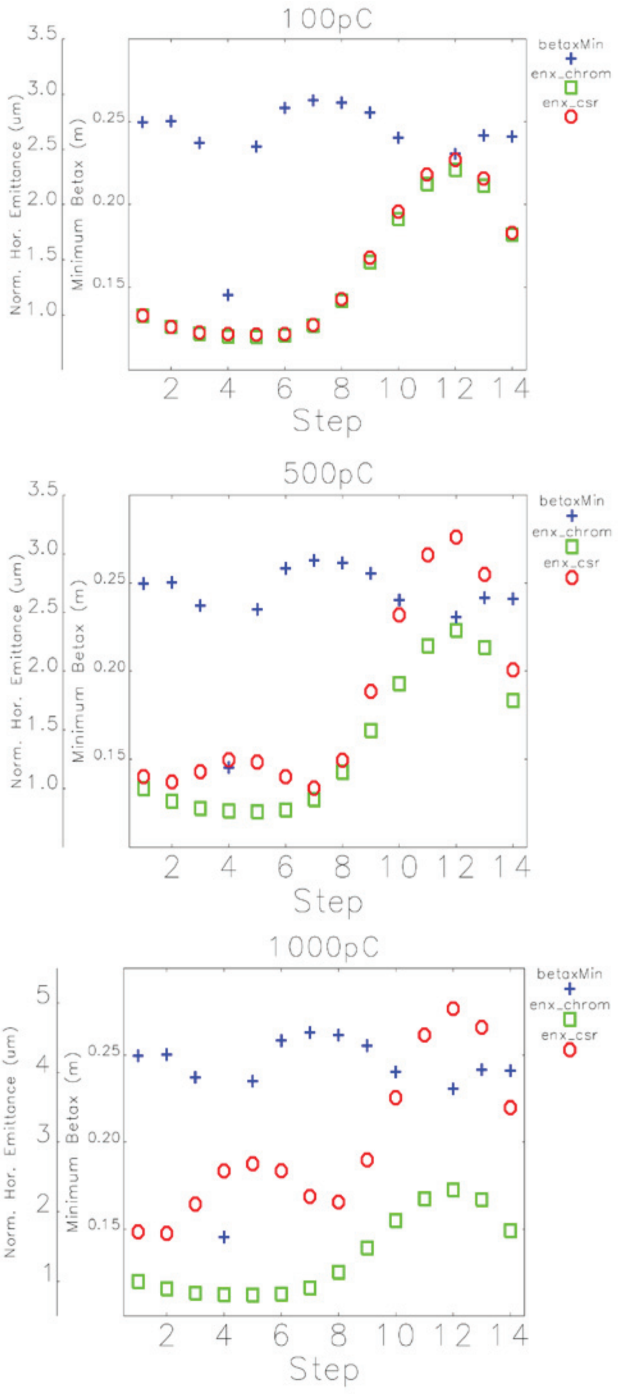

Figure 3: (Colour on-line) Normalized rms emittance and minimum betatron function in the arc dipoles, in the bending plane, at different simulation steps. Each step corresponds to a different periodic optics along the arc (Q1 strength is varied, see Fig.1). Bunch charge is 0.1, 0.5 and 1.0 nC from top to bottom; all other beam parameters are fixed (see Tab.1). Emittance is computed with (*enx_csr*) and without CSR (*enx_chrom*). Optical aberrations up to an including 3rd order and incoherent synchrotron radiation are computed in all simulations. CSR transient field and in drift sections following dipole magnets are included when CSR is turned on.

At 0.1 nC, the CSR effect is negligible over the entire range of β_2 considered; the emittance growth is dominated by chromatic aberrations (sextupole strengths are kept fixed during the scan, thereby aberrations are not corrected at each step).

At the higher charge of 0.5 nC, CSR starts degrading the emittance. The maximum CSR effect (in the figure, this is the distance between *enx_chrom* and *enx_csr*) corresponds to the lowest β_2's value over the scan. The CSR effect is almost fully suppressed at the optimum value of $\beta_2 = 0.26$ m, which is close to the theoretical expectation discussed above. There is a clear correlation between β_2 and the emittance value. It is a remarkable result that smallest β_2 values do not lead to optimum CSR suppression, in agreement with the analytical (and somehow counterintuitive) prediction of Eq.2.

At 1.0 nC, the CSR effect on emittance is visible at all values of β_2, as there is no exact cancellation of CSR kicks at any of the steps considered. The correlation between β_2 and the final emittance is apparent as in the 0.5 nC case.

Varying the Beam Energy

The same lattice considered above has been investigated for different beam mean energies, as shown in Fig.4. We assume that the field of the magnetic elements scale linearly with the beam energy. The difference of the final-minus-initial projected emittance (rms value) with CSR turned on and off is shown for beam charges of 0.1, 0.3 and 0.5 nC. Control of emittance growth at the 0.1 µm level is allowed at energies $E \geq 0.5$ GeV for 0.1 nC, at $E \geq 1$ GeV for 0.3 nC, and at $E > 2$ GeV for 0.5 nC.

For comparison, the analytical prediction based on Eqs.1–2 is shown in Fig.4. In this case, Eq.2 had to be evaluated for each DBA cell. At each cell, the rms energy spread due to CSR and the corresponding emittance increase is summed in quadrature with the energy spread and the emittance at the cell's entrance. After six cells, the difference between the final emittance and the emittance simulated without CSR is computed and plotted in Fig.4. It is worth noticing that for all charges considered, the rms energy spread induced by CSR in the first arc dipole magnet was assumed to be 1/3 of the energy spread predicted by the CSR steady-state emission of a longitudinal Gaussian charge distribution [10]. That value fits well all simulation results, and is supported by a current profile smoother than a Gaussian: it actually resembles a flat-top profile with Gaussian tails at the edges.

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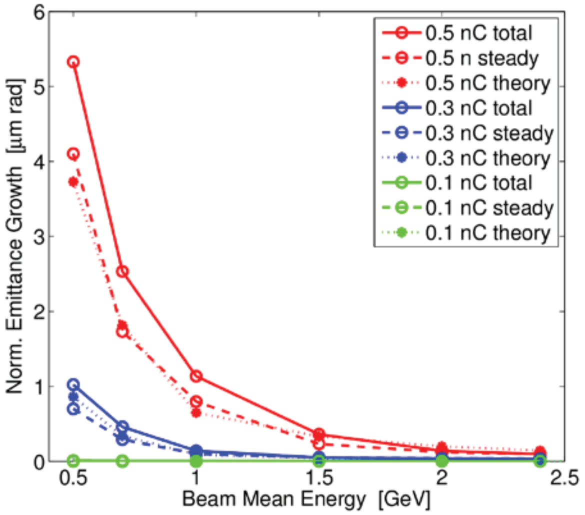

Figure 4: (Colour on-line) Quadratic difference of final-minus-initial normalized emittance (rms values) with CSR turned on and off, at different beam mean energies and bunch charges.

CONCLUSIONS

Linea transfer matrices associated to 1-D steady-state theory of CSR emission allow an estimation of chromatic emittance dilution due to CSR kicks in a multi-bend, 180 deg arc compressor, in a reasonable agreement with Elegant simulation results. The same theoretical background was used to design a compressor arc quasi-immune to CSR-induced emittance growth. The sensitivity of the compressed beam final emittance to optics, beam charge and mean energy has been investigated, and found to be in agreement with analytical expectations.

The simulation of finite dipoles' length, thus of variation of Twiss parameters in the dipoles, of transient CSR fields and CSR in drifts, all effects not included in the theoretical model, together with a partial cancellation of the CSR kicks along the lattice, lead to residual CSR-induced emittance growth at 1 nC (see Fig. 3), whereas it is still neutralized at 0.5 nC. A discrepancy as large as 20% in the final emittance value is found between theory and simulation at the highest charges and lowest beam energies. Nevertheless, the analytical prediction remains close to the numerical results when only CSR steady-state emission in dipoles is considered (see Fig. 4).

The most remarkable result is, in our opinion, the evidence of a correlation between β_2 and the final emittance value, and that the smallest β_2's value does not lead to optimum CSR suppression, in agreement with the analytical (and somehow counterintuitive) prediction of Eq.2. In conclusion, non-steady state CSR effects do not substantially affect the validity of the analysis; this can be used as a guidance for lattice design, possibly further improved by numerical optimization algorithms.

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