SASE FEL POLARIZATION CONTROL USING CROSSED UNDULATOR*

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Abstract

There is a growing interest in producing intense, coherent x-ray radiation with an adjustable and arbitrary polarization state. In this paper, we study the crossed undulator scheme for rapid polarization control in a self-amplified spontaneous emission (SASE) free electron laser (FEL). Because a SASE source is a temporally chaotic light, we perform a statistical analysis on the state of polarization using FEL theory and simulations. We show that by adding a small phase shifter and a short (about 1.3 times the FEL power gain length), 90° rotated planar undulator after the main SASE planar undulator, one can obtain circularly polarized light – with over 80% polarization – near the FEL saturation.

INTRODUCTION

Several x-ray free electron lasers (FELs) based on self-amplified spontaneous emission (SASE) are being developed worldwide as next-generation light sources [1, 2]. In the soft x-ray wavelength region, polarization control (from linear to circular) is highly desirable in studying ultrafast magnetic phenomena and material science. The x-ray FEL is normally linearly polarized based on planar undulators. Variable polarization could in principle be provided by employing an APPLE-type undulator [3]. However, its mechanical tolerance for lasing at x-ray wavelengths has not been demonstrated, and its focusing property may change significantly when its polarization is altered. An alternative approach for polarization control is the so-called "crossed undulator" (or "crossed-planar undulator"), which is the subject of this paper.

The crossed-planar undulator was proposed by K.-J Kim to generate arbitrarily polarized light in synchrotron radiation [4] and FEL sources [5]. It is based on the interference of horizontal and vertical radiation fields generated by two adjacent planar undulators in a crossed configuration (see Fig. 1). A phase shifter between the undulators is used to delay the electron beam and hence to control the final polarization state. For incoherent radiation sources, the radiation pulses generated in two adjacent undulators by each electron do not overlap in time. Thus, a monochromator after the second undulator is required to stretch both pulses temporally in order to achieve interference. The degree of polarization is limited by beam emittance, energy spread, and the finite resolution of the monochromator, as studied in a series of experiments at BESSY [6, 7]. On the other hand, for completely coherent radiation sources (such as generated from a seeded FEL amplifier or an FEL oscillator), the interference occurs due to the temporal overlap of two coherent radiation components in the second undulator as the relative slippage between them is typically a small fraction of the total pulse length [5]. A recent crossed-undulator experiment at the Duke storage ring FEL reported controllable polarization switches with a nearly 100% total degree of polarization [8].

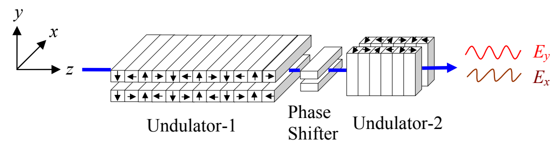

Figure 1: Schematic of the crossed undulator for polarization control

Due to the shot noise startup, a SASE FEL is temporally chaotic even though its transverse coherence can approach 100% near saturation. Thus, the effectiveness of the crossed undulator for polarization control in a SASE FEL deserves a detailed study. In this paper, starting with one-dimensional (1D) FEL theory, we calculate both radiation components and generalize the results of Ref. [5] to the case of SASE. We then determine the required length of the second undulator in order to produce the same average power as that produced in the first undulator and calculate the degree of polarization. The analytical results are compared with 1D SASE simulations after a proper statistical averaging. Finally, three-dimensional (3D) effects and simulation results are also discussed.

FIELD CALCULATION

Figure 1 shows a schematic of the crossed undulator applied to a SASE FEL. In the first planar undulator with a total length L_1, spontaneous radiation is amplified to generate horizontally polarized SASE field E_x. In the second undulator (of length L_2) that is rotated 90° with respect to the first one, E_x propagates freely without interacting with the electron beam, while a vertically polarized radiation field E_y is produced by the micro-bunched beam. A simple phase shifter such as a four-dipole chicane placing between the two undulators can slightly delay the electrons in order to adjust the relative phase of the two polarization components.

Let us consider the one-dimensional (1D) case where the electric field does not depend on the transverse coordinates. Let $E(z, t)$ be the complex but slowly varying electric field.

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We write

$$E(z,t) = \int \frac{\omega_1 d\nu}{\sqrt{2\pi}} E_\nu(z) e^{i\Delta\nu[(k_1+k_u)z-\omega_1 t]} , \quad (1)$$

where $\omega_1 = k_1 c$ is the fundamental resonant frequency corresponding to the average beam energy, $\nu = \omega/\omega_1$, $\Delta\nu = \nu - 1$ is the relative frequency detuning, $k_u = 2\pi/\lambda_u$ with λ_u the undulator period.

The horizontal field builds up from shot noise. In the small signal regime before FEL saturation, the solution for a cold beam without any energy spread is [9, 10]:

$$E^x_\nu(z) = \frac{-ieK[\mathrm{JJ}]n_0}{12\epsilon_0\gamma_0\rho k_u N_\lambda \mu_0} e^{-i\mu_0 2\rho k_u z} \sum_{j=1}^{N_e} e^{i\nu\omega_1 t_j} , \quad (2)$$

where K is the dimensionless undulator strength parameter, the Bessel function factor [JJ] is equal to $[J_0(\xi) - J_1(\xi)]$ with $\xi = K^2/(4+2K^2)$, n_0 is the electron volume density, ϵ_0 is the vacuum permittivity, γ_0 is the initial electron energy in units of mc^2, ρ is the dimensionless FEL Pierce parameter [11], and μ_0 is the exponential growth solution and is a function of the detuning parameter $\Delta\nu$:

$$\mu_0 \approx -\frac{1}{2}\left[1 - \frac{\Delta\nu}{3\rho} + \frac{(\Delta\nu)^2}{36\rho^2}\right] + i\frac{\sqrt{3}}{2}\left[1 - \frac{(\Delta\nu)^2}{36\rho^2}\right] . \quad (3)$$

The vertical field is created by the prebunched electron beam radiating in the second undulator. For a short undulator section, we have [12]

$$E^y_\nu(z_2) = E^x_\nu(L_1) e^{i(\phi-\psi/2)} \mathrm{sinc}\left(\frac{\psi}{2}\right) \times \frac{2i}{\mu_0^2}\left[\rho k_u z_2 - \mu_0 e^{i\alpha}(\rho k_u z_2)^2\right] . \quad (4)$$

where z_2 is the undulator distance from the beginning of the the second undulator, ϕ is the phase factor introduced by a weak chicane before the second undulator, $\psi = \Delta\nu k_u z_2$, $\mathrm{sinc}(x) = \sin(x)/x$, and

$$\alpha = \arctan\left[\frac{\sin(\psi/2)}{\mathrm{sinc}(\psi/2) - \cos(\psi/2)}\right] . \quad (5)$$

In order to generate circularly polarized light, we require that both E_x and E_y have the same average amplitude. From Eq. (4), this corresponds to the condition

$$\left|\frac{2i}{\mu_0^2}\left[\rho k_u L_2 - \mu_0 e^{i\alpha}(\rho k_u L_2)^2\right]\right| = 1 . \quad (6)$$

As the growth rate Im(μ_0) maximizes at $\Delta\nu = 0$, we have [12]

$$L_2 \approx 1.3 L_G , \quad \text{where} \quad L_G = \frac{\lambda_u}{4\pi\sqrt{3}\rho} \quad (7)$$

is the 1D power gain length.

DEGREE OF POLARIZATION

The interference of the two radiation components generated by the crossed undulator will produce flexible polarization. At the end of the second undulator when $z = L_1 + L_2$, these radiation fields in the time domain are

$$E_y(t) = \int \frac{\omega_1 d\nu}{\sqrt{2\pi}} E^y_\nu(z_2 = L_2) e^{i\Delta\nu[(k_1+k_u)(L_1+L_2)-\omega_1 t]} ,$$
$$E_x(t) = \int \frac{\omega_1 d\nu}{\sqrt{2\pi}} E^x_\nu(z = L_1) e^{i\Delta\nu[(k_1+k_u)L_1 + k_1 L_2 - \omega_1 t]} . \quad (8)$$

Note that we only used Eq. (1) for E_x at $z = L_1$ (and t_1) and applied the free space propagation phase factor $e^{i\Delta\nu[k_1 L_2 - \omega_1(t-t_1)]}$ in the second undulator as E_x does not interact with the electron beam there. Because of the chaotic nature of SASE radiation, we perform a statistical analysis to quantify the state of polarization.

The state of polarization can be described by the coherency matrix [13]

$$\mathbf{J} = \begin{bmatrix} J_{xx} & J_{xy} \\ J_{yx} & J_{yy} \end{bmatrix} = \begin{bmatrix} \langle E_x(t)E^*_x(t)\rangle & \langle E_x(t)E^*_y(t)\rangle \\ \langle E_y(t)E^*_x(t)\rangle & \langle E_y(t)E^*_y(t)\rangle \end{bmatrix} , \quad (9)$$

where * means complex conjugate, and the angular bracket refers to the ensemble average. For polarization control in the crossed undulator, we are particularly interested in the case when the average intensities of the two radiation components are the same: $\langle|E_x(t)|^2\rangle = \langle|E_y(t)|^2\rangle = \bar{I}$. Under this condition, the coherency matrix simplifies to

$$\mathbf{J} = \bar{I}\begin{bmatrix} 1 & |g_{xy}|e^{i\theta} \\ |g_{xy}|e^{-i\theta} & 1 \end{bmatrix} , \quad (10)$$

where

$$g_{xy} \equiv \frac{\langle E_x(t)E^*_y(t)\rangle}{[\langle|E_x(t)|^2\rangle\langle|E_y(t)|^2\rangle]^{1/2}} \quad (11)$$

is the first-order time correlation between E_x and E_y, and θ is the overall phase difference between E_x and E_y. When $\theta = \pm\frac{\pi}{2}$, the combined radiation is circularly polarized; when $\theta = 0$ or π, it is linearly polarized at $\pm 45°$ relative to the horizontal axis. The state of polarization is controllable by adjusting the phase shift ϕ in Eq. (4) so that the net phase in g_{xy} is $\theta = \pm\frac{\pi}{2}$ or $0/\pi$. With equal intensity in both transverse directions, the total degree of polarization is simply given by the amplitude of the x-y time correlation, i.e.,

$$P = |g_{xy}| . \quad (12)$$

In the x-ray wavelength region, the electron bunch duration is typically much longer than the coherence time of the SASE radiation. Thus, a SASE pulse consists of many random intensity spikes that are statistically independent. For a flattop current distribution (of width T), we can convert the ensemble average of Eq. (11) into a time average as

$$g_{xy} = \lim_{T\to\infty} \frac{1}{\bar{I}T}\int_{-T/2}^{T/2} dt E_x(t)E_y^*(t)$$
$$= \frac{1}{\bar{I}T}\int_{-\infty}^{\infty}\omega_1 d\nu E_\nu^x(L_1)E_\nu^{y*}(L_2)e^{-i\Delta\nu k_u L_2}\,, \quad (13)$$

where we have applied Eq. (8) and the Parseval relation in converting the time integration to the frequency integration. Assuming that the first undulator operates in the exponential gain regime, the frequency dependence of E_ν^x is approximately Gaussian with the rms relative bandwidth $\sigma_\nu = \sqrt{9\rho/(\sqrt{3}k_u z)}$ [9, 10]. Eq. (13) can be approximated as [12]

$$|g_{xy}| \approx \left| \int_{-\infty}^{\infty} d\bar{\nu} \frac{\exp\left(-\frac{\bar{\nu}^2}{2} - i\frac{\bar{\nu}\sigma_\nu k_u L_2}{2}\right)\mathrm{sinc}\left(\frac{\bar{\nu}\sigma_\nu k_u L_2}{2}\right)}{\sqrt{2\pi}\left[1 + (-\frac{1}{2} + i\frac{\sqrt{3}}{2})\frac{\bar{\nu}\sigma_\nu}{3\rho}\right]} \right|, \quad (14)$$

where $\bar{\nu} = \Delta\nu/\sigma_\nu$. In view of Eq. (7), we take $L_2 = 1.3L_G$ in Eq. (14) and obtain the total degree of polarization by computing $|g_{xy}|$.

Stokes parameters are also generally used to describe the partially polarized light. They are related to the coherency matrix as [13]:

$$\begin{aligned} S_0 &= J_{xx} + J_{yy}\,, \\ S_1 &= J_{xx} - J_{yy}\,, \\ S_2 &= J_{xy} + J_{yx} = 2\langle A_x(t)A_y(t)\cos(\theta(t))\rangle\,, \\ S_3 &= i(J_{yx} - J_{xy}) = 2\langle A_x(t)A_y(t)\sin(\theta(t))\rangle\,. \end{aligned} \quad (15)$$

Here $A_x(t)$ and $A_y(t)$ are the amplitudes of the radiation fields $E_x(t)$ and $E_y(t)$, respectively, and $\theta(t)$ is the phase difference between $E_x(t)$ and $E_y(t)$. Using these Stokes parameters, the total degree of polarization is given by

$$P = \frac{\sqrt{S_1^2 + S_2^2 + S_3^3}}{S_0}. \quad (16)$$

It is easy to see that when $\langle A_x\rangle = \langle A_y\rangle$, this definition reduces to Eq. (12). One can define the circular degree of the polarization P_c as:

$$P_c = \frac{|S_3|}{S_0}. \quad (17)$$

NUMERICAL SIMULATIONS

1D results

We first use a 1D FEL code to simulate the SASE radiation produced by the crossed undulator configuration and to analyze the degree of polarization. The parameters used are similar to the soft x-ray LCLS operation [1] (see Table 1). In the 1D simulations, the energy spread is set to zero since we want to compare with the previous analytical results.

Table 1: LCLS soft x-ray FEL parameters used in simulations.

Parameter	value	unit
electron beam energy	4.3	GeV
relative energy spread	0(0.023)	%
bunch peak current	2	kA
transverse norm. emittance	1.2	μm
average beta function	8	m
undulator period λ_u	3	cm
undulator parameter K	3.5	
FEL wavelength	1.509	nm
FEL ρ parameter	0.119	%
1D power gain length L_G	1.17	m
3D power gain length L_G^{3D}	1.48	m

Fig. 2 shows the average radiation power in both x and y directions produced by the cross undulator. The length of the first undulator is allowed to vary, while the second undulator length $L_2 = 1.3L_G \approx 1.53$ m is held constant. The phase shift ϕ is set up properly to maximize the circular degree of polarization. As predicted by Eq. (7), the power of the two radiation components are essentially the same in the exponential gain regime. Near saturation, the power of the vertical field is lower than that of the horizontal one because the FEL-induced energy spread starts to de-bunch the electron beam in the second undulator. We repeat the simulations 200 times for each L_1 with different random seeds to start the process and calculate the degree of the polarizaiton at the exit of the second undulator. Figure 3 shows the total degree and circular degree of polarization from the simulation results as well as the total degree from the numerical integration of Eq. (14) for a comparison. When the first undulator is less than a couple of gain lengths, the crossed undulator operates in the spontaneous emission regime, the amplitude of the x-y correlation and hence the degree of polarization are very small without the use of a monochromator. The degree of polarization increases in the exponential growth regime and reaches a maximum of 85% near the FEL saturation. In this regime and especially when the gain is very high, we see a good agreement between simulations and Eq. (14). In the saturation regime, the total degree of polarization is still preserved, while the circular degree of polarization decreases quickly as the vertical radiation intensity becomes smaller than the horizontal one. Since Eq. (14) derived from the linear theory is not applicable in this regime, only simulation results are shown in Fig. 3 for $L_1 \geq 20$ m.

There are two effects that prevent the degree of polarization to reach 100% in a crossed-undulator SASE FEL. First, there is relative slippage between E_x and E_y in the second undulator. Since E_x stops interacting with the electron beam after the first undulator, the group velocity of E_x is the speed of light c. However, the group velocity of E_y is slower than c because it is generated by the pre-bunched beam that travels at the average longitudinal velocity $\beta_\parallel c$. In fact, 1D simulations indicate that the group velocity of

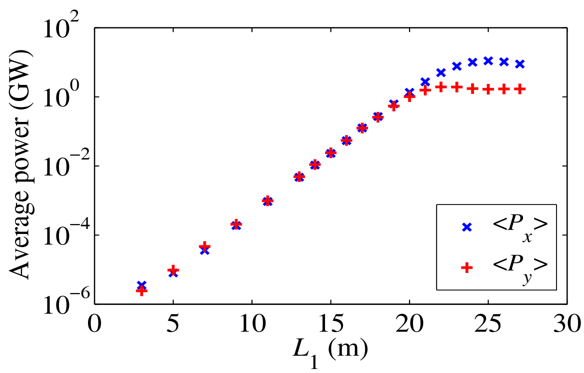

Figure 2: 1D simulations of the average SASE power at 1.5 nm from the first (blue cross) and the second (red plus) undulator. Here L_1 is the length of the first undulator, $L_2 = 1.3L_G = 1.53$ m is the length of the second undulator.

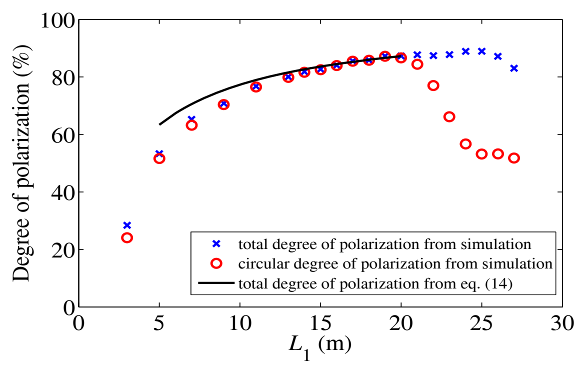

Figure 3: Total degree of polarization from 1D simulations (blue cross) and from analytical formula (black curve), and circular degree of polarization (red circle) from 1D simulations.

E_y is almost the same as that of the electrons within the short second undulator section. In addition to a simple time delay due to slippage, the temporal profile of the vertical radiation field is also distorted from the horizontal one because of the lethargy during the initial radiation buildup process in the second undulator. This accounts for the additional depolarization effect in a crossed undulator SASE FEL. A monochromator after the second undulator may be used to select a single frequency mode and to improve the temporal coherence. This in principle should increase the degree of polarization, but the shot-to-shot intensity fluctuation will be nearly 100%.

3D Discussions

A remarkable feature of a SASE FEL is its transverse coherence. At a sufficiently high gain, a single transverse mode with the largest growth rate will dominate over all other transverse modes for a typical SASE FEL. Thus, we expect the previous 1D analysis still applies to 3D situations in the high gain limit, with the maximum polarization obtainable at the end of the exponential growth regime. Since the length of the second undulator is short, the diffraction effects for the free-propagating E_x in the x-ray wavelength regime is expected to be small. Thus, the 3D effects such as emittance and diffraction do not play significant roles in determining the degree of polarization for a crossed undulator SASE FEL.

We use the 3D FEL code GENESIS 1.3 [14] to check these expectations. The electron beam is dumped at the end of the first undulator and is used to generate E_y in the second undulator. E_x propagates in the same length of the second undulator but without any undulator magnetic field. We use the same soft x-ray FEL example listed in Table 1 as the 1D case but with a relative energy spread of 0.023%, which roughly corresponds to the LCLS soft x-ray parameters. The length of the first undulator is chosen to be 23 m and is about 3 m before the saturation point. A 2-m short second undulator is necessary to produce the same radiation power for the vertical field. The 3D power gain length corresponding to these parameters is $L_G^{3D} = 1.48$ m, so Eq. (7) approximately holds in this 3D case. We use the on-axis far-field radiation intensity and phase from GENESIS simulations to calculate the total degree of polarization. Instead of performing many statistical runs for the ensemble average, we average the result over hundreds of intensity spikes within the radiation pulse in order to save on simulation effort. The total degree of polarization from this 3D calculation is 87%, very close to the 1D prediction.

CONCLUSIONS

The statistical analysis presented in this paper shows that the crossed-planar undulator is an effective method for polarization control in a SASE FEL. The maximum degree of polarization is over 80% from both theory and simulations. If fast pulsed magnets are employed in the phase shifter chicane, the relative phase between the two radiation components from the crossed undulator can vary at hundreds of Hz, hence enabling fast polarization switching for many scientific applications.

REFERENCES

[1] LCLS Conceptual Design Report, SLAC-R-593, 2002.
[2] TESLA Technical Design Report, TESLA FEL 2002-09, 2002.
[3] S. Sasaki, Nucl. Instrum. Methods A **347**, 83-86 (1994).
[4] K.-J Kim, Nucl. Instrum. Methods A **219**, 425 (1984).
[5] K.-J. Kim, Nucl. Instrum. Methods A **445**, 329 (2000).
[6] J. Bahrdt *et. al.*, Rev. Sci. Instrum. **63**, 339 (1992).
[7] J. Bahrdt, private communication.
[8] Y.K. Wu *et. al.*, Phys. Rev. Lett. **96**, 224801 (2006).
[9] K.-J Kim, Nucl. Instrum. Methods A **250**, 396 (1986).
[10] J. -M. Wang, L. -H. Yu, *ibid*, 484 (1986).
[11] R. Bonifacio, C. Pellegrini and L.M. Narducci, Opt. Commun. **50**, 373 (1984).
[12] Y. Ding, Z. Huang, Phys. Rev. ST-AB **11**, 030702 (2008).
[13] M. Born, E. Wolf, *Principles of Optics* (1999).
[14] S. Reiche, Nucl. Instrum. Meth. A **429**, 243 (1999).