# PHASE STABILITY OF A MICROTRON DRIVING A TERAHERTZ FEL * 

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## Abstract

The phase stability of bunches accelerated by a magnetron-driven microtron-injector of a terahertz Free Electron Laser (FEL) has been studied to optimize the microtron regimes providing good operation of the FEL. The study is based on a simulation of the beam dynamics in the microtron considering 2-D motion of the electrons in the median plane. This allows the computation of the current loading the accelerating cavity as well as the output microtron current. The loading current has been used to calculate the frequency deviations caused by the incremental loading in the accelerating cavity coupled with the magnetron. Further computations using the 2-D simulation show noticeable phase oscillation of the accelerated bunch leaving the microtron on the macropulse front. The phase oscillation is in agreement with the measured one and affects the lasing in the microtronbased FEL. Optimization of the microtron regimes allows one to minimize the effect. As a result, the terahertz microtron-based FEL provides radiated macro-pulse energy up to 0.2 mJ tunable in the range of $0.85-3 \mathrm{THz}$ with good stability. Results of the simulation and the measurements are presented in this article.

## INTRODUCTION

An S-band, 12-orbit, magnetron-driven, high-current classical microtron has been developed for injection into a terahertz FEL. The microtron, using I-type internal injection with a thermionic cathode, [1], provides stable operation of the FEL, tunable in the sub-millimeter wavelength range with macro pulse power up to 50 W . Such parameters of the FEL require operation of the microtron with minimal bunch repetition rate instability of the accelerated beam. The instability depends on the frequency stability of the RF generator and the phase stability of the accelerated bunches. To improve the first one we stabilized the magnetron frequency by employing the microtron accelerating cavity as an external stabilizing resonator. Minimization of the bunch phase instability was provided by optimization of the microtron operating regimes. Stability of the frequency of the feeding RF and the phase of the accelerated bunches was studied by the simulation of the acceleration process considering the beam loading of the accelerating cavity and by direct measurements. Results of the simulation and comparison with measured results are presented and discussed in this article.

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## SIMULATION OF TRACKING IN THE MICROTRON

The simulation has been done considering 2D tracking of the electrons in the microtron median plane ( $\vec{z}^{0}, \vec{x}^{0}$ ) up to the last (12-th) orbit using the Lorentz equation:

$$
\begin{equation*}
\frac{d \vec{p}}{d t}=e \cdot\{\vec{E}(\vec{r}, t)+[\vec{v} \times \vec{B}]\} . \tag{1}
\end{equation*}
$$

Here: $\vec{E}(\vec{r}, t)$ is the electric field acting on the electron at the point with coordinate $\vec{r}$ and the time $t, \vec{v}$ is the velocity of the electron at this point at time $t, \vec{B}$ is the magnetic field acting on the electron ( $\vec{B}$ includes the RF component in the accelerating cavity and the permanent microtron field). The electric field in the cavity corresponds to the $\mathrm{E}_{010}$-mode of a cylindrical resonator with radius $R$ :

$$
\stackrel{\rightharpoonup}{E}(\stackrel{\rightharpoonup}{r}, t)=\vec{z}^{0} \cdot E_{0} \cdot J_{0}\left(\frac{\chi_{01} \cdot r}{R}\right) \cdot \cos \left(\omega t+\varphi_{0}\right)
$$

where $J_{0}$ is the Bessel function of the first kind, $\chi_{01}=2.405$ is first zero of the Bessel function, $\varphi_{0}$ is the initial phase of an accelerated particle. The amplitude $E_{0}$ is expressed via amplitude of the cavity voltage, $V_{C}$ :

$$
E_{0}=\frac{V_{C}}{L \cdot T T F},
$$

where: $L$ is the cavity length, $T T F=\frac{\sin (k L / 2)}{(k L / 2)}$ is the transit-time factor, $k=2 \pi / \lambda=\omega / c$, and $\omega$ is the angular frequency of the accelerating field.
The equation of the motion of the electron, Eq. (1), was solved assuming a constant cavity voltage $V_{C}$ at various values of the voltage.
From the tracking the amplitude and phase of the current loading of the accelerating cavity have been determined. This was used in the abridged equations to simulate a transient state of the system including coupling of the magnetron and the microtron cavities. The loading current was calculated as follows from [2]:

$$
\begin{equation*}
\tilde{J}_{C}=i_{C}(t) \cdot \frac{1}{w^{*}} \cdot \int_{V} \overrightarrow{\widetilde{J}}_{1}(\vec{r}) \cdot \vec{e}(\vec{r}) d V, \tag{2}
\end{equation*}
$$

where: $i_{C}(t)$ is the normalized dimensionless timedependent measured emission current, $\overrightarrow{\widetilde{J}}_{1}(\vec{r})$ is the first harmonic of the loading current, which depends on the emission current and the coordinates and velocities of all
electrons passing through the cavity, and $\vec{e}(\vec{r})$ is the distribution of the normalized electric field in the cavity.

$$
w^{*}=\int_{-L / 2}^{L / 2} e_{z}\left(\vec{r}_{\perp}=0, z\right) \cdot \exp \left(-i \omega \cdot \frac{z}{v_{0}}\right) d z
$$

where $v_{0}$ is the average velocity of the electron $\left(v_{0} \cong c\right)$.
The distribution of the normalized electric field $\mathrm{E}_{010^{-}}$ mode in the cylindrical cavity with radius $R$ and length $L$ is expressed as:

$$
\vec{e}(\vec{r})=\vec{z}^{0} \cdot\left\langle-\sqrt{\frac{1}{\pi L}} \cdot \frac{1}{R} \cdot \frac{1}{J_{1}\left(\chi_{01}\right)} \cdot J_{0}\left(\frac{\chi_{01} \cdot r}{R}\right)\right\rangle
$$

Here: $J_{1}$ is the Bessel function of the first kind.
The first harmonic of the loading current, $\overrightarrow{\tilde{J}}_{1}(\vec{r})$, was computed directly from electron trajectories for obtained amplitude of the cavity voltage, $V_{C}$. In the computation we considered all electrons, synchronous and nonsynchronous as well.

The computed amplitudes of the loading current, $J_{C}$, and the accelerated current extracted from the 12-th orbit, $J_{12}$, (the microtron output current) as functions of the amplitude of the accelerating voltage, $V_{C}$, are plotted in Fig. 1 by red and blue colors, respectively.


Fig. 1. Normalized amplitudes of the loading current, $J_{C}$, and the accelerated current extracted from the 12-th orbit, $J_{12}$, (in relative units) vs. the accelerating cavity voltage. In the inset, (2), the curves $J_{C}$ and $J_{12}$ are plotted in detail in the neighborhood of the microtron operating conditions.

Fig. 2 shows the computed time-dependent phases of the loading current, $\varphi_{J_{C}}$, and of the extracted current from the 12 -th orbit, $\varphi_{J_{12}}$, as functions of the accelerating cavity voltage. In the inset, (2), are shown detailed plots of the loading current and the extracted current phases, $\varphi_{J_{C}}$ and $\varphi_{J_{12}}$, respectively, in the neighborhood of the microtron operating conditions.

Note that the dependence of the loading current phase on $V_{C}$, in the neighborhood of the microtron operating
conditions as shown in inset of Fig. 2, is quite weak because it is a result of the contribution of all bunches on all orbits and therefore it is the resulting phase averaged over all orbits; i.e. this is a collective effect. Previous computations in Ref. [3] show that just this collective effect is responsible for the frequency of oscillation in the accelerating cavity. Therefore it determines the magnetron frequency due to stabilization through the reflected wave from the accelerating cavity of the microtron. Phase oscillations of a single bunch do not play a noticeable role in the collective effect and thus do not deteriorate the frequency stability of the accelerating voltage in the microtron-FEL injector.


Fig. 2. Phases of the loading current, $\varphi_{J_{C}}$, and the extracted current, $\varphi_{J_{12}}$, as functions of the accelerating cavity voltage $V_{C}$.

Unlike the weak dependence of the loading current phase, $\varphi_{J_{C}}$, on $V_{C}$, dependence of the phase of the extracted current, $\varphi_{J_{12}}$, on $V_{C}$ as shown in inset of Fig. 2 is much stronger. This phenomenon does not make worse acceleration in the high-current microtron. However, as it is caused by the transient process in the equilibrium phase establishment, it results in an additional phase instability of the extracted bunches and causes an additional detuning of the FEL optical resonator affecting the FEL operation in the beginning of the macro-pulse. The amplitude and the phase of the extracted current computed as functions of the amplitude of the accelerating voltage allow the calculation of the phase deviations of the bunches extracted from the microtron and passed into the FEL undulator.
We computed the amplitude and the phase of the extracted current as functions of the amplitude of the cavity voltage, assuming the voltage after transition process as a given. The cavity voltage, $\tilde{V}_{C}$, depending on time, was determined solving the abridged equations describing the transient state of the coupled microtron and magnetron cavities, [3]. The cavity voltage one can write in following form:

$$
\begin{equation*}
\tilde{V}_{C}(t)=V_{C}(t) \cdot \exp \left[i \cdot \varphi_{C}(t)\right] \tag{3}
\end{equation*}
$$

where $V_{C}(t)$ is the time-dependent module of the cavity voltage and $\varphi_{C}(t)$ is the time-dependent phase of the cavity voltage. The extracted current is proportional to the time-dependent emission current, the dimensionless amplitude of the accelerated current from the 12-th orbit, depending on $V_{C}(t)$, and depends on the phase, $\varphi_{C}(t)$. The phase of the bunch at the entrance of the extracting channel depends on time as:

$$
\begin{equation*}
\varphi_{12}(t)=\varphi_{C}(t)+\varphi_{J_{12}}\left[V_{C}(t)\right] \tag{4}
\end{equation*}
$$

Deviations of the frequency in the microtron accelerating cavity during the macro-pulse are computed using following expression:

$$
\begin{equation*}
\Delta F_{C}(t)=\frac{1}{2 \pi} \cdot \frac{d \varphi_{C}(t)}{d t} \tag{5}
\end{equation*}
$$

It is obvious that the input microtron cavity voltage and the magnetron frequency have the same deviations.

Deviations of the bunch repetition rate caused by the phase motion of the extracted bunches are computed as:

$$
\begin{equation*}
\Delta F_{12}(t)=\frac{1}{2 \pi} \cdot \frac{d \varphi_{12}(t)}{d t} \tag{6}
\end{equation*}
$$

The time-dependent deviations of the repetition rate of the extracted bunches computed for the accelerating cavity detuning parameter, $\varepsilon=0.74$, are shown in Fig. 3 as curve C . The detuning parameter for the cavity operating with the frequency $\omega$ is given by $\varepsilon \cong-2 Q_{L}\left(\omega-\omega_{0}\right) / \omega_{0}$, where $Q_{L}$ is quality factor of the loaded microtron cavity, and $\omega_{0}$ is the circular eigenfrequency of the cavity. For comparison in Fig. 3 curve B shows deviations of the accelerating frequency caused by phase deviations of the current loading the accelerating cavity during the macro-pulse. Just these deviations determine the stability of the frequency of the accelerating voltage in the microtron-FEL injector.


Fig. 3. B: Deviations of the frequency in the microtron accelerating cavity, C : Bunch repetition rate deviations of the extracted current.

Analysis of these plots shows that the average increment in both curves is caused by incremental loading
of the accelerating cavity because of a back bombardment of the cathode surface by non-synchronous electrons, [4]. The oscillations in curve B with an amplitude of $\approx 30 \mathrm{kHz}$, that is $\approx 10^{-5}$ of the magnetron frequency, are caused by deviations of the magnetron current because of the frequency pushing in the magnetron during the macropulse. Oscillations in the bunch repetition rate, curve C , in the beginning of the macro-pulse are caused, as noted above, by the transition process during the establishment of the equilibrium phase due to an increase of the $\tilde{V}_{C}$ amplitude from a minimum value, acceptable for acceleration of the electrons over all orbits in the microtron and corresponding to 0.557 MV to the operating value, corresponding to $\leq 0.572 \mathrm{MV}$, as it shown in Fig. 2-2.

## BUNCH REPETITION RATE STABILITY OF THE HIGH-CURRENT MICROTRON AND ITS EFFECT IN A LASING

We have studied this phenomenon experimentally by employing a low Q-factor measuring cavity, Fig. 4, integrated into the microtron beamline, [5].


Fig. 4. Layout of the measuring cavity: 1- measuring cavity cover, 2- body of the measuring cavity, 3- coupling loop, 4- ceramic pivot, 5- bellows, 6- ceramic pivot support, 7-moving carrier, 8- beam line pipe.

The cavity body and covers have been made from stainless steel with vacuum sealing by indium wires. The cavity is undercoupled with the coupling loop; the beamline pipes serve as below-cutoff waveguides, thus the measured cavity Q-factor of $\approx 1000$ is determined by losses in walls. The ceramic pivots are movable along the axis via carrier and flexible bellows; they are used to tune the cavity.

The electron beam passing through the cavity along the axis causes $\mathrm{E}_{010}$-mode oscillations which are mixed with the 2.797 GHz synthesizer signal. Using a $2 \mathrm{Gs} / \mathrm{s}$ digital oscilloscope we measured periods of the intermediate frequency of about 8 MHz during the macro-pulse, in fact, the bunch repetition rate deviations in the intermediate frequency. The method allows detection of frequency deviations of a few kHz in the range of 3 GHz in a time interval of $\sim 0.1 \mu \mathrm{~s}$. With a 20 dB directional coupler integrated into the microtron RF system we measured deviations of the frequency of the magnetron feeding the accelerating cavity with the same technique, [3].

The deviations measured during the macro-pulse for two values of the microtron cavity detuning parameter are shown in Fig. 5.


Fig. 5. Measured deviations in the bunch repetition rate, solid line with error bar, and in the magnetron frequency, dots, during the macro-pulse for two values of the detuning parameter.

During the measurements the detuning parameter value was varied by mechanical tuning of the magnetron.

The plotted curves show that measured deviations of the magnetron frequency with period of $\sim 0.6 \mu \mathrm{~s}$ and amplitude of $\sim 30 \mathrm{kHz}$ in fact are similar in shape for both values of the detuning parameter. However the measured repetition rate of the extracted bunches in the first half of the accelerated macro-pulse current demonstrates oscillations noticeably different from deviations of the magnetron frequency. The amplitude of the oscillations is strongly depends on the detuning parameter. The phenomenon, as was shown above, is caused by variation of the equilibrium phase in the process of acceleration. An increase of the detuning parameter of the accelerating cavity increases the variation of the equilibrium phase that increases the amplitude of the phase oscillations of the extracted bunches during the transient process.

Note that the phenomenon causes an additional detuning of the optical FEL resonator that increases the build-up time for lasing, Fig. 6-a, and results in a decrease of the macro-pulse energy of the microtron-based FEL, Fig. 6-b.


Fig. 6. a. 1- Shape of the macro-pulse current at the FEL undulator entrance (right scale), 2 and 3- Shapes of the

FEL macro-pulse power (left scale) measured at the wavelength of $110 \mu \mathrm{~m}$ at the accelerating cavity detuning parameter of $\varepsilon=0.52$, and $\varepsilon=0.88$, respectively. bMeasured lasing macro-pulse energy vs. the detuning parameter (dots).

From measurements the beam current at the entrance of undulator was kept constant with an accuracy of $3 \%$. The lasing macro-pulse shape was measured with a quasioptical Schottky-barrier detector, [6], at the lasing wavelength of $110 \mu \mathrm{~m}$. Plotted in Fig. 6-b the dots point to the necessity of optimizing of the microtron operating regime to provide operation of the microtron-based FEL with minimal detuning parameter of the accelerating cavity at a given value of the accelerated current.

The obtained results allow optimization of the microtron parameters providing operation of the tunable in wide band terahertz FEL at a lasing energy of $\approx 0.2 \mathrm{~mJ}$ in the macro-pulse with duration of $\approx 4 \mu \mathrm{~s}$. The standard deviation of the lasing energy during long-time operation is less than $10 \%$.

## SUMMARY

2D tracking has been employed in considering the transient process in the microtron accelerating cavity coupled with the magnetron. The simulation showed that the transient process involves the phase motion of all bunches on all orbits as well as the establishment of the equilibrium phase. The first phenomenon is the oscillation averaged over all phases and in fact does not affect the phase stability of the separate extracted bunch unlike the second one. The simulation and measurements show that minimization of the detuning parameter at a given accelerated current minimizes the equilibrium phase motion effect. Optimization was realized in the microtronbased terahertz FEL that allowed stable and effective operation of the facility.

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