# GUN EMITTANCE AND COMPACT XFEL \*

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### Abstract

Role of gun emittance is discussed to build a compact XFEL machine. It is shown that low gun emittance plays the critical role of reducing the XFEL machine size.

## **INTRODUCTION**

X-ray free electron laser (XFEL) based on self amplified spontaneous emission (SASE) [1, 2] is considered the next generation light source. However, the XFEL machine is so huge and generally costs very high. It is a natural attempt to find the possibility of building an XFEL machine of lower electron energy and a reasonably modest (compact) size, without degrading the radiation quality. The difficulty of designing a compact and low electron energy (E) XFEL can be summarized as following.

- 1. As *E* is lowered, the FEL parameter  $\rho$  is also lowered depending upon *E*. However, since the relative electron beam energy spread  $\sigma_E/E$  gets lager, the lasing condition  $\sigma_E/E < \rho$  is hardly met. As a result, the radiation power and quality are poor.
- 2. The linac size decreases as *E* decreases. However, the needed undulator length does not decrease as fast as *E*, as will be shown below.
- 3. In a low electron energy XFEL, the undulator gap tends to be small. The undulator wakefield problem is more serious.

We will show, in this paper, that these problems can be solved by using a low emittance gun [3, 4]. A normalized beam emittance ( $\epsilon_n$ ) of 1-1.2 mm mrad has been a widely used number for a photo-cathode RF gun. However, recent development of technology makes it a realistic goal in the near future to generate even lower gun emittance. There are a few schemes under intensive R&D. A well known example is the single crystal thermionic gun that is going to be used in the SPring-8 Compact SASE Source (SCSS) [5]. Its emittance is expected to be around  $\epsilon_n = 0.6$  mm mrad, although this goal is not achieved yet [6]. Furthermore, a field emitter array gun that is now under development in Paul Scherrer Institute (PSI) is expected to achieve a lower emittance even down to  $\epsilon_n = 0.1 \text{ mm mrad } [7]$ , although it is still at the very beginning stage. Besides these new type of guns, conventional photo-cathode guns are still under progress toward a low emittance [8, 9]. For example, the slice emittance of the LCLS photo injector was recently measured to be 0.9 mm mrad with 1 nC charge, a promising result [8].

Therefore, it is now a good time to study the impact of a low emittance gun on the XFEL machine, although its practical application is in the future. In this paper, we study the feasibility of a compact XFEL machine with variable  $\epsilon_n$ down to nearly 0.1 mm mrad and also variable gun current. Note that  $\epsilon_n$  here refers to the theoretical emittance used in the FEL physics, that is, the slice emittance.

### **E-DEPENDENCE OF PARAMETERS**

The method of energy scaling in XFEL is to lower the electron beam energy (E) while keeping the undulator resonant condition,

$$\lambda_r = \frac{\lambda_u}{2\gamma^2} \left( 1 + \frac{K^2}{2} \right),\tag{1}$$

where  $\lambda_r$  is the resonant wavelength,  $\gamma = E/(mc^2)$  is the Lorentz factor, and K is the undulator parameter defined by

$$K = 0.934 B_0 [\text{Tesla}] \lambda_u [\text{cm}]. \tag{2}$$

 $B_0$ , the undulator peak magnetic field, depends not only on the undulator gap and period but also on the magnet material. If we consider a hybrid undulator with vanadium permendur, it is given by

$$B_0 = 3.694 \exp\left[-5.068 \frac{g}{\lambda_u} + 1.520 \left(\frac{g}{\lambda_u}\right)^2\right] \quad (3)$$

with g denoting the undulator gap. To build a compact XFEL,  $B_0$  should never be decreased, otherwise the saturation length and thus the undulator length would increase. We will fix  $g/\lambda_u$  to keep  $B_0$  unchanged and will adjust  $\lambda_u$  in Eq. (1) to keep  $\lambda_r$  unchanged, while lowering E from the LCLS energy 14.35 GeV. The LCLS wavelength,  $\lambda_r = 1.5$  Å, will be kept and  $\lambda_u = 3$  cm, g = 0.65 cm will be adjusted with the ratio kept. To find out how  $\lambda_u$  should be changed to keep  $\lambda_r = 1.5$  Å and  $B_0$  at the energy scaling, note that Eq. (1) is a cubic equation for  $\lambda_u$  for given  $\lambda_r$  and  $B_0$ . Arranging Eq. (1) for  $\lambda_u$  gives

$$\lambda_u^3 + \frac{2}{a^2}\lambda_u = \frac{4\lambda_r \gamma^2}{a^2},\tag{4}$$

where  $a = 0.934B_0$ . Solving this cubic equation, we obtain  $\lambda_u$  as a function of  $\gamma$  (or *E*). The graph of  $\lambda_u$  versus *E* is shown in Fig. 1. Since  $B_0 \approx 1$  T, we see that  $a \approx 1$  cm<sup>-1</sup> and  $\lambda_u > a^{-1}$  for  $\lambda_u > 1$  cm, which is usually the case. Equation (4) can then be roughly approximated to

$$\lambda_u^3 \approx \frac{4\lambda_r \gamma^2}{a^2},\tag{5}$$

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from which we can derive the rough dependence of  $\lambda_u$  on  ${\cal E}$ 

$$\lambda_u \propto E^{2/3}.$$
 (6)

Hence, as E decreases from the LCLS energy in the graph,  $\lambda_u$  decreases almost linearly. Since  $g/\lambda_u$  is fixed to 0.217,  $g = 0.217\lambda_u$  also decreases making an in-vacuum undulator the inevitable choice at lower electron energies. Eversus  $\lambda_u$  is shown in Fig. 1.



Figure 1:  $\lambda_u$  as a function of E to keep the resonance condition for 1.5 Å hard X-ray in the energy scaling. The undulator peak field  $B_0$  is fixed in the scaling.

The rough dependence of  $\lambda_u$  on E is used to derive the rough E dependence of other parameters. First of all, the FEL parameter  $\rho$  that is defined by

$$\rho = \frac{1}{2\gamma} \left[ \frac{I_{pk}}{I_A} \frac{\lambda_u^2 K^2 [JJ]^2}{8\pi^2 \sigma_x^2} \right]^{1/3},\tag{7}$$

where  $I_A = 17.045$  kA is the Alfen current,  $I_{pk}$  is the peak current,  $\sigma_x$  is the cross sectional beam size, and [JJ] is defined as

$$[JJ] = J_0\left(\frac{K^2}{4+2K^2}\right) - J_1\left(\frac{K^2}{4+2K^2}\right).$$
 (8)

In Eq. (7), note that  $\sigma_x^2 = \beta \epsilon_n / \gamma$  where  $\beta$  is the betatron function.  $\beta$  is an independent parameter we can choose freely. It is usual to choose the optimal  $\beta$  that gives the shortest saturation length. The optimal  $\beta$  was evaluated in [12] and is given by

$$\beta_{opt} = 11.2 \left(\frac{I_A}{I_{pk}}\right)^{1/2} \frac{\epsilon_n^{3/2} \lambda_u^{1/2}}{\lambda_r K[JJ]}.$$
(9)

Using  $\beta_{opt}$ ,  $\rho$  is completely described by the known parameters as in

$$\rho = \frac{1}{2} K[JJ] \left( \frac{I_{pk}}{I_A} \frac{\lambda_u}{\epsilon_n} \right)^{1/2} \left( \frac{\lambda_r}{89.6\pi^2 \epsilon_n \gamma^2} \right)^{1/3}.$$
 (10)

X-ray FELs

122

Since K has the same E dependence as  $\lambda_u$ , it is easy to see the rough dependence of  $\rho$  as

$$\rho \propto \left(\frac{E}{\epsilon_n}\right)^{1/3} \left(\frac{I_{pk}}{\epsilon_n}\right)^{1/2}.$$
(11)

When  $\epsilon_n$  is fixed to 1.2 mm mrad, the *E*-dependence of  $\rho$  is shown in Fig. 2. For the beam peak current, we used the LCLS value  $I_{pk} = 3.4$  kA.  $\rho$  decreases as *E* decreases. This degrades the machine performance and the radiation quality. The requirement  $\sigma_E/E < \rho$  gives a severe restriction. The LCLS initial value of  $\sigma_E/E$  is approximately 0.01% [10], which means  $\sigma_E \approx 1.4$  MeV. Obviously, the lower *E* is, the larger  $\sigma_E/E$  is. As *E* decreases in the scaling,  $\sigma_E/E$  is comparable to  $\rho$  at around E = 4.5 GeV, where the lasing barely happens.



Figure 2: FEL parameter  $\rho$  as a function of E with  $\epsilon_n = 1.2$  mm mrad.

Another potential problem is the undulator wakefield, which is inversely proportional to the undulator gap. The undulator wakefield creates relative energy spread between the slices, the rms of which is given by [10]

$$\sigma_w = -\frac{e^2 N L(W_z)_{rms}}{E},\tag{12}$$

where L is the undulator length, and  $(W_z)_{rms}$  is the rms of the wakefield over a bunch. For a Gaussian bunch, we have [10]

$$(W_z)_{rms} \approx 1.02 \frac{\Gamma(3/4)}{2\sqrt{2}\pi^2} \frac{c}{\sigma_z^{3/2}g} \left(\frac{Z_0}{\sigma}\right)^{1/2}.$$
 (13)

L is obviously almost equal to  $L_{sat}$ . Since  $\sigma_w$  is inversely proportional to E, it is supposed to grow and give more power reduction for lower E. This is one of the difficulties to make a compact XFEL. However, using Eq. (19) and the fact that eN is proportional to  $I_{pk}$ , we can find the rough dependence of  $\sigma_w$  under the energy scaling as

$$\sigma_w \propto \left(\frac{\epsilon_n^2}{E}\right)^{2/3} \left(\frac{I_{pk}}{\epsilon_n}\right)^{1/2} g^{-1}.$$
 (14)

# **ROLE OF GUN EMITTANCE**

Equation (11) clearly shows that the reduction of  $\rho$  at a lower energy can be compensated by using lower emittance gun.  $\rho$  also depends on the ratio  $I_{pk}/\epsilon_n$ , which measures the effectiveness or performance of a low emittance gun, although  $I_{pk}$  is determined by not only the gun performance but also the bunch compression. The higher the ratio is, the more effective the gun is in reducing the machine size. We will use f to denote the ratio as in

$$f = \frac{I_{pk}}{\epsilon_n}.$$
 (15)

For a given f, low  $I_{pk}$  can be allowed if  $\epsilon_n$  is low enough.  $\rho$  can be kept unchanged in the scaling if we decrease  $\epsilon_n$  as obtained by solving Eq. (10),

$$\epsilon_n = \left[ \left( \frac{K[JJ]}{2\rho_0} \right)^6 \left( \frac{I_{pk}}{I_A} \lambda_u \right)^3 \left( \frac{\lambda_r}{89.6\pi^2 \gamma^2} \right)^2 \right]^{1/5}, \quad (16)$$

where  $\rho_0$  is the constant value of  $\rho$ . When  $\rho_0$  takes the LCLS value,  $5.5 \times 10^{-4}$ , and this  $\epsilon_n$  is used in the energy scaling, we have the same  $\rho$  and thus comparable radiation quality and power as LCLS at lower electron energies.  $\epsilon_n$  as given in Eq. (16) is plotted in Fig. 3. If even lower  $\epsilon_n$  is used at each energy,  $\rho$  will be even larger. It is possible to recover what was lost in working at a lower energy by using a lower gun emittance solving the first problem mentioned in the introduction.



Figure 3: Graph of  $\epsilon_n$  that cancels the *E* dependence of  $\rho$  and makes it constant in the energy scaling.

To build a compact XFEL, not only the linac size but also the undulator size should be reduced. Actually, the energy scaling reduces the undulator size, too. This is easily seen by the behavior of the one dimensional gain length defined by

$$L_G = \frac{\lambda_u}{\sqrt{3}\pi\rho} \tag{17}$$

or more accurately of the saturation length given by

$$L_{sat} = L_G(1+\eta) \ln\left(\frac{P_{sat}\lambda_r}{2\rho^2 Ec}\right),\tag{18}$$

X-ray FELs

where  $\eta$  is the famous fitting formula by Ming Xie [11] and  $P_{sat} = 1.6\rho IE/e(1+\eta)^2$  is the saturated peak power. Since the logarithm is insensitive to the variation of its variable, the behavior of  $L_{sat}$  under the scaling is mostly given by the behavior of  $L_G$ . Using Eqs. (6) and (11), we obtain the rough dependence

$$L_{sat} \propto (E\epsilon_n)^{1/3} f^{-1/2},\tag{19}$$

where f is the previously defined ratio of  $I_{pk}$  to  $\epsilon_n$ . Equation (19) shows that the saturation length (thus the undulator length) decreases, as E decreases, only as  $E^{1/3}$ . However, it also shows that  $L_{sat}$  can also be reduced further by using low gun emittance. For example, if  $\epsilon_n$  varies as in Fig. 3,  $L_{sat}$  decreases almost as linearly as E decreases as shown in Fig. 5. Hence, a low gun emittance solves the 2nd problem.



Figure 4: Saturation length in the energy scaling when  $\epsilon_n$  moves as in Fig. 3.

Finally, we see from Eq. (14) that the growth of  $\sigma_w$  by lowering E can be canceled by using a low emittance gun. The equation also shows that even the growth of  $\sigma_w$  due to the small undulator gap can be canceled by  $\epsilon_n$  low enough. Another critical role of  $\epsilon_n$  to realize a compact XFEL. It solves the third problem.

### **OPTIMAL GUN EMITTANCE**

As far as  $\epsilon_n$  is concerned, it is not true that the lower, the better.  $\epsilon_n$  should be chosen to give the maximal transverse coherence to the XFEL radiation. The XFEL degree of transverse coherence was obtained as a function of  $z = 2\pi\epsilon_n/(\lambda_r\gamma)$  [12]. According to this result, the maximal transverse coherence is achieved at around  $z \approx 1$ . For LCLS, z = 1.8 is close enough to 1, but as the energy is scaled down z grows substantially and the transverse coherence of the radiation degrades. This degradation at a low energy can be prevented or the transverse coherence can even be improved by using a gun emittance low enough. In terms of  $\epsilon_n$ , the condition z = 1 becomes

$$\epsilon_n = \frac{\lambda_r}{2\pi}\gamma. \tag{20}$$

For given  $\lambda_r$  and  $\gamma$ , this may be called the optimal  $\epsilon_n$ . This optimal  $\epsilon_n$  versus *E* is plotted in Fig. 5 for two hard X-ray wavelengths, 1 and 1.5 Å.



Figure 5: E and  $\epsilon_n$  that gives the maximal transverse coherence for two different wavelengths, 1 and 1.5 Å.

#### CONCLUSION

Simply scaling down the LCLS energy degrades the machine performance and the radiation quality. Specifically, it reduces the radiation power, deteriorates the transverse coherence, and increases the power reduction due to the undulator wakefield. However, the performance and the radiation quality can be recovered or even be improved by using a gun emittance low enough. Furthermore, the low gun emittance reduces the undulator length. And, it also reduces the undulator wakefield effect. This paper has shown that a compact hard X-ray FEL can be constructed only by adopting a lower emittance gun. The necessary technology for the low emittance gun is not at hand but under development.

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X-ray FELs

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