RMS MODE SIZE OF THE XFEL RADIATION*

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Abstract

Formula of the rms transverse mode size of the XFEL radiation is derived. Derivation uses information on the XFEL transvese coherence. Derived formula agrees well with the simulation result.

DERIVATION

In this paper, the rms transverse mode size of the X-ray free electron laser (XFEL) radiation based on self amplified spontaneous emission (SASE) [1, 2, 3] is derived using the information on the transverse coherence of the radiation [4]. In general, the rms mode size of a coherent radiation is given by

$$\sigma_c = \sqrt{\lambda/(4\pi)Z_R}, \quad (1)$$

where λ is the radiation wavelength and Z_R is the Rayleigh length. Obviously, the XFEL mode size (σ_r) must have the form of σ_c with proper Z_R. $Z_R = 2L_G$, where $L_G = \lambda_u/(4\pi\sqrt{3}\rho)$ is the one dimensional gain length, has been generally used to give rough estimation for σ_r [5], which we denote as

$$\sigma_d = \sqrt{\frac{\lambda}{4\pi}\frac{2\lambda_u}{4\pi\sqrt{3}\rho}}. \quad (2)$$

Although σ_d is often used as a substitute for σ_r, it will be shown below that it is not. The derivation here uses the information of the transverse coherence of the XFEL radiation. We will consider only the cases in which σ_r is comparable to the rms electron beam size σ_x, not including the two extreme cases, $\sigma_x \gg \sigma_r$, $\sigma_r \gg \sigma_x$, which were discussed in literatures [5].

Saldin et al. [6] computed the XFEL degree of transverse coherence as a function of

$$z = \frac{2\pi\epsilon}{\lambda} = \frac{2\pi\sigma_x^2}{\lambda\beta_0} \quad (3)$$

where β_0 is the β-function that is optimized to give shortest saturation length. The result is shown in Fig. 1 and β_o is given by [6]

$$\beta_o = \frac{11.2}{\sqrt{2}}\left(\frac{I_A}{I}\right)^{1/2}\frac{(\gamma\epsilon)^{3/2}\lambda_u^{1/2}}{\lambda K[JJ]}, \quad (4)$$

where $[JJ]$ denotes the following combination of bessel functions,

$$[JJ] = J_0\left(\frac{K^2}{4+2K^2}\right) - J_1\left(\frac{K^2}{4+2K^2}\right). \quad (5)$$

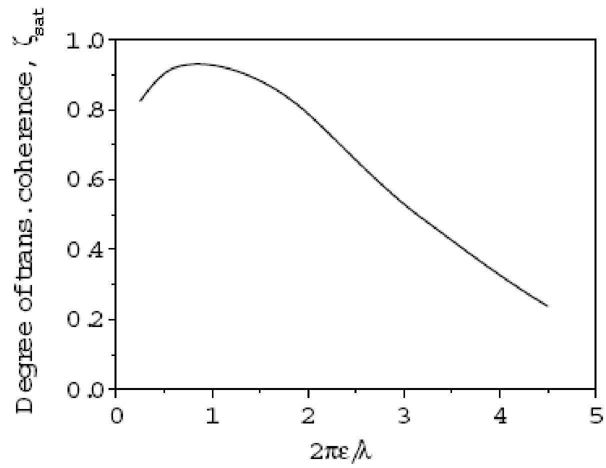

Figure 1: Graph of the degree of the transverse coherence versus $z = 2\pi\epsilon/\lambda$. (Ref. 6)

To make use of Fig. 1 to derive σ_r, we fix λ to a hard X-ray value. Then, z is proportional to σ_x^2. As z grows σ_x grows while σ_r would not grow much since λ is fixed. Hence, when z is large (far right side of Fig. 1), σ_x must be much larger than σ_r and the degree of transverse coherence is low. When z is very small (far left side of Fig. 1), σ_x must be much smaller than σ_r and the degree of transverse coherence is low again. The peaked part of the graph must correspond to that σ_x is nearly same as σ_r. Note, from Fig. 1, that the maximal transverse coherence is achieved at

$$\frac{2\pi\sigma_x^2}{\lambda\beta_o} = 0.8. \quad (6)$$

It is quite reasonable to assume that this maximal transverse coherence is achieved when $\sigma_r = \sigma_x$. Therefore, we see that

$$\sigma_r^2 = \sigma_x^2 = 0.8 \times \frac{\lambda\beta_o}{2\pi}. \quad (7)$$

To compare this expression with σ_d, note that the FEL parameter ρ is given by

$$\begin{aligned} \rho &= \left[\frac{1}{8\pi}\frac{I}{I_A}\left(\frac{K[JJ]}{1+K^2/2}\right)^2\frac{\gamma\lambda^2}{2\pi\sigma_x^2}\right]^{1/3} \\ &= \left[\frac{1}{8\pi}\left(\frac{I}{I_A}\frac{K^2[JJ]^2}{\gamma^3}\right)\frac{\lambda_u^2}{8\pi\sigma_x^2}\right]^{1/3}, \end{aligned} \quad (8)$$

where we used the resonance condition

$$\lambda = \frac{\lambda_u}{2\gamma^2}\left(1+\frac{K^2}{2}\right). \quad (9)$$

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Using Eq. (4) and (6), we find that

$$\frac{I}{I_A}\frac{K^2[JJ]^2}{\gamma^3} = \frac{11.2^2\lambda_u\epsilon^3}{2\lambda^2\beta_o^2} = \frac{62.7\lambda_u\sigma_x^2}{\beta_0^3}\left(\frac{0.8}{2\pi}\right)^2. \quad (10)$$

Then ρ is simplified to

$$\rho = \left[\frac{1}{(4\pi)^3}\frac{40.1}{4\pi}\left(\frac{\lambda_u}{\beta_0}\right)^3\right]^{1/3} = \frac{1}{4\pi}\left(\frac{40.1}{4\pi}\right)^{1/3}\frac{\lambda_u}{\beta_o}, \quad (11)$$

and β_o is simply given by

$$\beta_0 = \frac{1}{4\pi}\left(\frac{40.1}{4\pi}\right)^{1/3}\frac{\lambda_u}{\rho}. \quad (12)$$

Using this result in Eq. (7), it is straightforward to see that

$$\sigma_r^2 = 2.0 \times \frac{\lambda}{4\pi}\frac{2\lambda_u}{4\pi\sqrt{3}\rho} = 2.0\sigma_d^2. \quad (13)$$

Therefore, we conclude that

$$\sigma_r = 1.4\sigma_d. \quad (14)$$

Table 1: Parameters of the PAL-XFEL.

Electron energy (GeV)	3.7
Peak current (kA)	3
Normalized emittance (μrad)	1.0
Undulator period, λ_u (cm)	1.5
Undulator parameter, K	1.49
Average β function (m)	10
Radiation wavelength (Å)	3
FEL parameter, ρ	5.7×10^{-4}
1-D gain length, L_G (m)	1.19
Saturation length, L_{sat} (m)	45

COMPARISON WITH SIMULATION

Although this was obtained with the condition $\sigma_r = \sigma_x$, it does not mean that Eq. (14) is valid only when $\sigma_r = \sigma_x$. It is important to note that the condition $\sigma_r = \sigma_x$ was used only to derive the mode size using Fig. 1 and Eq. (14) is actually applicable to a wider range of σ_r. This is verified by simulation. As a demonstration, we consider PAL-XFEL that was under proposal at Pohang Accelerator Laboratory, Korea [7]. According to the PAL-XFEL parameters listed in Table 1, we find

$$\left(\frac{2\pi\epsilon}{\lambda}\right)_{PAL} = 2.89. \quad (15)$$

From Fig. 1, we see that the PAL-XFEL degree of transverse coherence is approximately 5.6, which is fairly off the maximal transverse coherence. It is easy to get

$$(\sigma_d)_{PAL} = \sqrt{\frac{\lambda}{4\pi}\frac{2\lambda_u}{4\pi\sqrt{3}\rho}} = 7.5\ \mu\text{m}, \quad (16)$$

and

$$(\sigma_r)_{PAL} = 10.5\ \mu\text{m}. \quad (17)$$

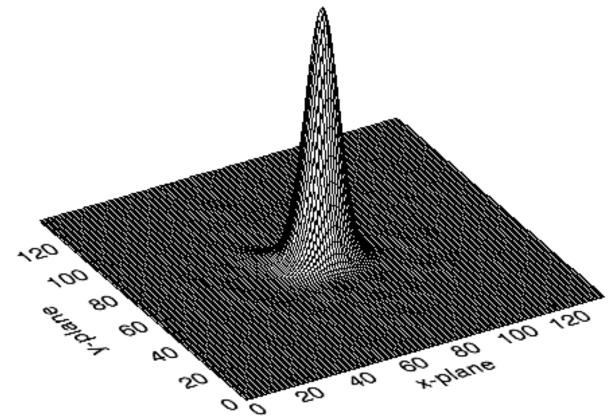

Figure 2: Simulated radiation profile of the PAL-XFEL.

The simulated radiation profile of PAL-XFEL is shown in Fig. 2 and its mode size is shown in Fig. 3. Both were simulated by GENESIS code [8]. Measuring the mode size from Fig. 3, we obtain

$$\sigma_r \cong 10\ \mu\text{m}. \quad (18)$$

Note that this number agrees well with the above $(\sigma_r)_{PAL}$ obtained by the formula of Eq. (14).

CONCLUSION

Comparison with the simulation shows that the derived formula Eq. (14) is fairly accurate.

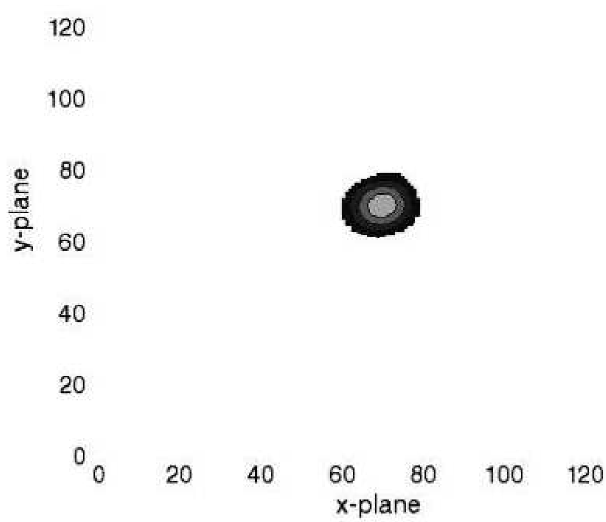

Figure 3: Simulated transverse mode size of the PAL-XFEL.

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