SHOT NOISE CONTROL AND REDUCTION BY COLLECTIVE COULOMB INTERACTIONS: 3D SIMULATIONS EVIDENCE

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Abstract

We present 3D simulations that verify a predicted current shot noise suppression effect. It was verified numerically that minimal current shot-noise is attainable in a drift length of quarter wavelength plasma oscillation as predicted by the 1D single mode model. We identify the parameter range, in which 3D deterioration effects are avoided, and the single mode model is valid. The noise reduction process may be applicable for controlling micro-bunching instabilities as well as enhancement of free electron laser coherence below the classical shot noise level.

INTRODUCTION

The current shot noise of a charged particle beam is the current fluctuations due to the corpuscular nature of the particles and their random arrival at the current measurement point. The shot noise is the source of incoherent radiation (spontaneous emission) in electron beam radiation sources of great interest such as Self Amplified Spontaneous Emission (SASE) free electron lasers (FELs).

It is thus of significant interest that a beam-dynamic collective interaction process exists, that enables control of the e-beam current shot-noise input into the radiation device [1],and makes it possible to suppress its incoherent output radiation power [2], below the classical shot-noise limitation level. The present theory for the noise suppression effect is based on an extended 1-D (single mode Langmuir plasma wave) small signal linear model [1]. According to this model, longitudinal Coulomb-collective interaction can take place along the transport line of an intense high brightness e-beam in a particular regime of transport parameters. At short collective interaction lengths, this process explains the Coherent Optical Radiation (COTR) radiative power enhancement that was measured in LCLS [3] and other laboratories [4], and the occurrence of microbunching instabilities in dispersive sections [5].

Contrary to other works on charge granularity and microbunching dynamics in e-beams [3], our model applies also to long interaction lengths. In this limit the velocity modulation of the beam by the longitudinal space charge field Coulomb forces, exceed the linear (as function of propagation distance) modulation range, and a process of random plasma oscillation starts taking place.

Focusing on the simple case of a uniform drift section, the solution of the cold beam linearized plasma fluid equations in the frequency domain, results in the following expression for the evolution of the current and velocity spectral components of the beam [1]:

$$i\left(L_d,\omega\right)=\left[\cos\phi_p\breve{i}\left(0,\omega\right)-i\left(\sin\phi_p/W_d\right)\breve{V}\left(0,\omega\right)\right]e^{i\phi_b\left(L_d\right)} \quad (1)$$

$$V\left(L_d,\omega\right)=\left[-iW_d\sin\phi_p i\left(0,\omega\right)+\cos\phi_p\breve{V}\left(0,\omega\right)\right]e^{i\phi_b\left(L_d\right)} \quad (2)$$

where L_d is the drift length in the laboratory frame, $\phi_b = L_d\omega/v_z$ is the plasma wave optical-phase, $\phi_p = \theta_{pr}L_d$ is the plasma longitudinal oscillation phase, $\theta_{pr} = r_p\omega_{pl}/v_0$ is the plasma longitudinal oscillation wave-number, $\omega_{pl} = \left(e^2 n_0/m\varepsilon_0\gamma^3\right)^{1/2}$ is the relativistic longitudinal plasma oscillation frequency, $W_d = r_p^2\sqrt{\mu_0/\varepsilon_0}/k\theta_{pr}A_e$ is the beam modulation impedance, r_p is the plasma reduction factor, $k = \omega/c$ is the optical wave-number, A_e is the e-beam area, $\breve{V}\left(z,\omega\right)$ is the spectral kinetic voltage (following Chu [6]):

$$\breve{V}\left(\omega\right)=-\gamma_0^3 v_0\left(m/e\right)\breve{v}\left(\omega\right)=-\left(mc^2/e\right)\breve{\gamma}\left(\omega\right) \quad (3)$$

The main conditions for keeping this single mode longitudinal interaction model valid are that the beam would be in the cold beam regime, namely the optical frequency modulation phase will not be washed out due to longitudinal velocity spread: $\Delta\phi_b = kL_d\Delta(1/\beta_z) << \pi$ (where $\Delta\beta_z$ is the axial velocity spread), and that higher order Langmuir plasma wave transverse modes will not be excited in the beam and disrupt the single mode process. These requirements set conditions on the beam (slice) energy spread and emittance parameters, the beam cross-section dimensions and other parameters [1].

If these conditions are maintained for a long enough distance to satisfy the quarter plasma oscillation condition: $\phi_p = \theta_p L_d = \pi/2$, then it appears from the cold beam linearized plasma fluid equations (1) and (2) that at this distance, full transformation of velocity noise into density noise takes place and vice versa:

$$\overline{\left|\breve{i}\left(L_d,\omega\right)\right|^2}=\overline{\left|\breve{V}\left(0,\omega\right)\right|^2}/W_d^2 \quad (4)$$

$$\overline{\left|\breve{V}\left(L_d,\omega\right)\right|^2}=\overline{\left|\breve{i}\left(0,\omega\right)\right|^2}W_d^2 \quad (5)$$

If the beam is dominated by current shot noise before entering the collective interaction region (which is usually the case in relativistic high current e-beams), namely:

$\overline{\left|\breve{i}(0,\omega)\right|^2} >> \overline{\left|\breve{V}(0,\omega)\right|^2}/W_d^2$, then, the current noise gain is much smaller than 1:

$$f_i = \frac{\overline{\left|\breve{i}_{out}(\omega)\right|^2}}{\left|\breve{i}(0,\omega)\right|^2} = \frac{\left|\breve{V}(0,\omega)\right|^2/W_d^2}{\left|\breve{i}(L_d,\omega)\right|^2} << 1 \qquad (6)$$

NUMERICAL SIMULATIONS

In the 3-D simulation studies, the beam dynamics in the collective interaction region was computed in the rest frame of the electron beam (which moved relatively to the lab frame with velocity v_0), by solving the motion equations of all sample particles under the Coulomb field forces applied on them by all other particles in a finite dimensions bunch of electrons (long enough regard to the bunch radius as a caustic beam and ignore coherent edge effects).

The simulations were carried out using General Particle Tracer (GPT) code starting from a uniform random distribution of sample particle in a cigar shaped charge bunch. The positions and velocities (**r**', **v**') were calculated for each particle (*j*) as a function of the time (*t'*). These variables were then transformed to the lab frame (*t*,**r**, **v**), using Lorentz transformation and calculated as a function of the position of the center of the bunch $z = v_0 t$.

Using $t_j(z) - t = z_j'/\gamma v_j(z)$ the current and velocity noise are calculated as a function of z – the transport distance of the bunch center:

$$\left|\breve{i}(\omega,z)\right|^2 = (q_e S)^2 \left|\sum_{j=1}^{N} \exp[i\omega t_j(z)]\right|^2 \qquad (7)$$

$$\left|\delta\breve{v}(\omega)\right|^2 = \frac{1}{(n_0 v_0 S)^2} \left|\sum_{j=1}^{N} (v_j - \overline{v}_{0j}) \exp[i\omega t_j(z)]\right|^2 \qquad (8)$$

where q_e is the charge of one macroparticle, $\overline{v}_{0j}$ is the average velocity of the electrons within a wavelength range around the j^{th} particle (this is done in order to eliminate an average energy chirp effect along the bunch length due to the average space charge). The summation is performed on all the macroparticles within the pulse.

Beam Parameters and Simulation Method

The simulations of drift length require a long straight acceleration section. We took for our exemplary computations the parameters of a drift section that can be generated after the injector in the FERMI@Elletra facility (Table 1). We assumed a flat top current density distribution beam. The emittance and energy spread were assumed to be small enough to satisfy the cold beam condition and a negligible initial velocity noise.

Table 1: Simulation Beam Parameters

Energy	100[MeV]
Current, Pulse length	80[A], 9[pS]
Beam Radius	1[mm]
Drift Length	31[m]
Drift Time	5.3e-10[S]

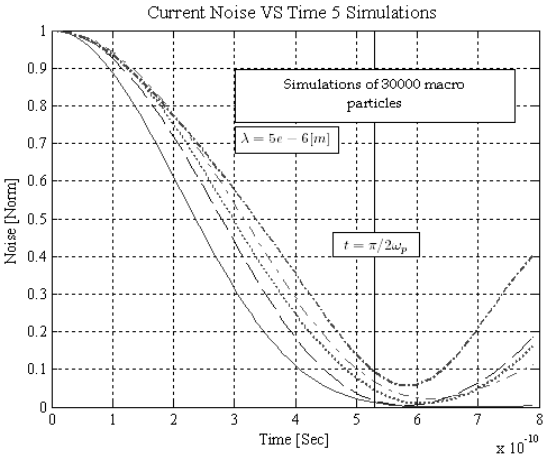

Figure 1: Current shot noise at 5 µm. Curves obtaine from 5 different 30k macro-particles GPT simulations The vertical line represents the theoretical quarter plasm oscillation time (in lab frame)- $t = \pi/2\omega_{pr}$

The current and velocity spectral noise parameters were calculated from eqs. (7), (8) for different frequencies (wavelengths) using a MatLab program. The range of wavelengths, for which calculations were made, was restricted to longer wavelengths in order to satisfy the conditions $n_0 A_e \lambda \beta_0 >> 1$, and $\lambda \gamma_0 \beta_0 << 2r_b$ [1]. The first is an obvious requirement - to have a multitude of particles (in the present case – macro-particles) per wavelength. The second condition is to assure operation in the Langmuir wave single transverse mode regime. When this condition was violated, 3-D effects washed out the looked-for effect, and the simulations did not produce any significant noise reduction results. This observation agrees with the findings of Venturini in the short interaction regime [7].

Results

The simulations run in the GPT program for several sets of random starting distribution of 30000 macro-particles. Because of the random nature of the shot noise, the initial shot noise in each set was different. Therefore, in order to show the characteristic behaviour of the noise evolution, in all cases the initial noise normalized to 1 (Figure 1).

Despite the variance between the different random starting particle sets, it is evident that there is noise

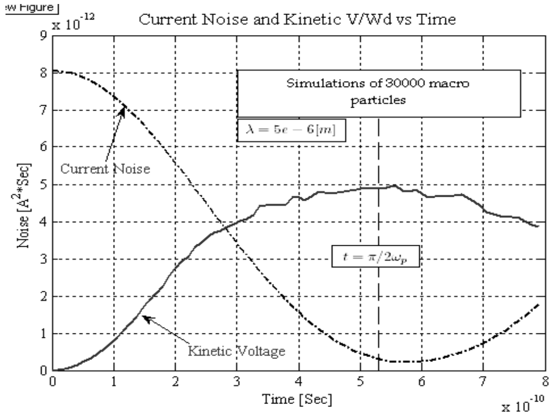

Figure 2: Spectral Current noise and kinetic voltage noise (normalized by the beam impedance) at 5 µm. The curves were obtained from a 30k macro-particles GPT simulation. The vertical line represents the theoretical quarter plasma oscillation time (in lab frame) - $t = \pi/2\omega_{pr}$

reduction in all cases. Moreover, it is clear that the noise minima fall slightly beyond, but very close to the calculated quarter plasma oscillation time (marked on the drawing in a vertical dashed line).

We also calculated the velocity noise (8) and the corresponding kinetic voltage noise for one of the sets (Figure 2). It reached its maximum value at the same time that the current noise reaches its minimum (quarter plasma oscillation time). This too provides good confirmation to the analytical linear single mode theory.

3-D effects and the Plasma reduction factor

When condition $\lambda\gamma_0\beta_0 << 2r_b$ is satisfied, the single mode model holds, however, due to the finite dimensions of the beam the plasma wave frequency deviates from the 1-D plasma wave frequency by the plasma reduction factor $r_p < 1$. The reason for this is the fringing of the microbunching space charge field lines at the periphery of the beam cross-section. This reduces the effective strength of the space-charge field which causes the plasma oscillation. Therefore, in this case, the quarter-plasma oscillation length is longer. Yet, this operating regime is the desirable operating regime, since in the opposite limit (for which $r_p = 1$) there is excitation of higher order Langmuir plasma wave modes and the transverse coherence of the bunching breaks down. In fact, it was shown by Venturini [7], that even in the short interaction length, 3-D effects wash out any transverse coherence of the bunching.

These theoretical observations are well confirmed by the calculated current noise evolution with time at different wavelengths for a particular particles simulation set (Figure 3). The results confirm that the minimal noise point is shifted to longer time for longer wavelengths.

Based on Venturini [7], we estimate the plasma reduction factor of the Langmuir wave fundamental mode from:

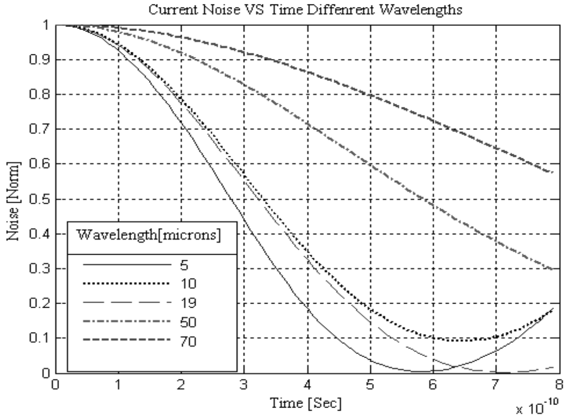

Figure 3: Current shot noise at different wavelengths (from 5 to 70 µm). Results obtained from a particular 30k macro-particles GPT simulation

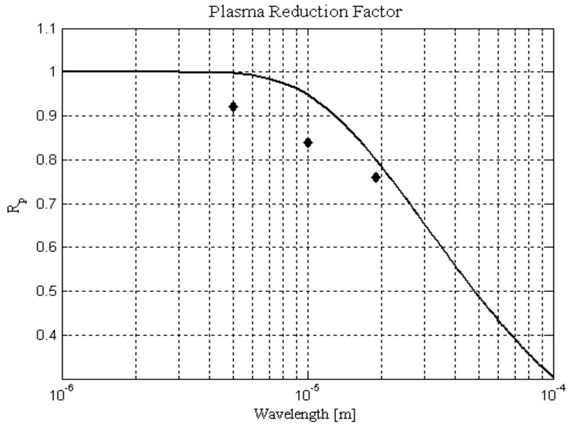

Figure 4: Plasma reduction factor dependence on wavelength: following Venturini [7] (solid curve) and from the minima of the current shot noise time (black dots).

$$r_p^{\,2} = 1 - \frac{kr_b}{\gamma} K_1\left(\frac{kr_b}{\gamma}\right) \qquad (9)$$

where $K_1(x)$ is the modified Bessel function and $k = 2\pi/\lambda$ is the optical wave number (Figure 4). By taking the ratio between the 1-D quarter plasma oscillation time and the minimum current-noise amplitude time (when the minimum is attained) for three different wavelengths, we calculated the plasma reduction factor for these wavelengths (Figure 4). The calculated points fall quite close to the theoretical curve and confirm its down-fall trend.

CONCLUSIONS

We have shown by 3-D numerical simulations that it is possible to control and decrease the optical frequency current shot noise in a drift section by setting proper beam parameters, as was predicted by a 1-D extended analytical model. In a case of a uniform drift section a drift length of quarter plasma oscillation $L_d = \pi/2\theta_{pr}$ produces the minimal current shot noise. Plasma reduction factor effects are investigated, and the results are in good agreement with the theoretical prediction – longer drift length for longer wavelengths. These provide a strong evidence for the validity of the single mode analytical model for current shot noise reduction in the appropriate beam parameter regime. This is also a necessary condition for spontaneous emission control and radiation noise reduction in FEL. Hence it indicates that satisfying conditions for realizing coherence enhancement of FELs below the shot-noise limit level [2] can be feasible at optical frequencies.

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