TEMPORAL AND CONVECTIVE ANALYSIS OF FREE-ELECTRON LASER IN HELICAL WIGGLER AND GUIDE MAGNETIC FIELDS

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Abstract

The full dispersion relation obtained for free-electron laser using helical wiggler circularly polarized magnetic field and an axial guide magnetic field using particle trajectories, their interaction with electric field by method of characteristics making the treatments quite general is reduced to Raman regime approximations in case of tenuous electron beam. The temporal and convective growth rates have been compared between full dispersion relation and Raman dispersion relation in microwave region. Results show the maximum of temporal growth in full dispersion relation and Raman dispersion relation is at the same locations. Whereas the maximum of convective growth rate in full dispersion relation is slightly deviated with respect to the Raman dispersion relation. The growth rates in Raman regime in both the cases are enhanced with respect to full dispersion relation for the same plasma frequency and cavity parameters.

Key words: free-electron laser, Raman regime, temporal growth rate, spatial growth rate, dispersion relation

INTRODUCTION

Sources of tunable coherent high power electromagnetic radiation are highly desirable so far as its applications are concerned. The applications are far more ranging from medical, biological, industrial, chemical, and physical including military. The radiation properties of relativistic electron beam having nonlinear wave particle interaction give rise to the birth of a new class of powerful laser called free-electron laser. In free-electron laser, a pump field is scattered from a relativistic electron beam and is of great interest as a potential high power, tunable source of coherent radiation, ranging from microwave to ultraviolet spectral regions. The concept involves the stimulated back scattered of a pump wave from a relativistic electron beam. The pump wave may be either an electromagnetic wave or a static periodic electric or magnetic field. When the electron beam is sufficiently intense, collective (Raman) effects become important. The axial guide field not only enhances focussing of intense electron beam but also increases the gain and efficiency of interaction due to enhanced transverse wiggler velocity. Free-electron lasers have been widely studied using hydrodynamic approach and computer simulation techniques with and without guide magnetic field [1-8]. Roberson and Sprangle [9] reviewed the theory of free-electron laser in Compton and Raman regimes in the presence of helical wiggler with and without guide magnetic field using hydrodynamic and vector potential approach. Davies et al. [10-11] have presented the kinetic description of Compton and Raman free-electron lasers stability properties for a cold and warm electron beam propagating through helical magnetic wiggler without guide magnetic field using vector potential approach. Kwan and Cary [12] have simulated computationally the absolute and convective instabilities of free electron lasers using a two and one half dimensional fully electromagnetic relativistic particle code. D. Li [13] has investigated the effect of dissipative loss in the grating surface on the Smith-Purcell free-electron laser with the help of a two-dimensional particle-in-cell Simulation. It has been shown that such a device can oscillate on both the convective and absolute instability when ignoring the surface-loss.

Therefore, in the present article, kinetic analysis of free-electron laser is studied by the method of characteristics using details of particle trajectories having variations in phase space deviating from vector potential approach. In this approach instead of vector potential, all the components of electric fields are taken and complete dielectric tensor is obtained [14]. The resulting full dispersion relation is further reduced to Raman dispersion relation. Expressions for temporal and convective growth rates are analyzed for both full and Raman dispersion relations. Consecutively results and discussions are presented.

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FULL DISPERSION RELATION

The externally imposed static magnetic field is taken to be:

$$B = \mathrm{B}_{\parallel}\,\hat{z} + \mathrm{B}_w\,(\hat{x}\cos \mathrm{k}_0 z + \hat{y}\sin \mathrm{k}_0 z) \tag{1}$$

Where $B_{||}$ and B_w are constants that are the measure of strength of homogeneous and rippled magnetic field, respectively and k_0 is the wiggler wave number. The unperturbed trajectories have been obtained from relativistic equation of motion in the said field. The Vlasov-Maxwell field equations [15] by the method of characteristics gives the perturbed distribution function for any arbitrary equilibrium distribution function. Following standard method [15], the perturbed distribution function, current density, conductivity and then dielectric tensors are obtained. These dielectric tensors are further solved by using the equilibrium distribution function for tenuous electron beam given as:

$$f_j^0 = \frac{N_b}{2\pi P_\perp}\delta(P_\perp - \gamma_0 m_j v_{\perp 0})\,\delta(P_z - \gamma_0 m_j v_{z0}) \qquad (2)$$

Where $P_\perp$ is the canonical momentum transverse to the beam propagation direction and P_z is the mechanical momentum in the z-direction, N_b is the beam density and γ_0 and $v_{\perp 0}$ and v_{z0} are constants.
Giving rise to the following full dispersion relation:

$$\left[\omega^2 - k_z^2c^2 - \frac{\omega_p^2}{\gamma_0}\frac{(\omega - k_z v_{z0})^2}{\{(\omega - k_z v_{z0})^2 - \Omega^2\}}\right]$$
$$\times\left[\omega^2 - (k_z + 2k_0)^2c^2 - \frac{\omega_p^2}{\gamma_0}\frac{(\omega - k_z v_{z0})^2}{\{(\omega - k_z v_{z0})^2 - \Omega^2\}}\right]$$
$$\times\left[\{\omega^2 - (k_z + k_0)v_{z0}\}^2 - \frac{\omega_p^2}{\gamma_0\gamma_z^2}\right]$$
$$= -\frac{\hat{\omega}_c^2\,\omega_p^2}{\gamma_0(1-\hat{\Omega})^2}\left[\omega^2 - (k_z + k_0)^2c^2 - \frac{\omega_p^2}{\gamma_0}\right]$$
$$\times\left[\begin{matrix}\omega^2 - (k_z + k_0)^2c^2 - k_0^2c^2 \\ -\dfrac{\omega_p^2}{\gamma_0}\dfrac{(\omega - k_z v_{z0})^2}{\{(\omega - k_z v_{z0})^2 - \Omega^2\}}\end{matrix}\right] \qquad (3)$$

Where

$$\hat{\omega}_c = \frac{eB_w}{\gamma_0 mc^2 k_0},\quad \hat{\Omega} = \frac{\Omega}{k_0\, v_{z0}},\ \Omega = \frac{eB_{II}}{\gamma_0 mc} \qquad (4)$$
and $\gamma_z = (1 - v_z^2/c^2)^{-1/2}$

Now when guide field is put to zero, our Expression (3) for full dispersion relation (FDR) reduces exactly to that of Davies et al. [10] and Davidson [15]. When the beam density is sufficiently high, the longitudinal electric field E_z plays an important role in determining detailed properties of free-electron laser instability. In this case, a more accurate analysis of the full dispersion relation (3) is required which includes the effects of the longitudinal plasma oscillations occurring in the dielectric factor $[\{\omega - (k_z + k_0)v_{z0}\}^2 - \omega_p^2/\gamma_0\gamma_z^2]$. One approximates the full dispersion relation in the vicinity of growth rate maximum as:

$$\left[\omega - (k_z + k_0)v_{z0} + \frac{\omega_p}{\gamma_0^{1/2}\gamma_z}\right]\left[\omega - \left\{k_z^2c^2 + \frac{\omega_p^2}{\gamma_0}\frac{1}{(1-\hat{\Omega}^2)}\right\}^{1/2}\right]$$

$$= -\frac{\hat{\omega}_c^2\gamma_z}{4\gamma_0^{1/2}}\frac{\omega_p}{(1-\hat{\Omega})^2}\frac{k_0 c[(2k_z c + k_0 c) + \hat{\Omega}^2\omega_p^2/(1-\hat{\Omega}^2)]}{\left[k_z^2c^2 + \frac{\omega_p^2}{\gamma_0}\frac{1}{(1-\hat{\Omega}^2)}\right]^{1/2}}$$
$$= -R \qquad (5)$$

Equation (5) is often referred as the Raman-regime dispersion relation.
The solution of the dispersion relation (5) gives:

$$\omega = (k_z + k_0)v_{z0} - \omega_p/\gamma_0^{1/2}\gamma_z + \frac{\mu}{2} \pm \frac{i\{4R\quad \mu^2\}^{1/2}}{2} \qquad (6)$$

where

$$\mu = \left[k_z^2c^2 + \frac{\omega_p^2}{\gamma_0}\frac{1}{(1-\hat{\Omega}^2)}\right]^{\frac{1}{2}} - (k_z + k_0)v_{z0} + \frac{\omega_p}{\gamma_0^{1/2}\gamma_z} \qquad (7)$$

And μ is the frequency mismatch between the electrostatic and electromagnetic waves.
Equation (6) indicates that the electromagnetic wave is unstable and the width of the unstable spectrum is determined by the various parameters such as the beam energy and the strength of the wiggler field and the axial guide magnetic field.

Temporal Growth Rate of Free-Electron Laser

The dimensionless temporal growth rate is given by[15]:

$$\frac{\mathrm{Im}\,\omega}{ck_0} = \frac{i}{2ck_0}[4R - \quad \mu^2]^{1/2} \qquad (8)$$

$$\text{and } \mathrm{Re}\,\omega = (k_z + k_0)v_{z0} - \omega_p/\gamma_0^{1/2}\gamma_z + \frac{\mu}{2} \qquad (9)$$

From Equation (5), it is seen that R is proportional to $\hat{\omega}_c^2$; therefore, the width of unstable spectrum is linearly dependent on the strength of the helical wiggler magnetic field. When the axial guide magnetic field is withdrawn from the Equation (8), the expression reduces to that of Davies et al. [10] and Davidson [15].

Spatial Growth Rate of Free-Electron Laser

Dimensionless spatial growth rate is given as:

$$k_i/k_0 = -\frac{\mathrm{Im}\,\omega/ck_0}{1/c\,\frac{\partial\omega}{\partial k_z}} \qquad (10)$$

Expression for absolute instability in Raman regime obtained in Equation (8) and group velocity is determined from Equation (9) under approximation $\omega \cong ck_z$ gives the spatial growth rate with $\hat{\omega} = \omega/ck_0$ as:

$$\frac{k_i}{k_0}=\frac{\frac{1}{2}\left\{\frac{\hat{\omega}_c^2\hat{\omega}_p^2\gamma_z}{(1-\hat{\Omega})^2}\frac{\left[(1+2\hat{\omega})+\frac{\hat{\Omega}^2\hat{\omega}_p^2}{(1-\hat{\Omega})^2}\right]}{\left[\hat{\omega}^2+\hat{\omega}_p^2/(1-\hat{\Omega}^2)\right]^{1/2}}-\left[\left\{\hat{\omega}^2+\frac{\hat{\omega}_p^2}{(1-\hat{\Omega}^2)}\right\}^{1/2}-\frac{v_{z0}}{c}(1+\hat{\omega})+\frac{\hat{\omega}_p^2}{\gamma_z}\right]^2\right\}^{1/2}}{\left[\frac{\omega}{W}+\frac{v_{z0}}{c}Q\right]/[1-Q]} \quad (11)$$

Where $W=\left[\omega^2+\frac{\omega_p^2}{\gamma_0}\frac{\omega^2(1-v_{z0}/c)^2}{\{\omega^2(1-v_{z0}/c)^2-\Omega^2\}}\right]^{1/2}$

and $Q=\frac{\omega_p^2}{\gamma_0}\cdot\frac{\omega}{W}\frac{\Omega^2(1-v_{z0}/c)}{\{\omega^2(1-v_{z0}/c)^2-\Omega^2\}^2}$ (12)

RESULTS AND DISCUSSIONS

Equations (3), (8) and (11) are solved numerically by computer in order to obtain amplification/gain for complex ω and real k (Temporal growth rate) as well as for complex k and Real ω (Convective growth rate), respectively. The numerical calculations have been performed for both the growth rates for full dispersion relation and Raman dispersion relation for the various plasma input parameters shown in captions accompanied with discussions.

Following plasma parameters are considered:

For temporal growth rate:

$\hat{\omega}_p = 0.0707e-1, \hat{\omega}_c = 0.5(B_w = 5.29kG)$,

$\hat{\Omega} = 7.4(B_{II} = 58.7kG), v_{z0}/c = 0.75$,

$\gamma_0 = 2, \gamma_z = 1.41$

For convective (spatial) growth rate:

$\hat{\omega}_p = 0.0707e-1, \hat{\omega}_c = 0.5(B_w = 5.29kG)$,

$\hat{\Omega} = 9.15(B_{II} = 72.6kG), v_{z0}/c = 0.75$,

$\gamma_0 = 2, \gamma_z = 1.41$

The results are shown in Figures 1 and 2. The spatial growth rate is more than the temporal growth rate. The increase in the magnetic field increases the growth rates but produces the shift in maxima in case of full dispersion relation. The fact has been seen computationally and is in agreement with the results reported earlier 14]. The maxima in case of temporal instability depicted in figure 1 are at the same value of $k_z/k_0=3$ but magnitude is little less for full dispersion relation than the Raman dispersion relation. This may be due to the deviations produced in Doppler shifted frequency in the presence of guide magnetic field affecting the resonance condition. However, the guide field is enhancing the focusing in either case of temporal and spatial growth rates shown in figures 1 and 2, respectively. The maxima for full dispersion relation in case of convective growth rate are appearing at $k_z/k_0 \approx 3.96$ whereas in case of Raman regime, it is at $k_z/k_0 \approx 4$. Further, spatial growth rate in case of full dispersion relation is reduced by eight times than the Raman dispersion relation. This may be due to reduction in group velocity as well as Doppler shifted frequency in the presence of guide magnetic field.

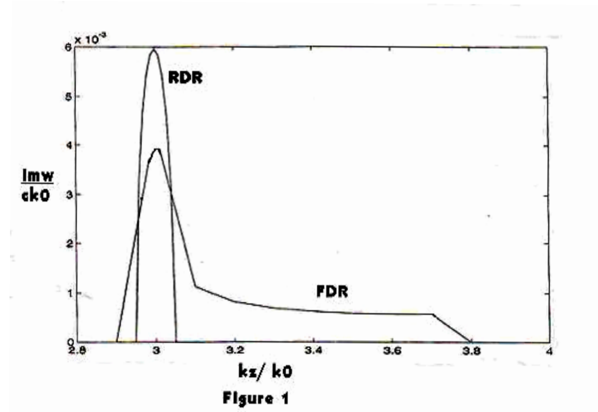

Figure 1: Dependence of dimensionless temporal growth rate on dimensionless wave number for full dispersion relation and Raman dispersion relation $(\hat{\omega}_p = 0.0707e-1, \hat{\omega}_c = 0.5, \hat{\Omega} = 7.4, v_{z0}/c = 0.75, \gamma_0 = 2)$.

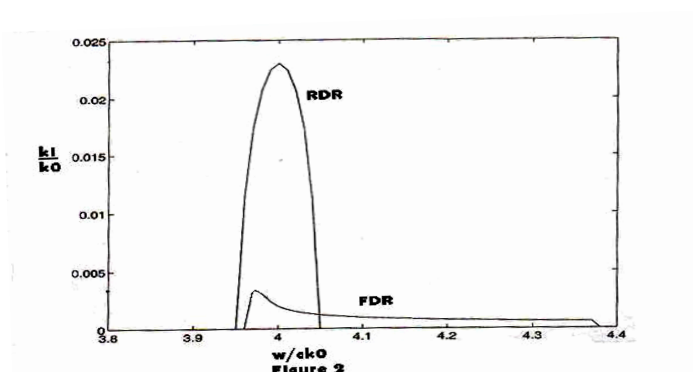

Figure 2: Dependence of dimensionless spatial growth rate on dimensionless wave number for full dispersion relation and Raman dispersion relation $(\hat{\omega}_p = 0.0707e-1, \hat{\omega}_c = 0.5, \hat{\Omega} = 9.154, v_{z0}/c = 0.75, \gamma_0 = 2)$.

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