# VARIABLE POLARIZED HARMONIC-UNDULATOR FREE ELECTRON LASER AND EFFECT OF BEAM ENERGY SPREAD

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*Abstract*

In the recent past Variable Polarized Harmonic Undulators are proposed for obtain radiation at all polarization. The scheme has been further modified to variable polarized harmonic undulator scheme. The scheme consists of two identical linear polarized magnets with high permeable shim in the gap of the undulator magnets. In this paper we include the important effects of beam energy spread and the variable polarized harmonic undulator radiation and free electron laser gain. We have considered both single and two-peak gaussian beam energy distribution to calculate intensity and gain reduction. We show that the variable polarized harmonic scheme compensates the undesired effects of the beam energy spread in comparison to the standard variable polarized undulator scheme.

## INTRODUCTION

In recent years many new and alternate undulator configuration have been proposed for better operation. These includes a harmonic undulator[1], variable polarized undulator[2-3], optical klystron undulator are some examples, which have attracted wide interest in this context. In this paper we have introduced harmonic field in the variable polarized undulator configuration with the inclusion of beam energy spread[4]. We have studied two-peak electron beam energy spread to analyze the spectrum broadening and gain reduction in the variable polarized harmonic undulator free electron lasers.

## UNDULATOR RADIATION AND ENERGY SPREAD

We assume the electron moves on-axis and its field is given by,

$$B_x = -B_0 a_1 \sin(\delta)\cos(k_u z + \delta)$$
$$B_y = B_0 a_1 \cos(\delta)\sin(k_u z + \delta) + B_0 a_2 \sin(h k_u z) \qquad (1)$$

which reduce to the linearly polarized case when $\delta = 0, h = 0$ and helically polarized case for $\delta = \pi/4, h = 0$. Here $k_u = 2\pi/\lambda_u$, where $\lambda_u$ is the undulator wavelength, $B_0$ is peak field strength, $h$ is an integer multiple and $\delta$ is the phase parameter. The velocity and trajectory can be evaluated by using the Lorentz force equation. This gives

$$\beta_x = -\frac{K_1}{\gamma}\cos(\delta)\cos(\omega_u t + \delta) - \frac{K_h}{\gamma}\cos(h\omega_u t)$$
$$\beta_y = \frac{K_1}{\gamma}\sin(\delta)\sin(\omega_u t + \delta)$$
$$\beta_z = \beta^* - \frac{K_1^2}{4\gamma^2}\cos(2\delta)\cos(2(\omega_u t + \delta)) - \frac{K_h^2}{4\gamma^2}\cos(2h\omega_u t)$$
$$-\frac{K_1 K_h}{2\gamma^2}\cos(\delta)\cos(\omega_u(1+h)t + \delta) - \frac{K_1 K_h}{2\gamma^2}\cos(\delta)\cos(\omega_u(1-h)t + \delta)$$

and

$$x(t) = -\frac{K_1 c}{\gamma\omega_u}\cos(\delta)\sin(\omega_u t + \delta) - \frac{K_h c}{\gamma h \omega_u}\sin(h\omega_u t)$$
$$y(t) = -\frac{K_1 c}{\gamma\omega_u}\sin(\delta)\cos(\omega_u t + \delta)$$
$$z(t) = \beta^* ct - \frac{K_1^2 c}{8\gamma^2\omega_u}\cos(2\delta)\sin(2(\omega_u t + \delta)) - \frac{K_h^2 c}{8\gamma^2 h\omega_u}\sin(2h\omega_u t)$$
$$-\frac{K_1 K_h c}{2\gamma^2\omega_u(1+h)}\cos(\delta)\sin(\omega_u(1+h)t + \delta) - \frac{K_1 K_h c}{2\gamma^2\omega_u(1-h)}\cos(\delta)\sin(\omega_u(1-h)t + \delta) \qquad (2)$$

where, $\beta^* = 1 - \frac{1}{2\gamma^2}\left[1 + \frac{K_1^2}{2} + \frac{K_h^2}{2}\right]$

and $K_1 = \frac{eB_0 a_1}{m_0 c\omega_u}$, $K_h = K_1\Delta$, $\Delta = (a_2/ha_1)$, $\omega_u = k_u c$, $K_1$ & $K_h$ defines the undulator parameter of the respective fields.

We recall that the energy radiated per unit solid angle and frequency interval, i.e., brightness[5] is given by

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2\omega^2}{4\pi^2 c}\left|\int_0^T \{\hat{n}\times(\hat{n}\times\vec{\beta})\}\exp\left[i\omega\left(t - \frac{z}{c}\right)\right]dt\right|^2 \qquad (3)$$

where the integration is carried over the undulator length, $T = \frac{2N\pi}{\omega_u}$ and $\omega$ is the emission frequency of the source. The oscillating part in above eq. can be written as,

$$\exp\left[i\omega\left(t - \frac{z}{c}\right)\right] = \sum_{m,n,p,q=-\infty}^{\infty}\exp[i\nu t] J_m(0,\xi_1) J_n(0,\xi_2) J_p(\xi_3,0) J_q(\xi_4,0)$$

with

$$\nu = \frac{\omega}{2\gamma^2}\left(1 + \frac{K_1^2}{2} + \frac{K_h^2}{2}\right) - \eta \qquad (4)$$

where,

$$\eta = (m\omega_u + nh\omega_u + p\omega_u(1+h) + q\omega_u(1-h))$$

$$\xi_1 = -\frac{\omega K_1^2}{8\gamma^2\omega_u}\cos(2\delta), \quad \xi_2 = -\frac{\omega K_h^2}{8\gamma^2 h\omega_u},$$
$$\xi_3 = -\frac{\omega K_1 K_h}{2\gamma^2\omega_u(1+h)}, \quad \xi_4 = -\frac{\omega K_1 K_h}{2\gamma^2\omega_u(1-h)}$$

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The brightness expression reduced to

$$\frac{d^2I}{d\omega\omega\Omega} = \frac{e^2\omega^2T^2}{4\pi^2c}\left\{\left|T_x^2\right| + \left|T_y^2\right|\right\}s(\nu)$$

where, $s(\nu) = \left((\sin(\nu t/2))/\nu\, t/2\right)^2$ and the intensity coefficients $T_x$ and $T_y$ are given by,

$$T_x = \left[\frac{K_1}{2\gamma}\cos(\delta)\{J_{m+1}(0,\xi_1) + J_{m-1}(0,\xi_1)\}J_n(0,\xi_2)J_p(\xi_3,0)J_q(\xi_4,0)\right.$$
$$\left. + \frac{K_h}{2\gamma}\{J_{n+1}(0,\xi_2) + J_{n-1}(0,\xi_2)\}J_m(0,\xi_1)J_p(\xi_3,0)J_q(\xi_4,0)\right]$$

$$T_y = -\frac{K_1}{2i\gamma}\sin(\delta)\{J_{m+1}(0,\xi_1) - J_{m-1}(0,\xi_1)\}J_n(0,\xi_2)J_p(\xi_3,0)J_q(\xi_4,0)$$

The resonance condition in a free electron laser is provided by $\nu = 0$. This provides the central emission frequency as,

$$\omega_{m,n,p,q} = \frac{2\gamma^2\eta}{1 + \left(K_1^{\,2}/2\right) + \left(K_h^{\,2}/2\right)} \quad (5)$$

The spectral line shape distribution induced by the beam energy having an energy distribution $f(\varepsilon)$ can be written as,

$$s(\nu) = \int_{-\infty}^{\infty} d\varepsilon\left[\frac{\sin((\nu + \partial\nu_\varepsilon)/2)}{((\nu + \partial\nu_\varepsilon)/2)}\right]^2 f(\varepsilon) \quad (6)$$

where $\partial\nu_\varepsilon = 4\pi N\varepsilon$ and defines the shift induced by the energy spread. For a Gaussian type energy distribution we write two-peak electron energy beam as,

$$f(\varepsilon) = \frac{1}{\sqrt{2\pi}}\sum_i \frac{\alpha_i}{\sigma_\varepsilon^i}\exp\left(\frac{-(\varepsilon-\varepsilon_i)^2}{2\sigma_\varepsilon^{i\,2}}\right) \qquad i = 1,2 \quad (7)$$

$\alpha_1 + \alpha_2 = 1.0$

In Eq(7) $\sigma$ is the r.m.s relative energy spread and $\varepsilon = \delta\gamma/\gamma$ , $\delta\gamma = \gamma - \gamma_0$ , $\gamma_0$ being the nominal energy of the electron beam. Therefore Eq (6) can be transformed in to

$$s(\nu) = \alpha_1\int_{-\infty}^{\infty} d\varepsilon\left[\operatorname{sinc}((\nu + \partial\nu_\varepsilon)/2)\right]^2 \frac{1}{\sqrt{2\pi}\sigma_\varepsilon^1}\exp\left(\frac{-(\varepsilon-\varepsilon_1)^2}{2\sigma_\varepsilon^{1^2}}\right) + \quad (8)$$
$$\alpha_2\int_{-\infty}^{\infty} d\varepsilon\left[\operatorname{sinc}((\nu + \partial\nu_\varepsilon)/2)\right]^2 \frac{1}{\sqrt{2\pi}\sigma_\varepsilon^2}\exp\left(\frac{-(\varepsilon-\varepsilon_2)^2}{2\sigma_\varepsilon^{2^2}}\right)$$

Using Eq (8) is solved to get the line shape function for the fundamental and higher harmonics for such two peak beam given as,

$$s(\nu) = 2\alpha_1\int_0^1 dt(1-t)\cos(\nu t + 4\pi N\varepsilon_1 t)\exp\left(\frac{-m^2\mu_1^{\,2}\pi^2t^2}{2}\right) +$$

$$2\alpha_2\int_0^1 dt\,(1-t)\cos(\nu t + 4\pi N\varepsilon_2 t)\exp\left(\frac{-m^2\mu_2^{\,2}\pi^2t^2}{2}\right)$$

where

$\mu_1 = 4N\sigma_\varepsilon^1$ and $\mu_2 = 4N\sigma_\varepsilon^2$

## FREE ELECTRON LASER GAIN AND ENERGY SPREAD

To find small signal gain of harmonic undulator free electron laser under radiation field given by,

$$E = E_0\cos(\psi)\hat{x} + E_0\sin(\psi)\hat{y} \quad (9)$$

where $\psi = n_1k_1z - n_1\omega_1t + \phi$, $n_1$ is emissions harmonic number at which resonance occur. The small signal gain of the harmonic undulator free electron laser is derived by V. Gupta et al[6],

$$G = -\frac{j}{\nu^3}\left[2 - 2\cos(\nu) - \nu\sin(\nu)\right] \quad (10)$$

where $j$ is the dimensionless current density given as,

$$j = \Big[\frac{4\pi^2Ne^2K_1^{\,2}L_u^{\,2}n_e\eta}{m_0c^2\gamma^3}[\cos(\delta)+\sin(\delta)]^2\left\{(J_{m+1}(0,\xi_1)+J_{m-1}(0,\xi_1))^2\right\}$$
$$J_n^{\,2}(0,\xi_2)J_p^{\,2}(\xi_3,0)J_q^{\,2}(\xi_4,0)$$
$$+\frac{4\pi^2Ne^2K_h^{\,2}L_u^{\,2}n_e\eta}{m_0c^2\gamma^3}\left\{(J_{n+1}(0,\xi_2)+J_{n-1}(0,\xi_2))^2\right\}$$
$$J_m^{\,2}(0,\xi_1)J_p^{\,2}(\xi_3,0)J_q^{\,2}(\xi_4,0)\ \Big] \quad (11)$$

We write the gain as the derivative of the spontaneous emission line shape to get,

$$G = j\frac{\partial}{\partial\nu}\left[\operatorname{sin}c(\nu\ /2)\right]^2 \quad (12)$$

If the electron beam with two peak energy spread, then gain is

$$G = 2j\Big[\alpha_1\int_0^1 dt\,(1-t)t\sin(\nu t + 4N\pi\varepsilon_1 t)\exp\left(\frac{-m^2\mu_1^{\,2}\pi^2t^2}{2}\right) +$$
$$\alpha_2\int_0^1 dt\,(1-t)t\sin(\nu t + 4N\pi\varepsilon_2 t)\exp\left(\frac{-m^2\mu_2^{\,2}\pi^2t^2}{2}\right)\Big] \quad (13)$$

## RESULTS & DISCUSSION

In this paper we have considered the influence of beam energy spread on the Variable Polarized harmonic undulator radiation. To compare the results with Dattoli et al. we plot the intensity versus $\delta$ without including the effects of the harmonic field component and energy spread. The results agree as in Fig.1. Fig.2 illustrates the case of undulator radiation with beam energy spread for $\delta = 0, \pi/4, \pi/2$ respectively. Fig.2 reflects the intensity reduction for the fundamental and higher harmonics. Fig. 3 & 4 plots intensity versus frequency for various values of $\delta$ with harmonic field for h=3,5 respectively. For two-peak beam energy distribution, the reduction of the intensity spectrum broadening is displayed in Fig.5 & 6.

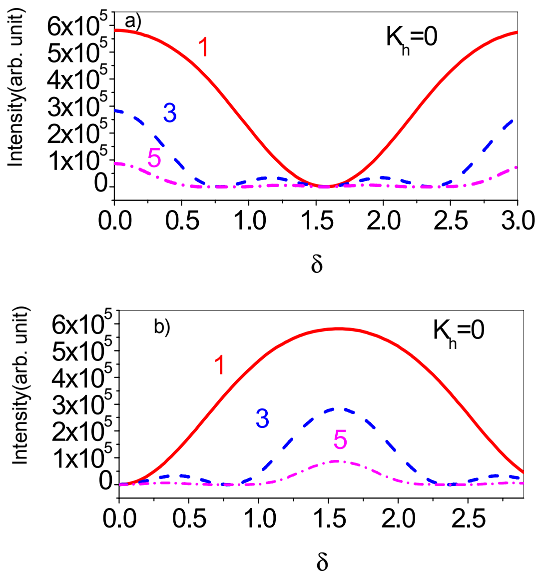


Figure 1: Horizontal and vertical component of the spectral intensity for first, third and fifth harmonic versus $\delta$ where $\delta$ is the phase parameter of VPHU.

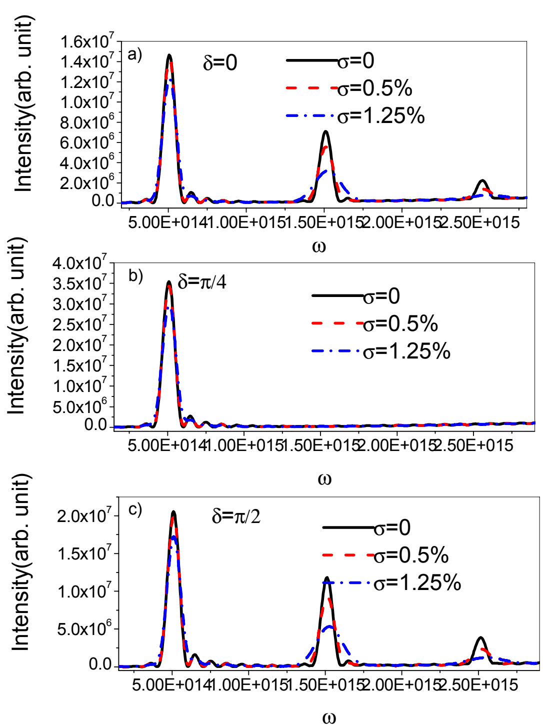


Figure 2: Intensity versus radiation frequency with one-peak beam energy distribution function for various values of $\delta$ without harmonic field.

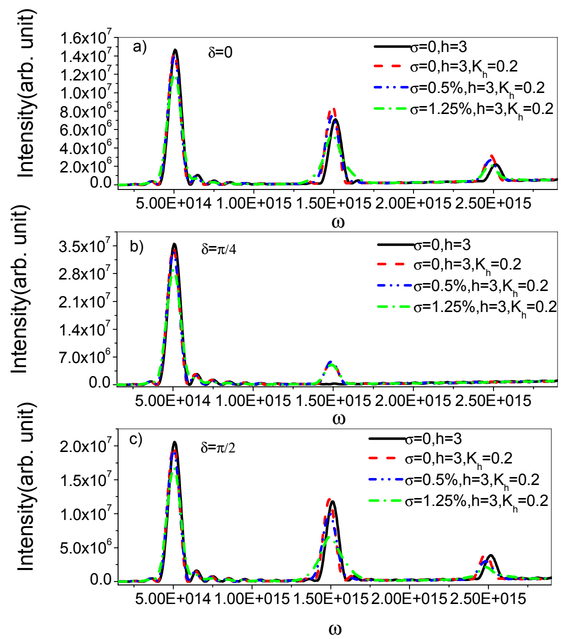


Figure 3: Intensity versus radiation frequency with one-peak beam energy distribution function for various values of $\delta$ with harmonic field.

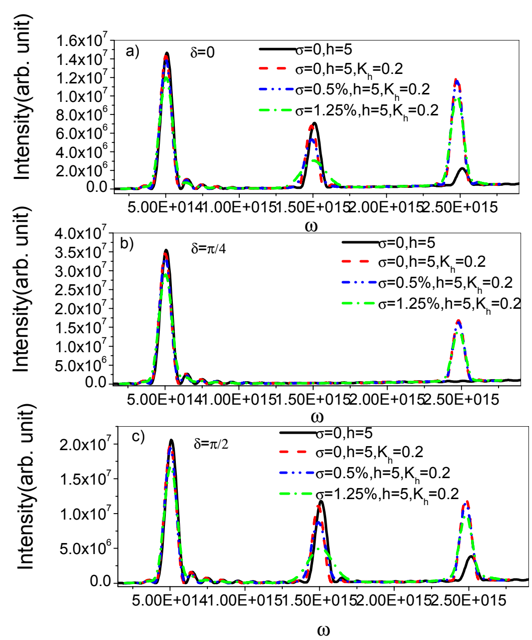


Figure 4: Intensity versus radiation frequency with one-peak beam energy distribution function for various values of $\delta$ with harmonic field.

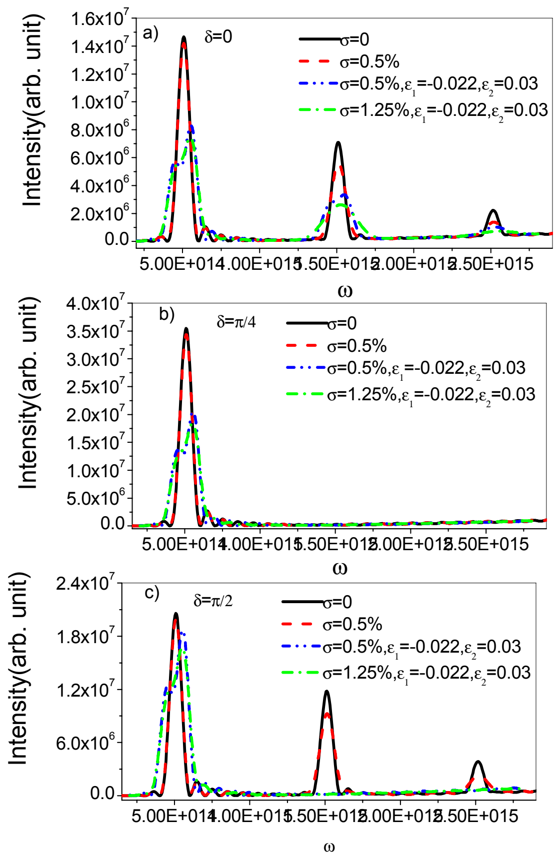


Figure 5: Intensity versus radiation frequency with two-peak beam energy distribution function for various values of $\delta$ without harmonic field.

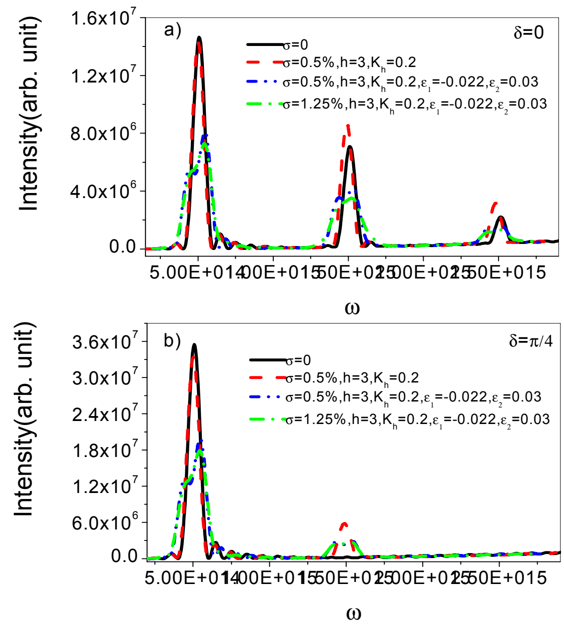


Figure 6: Intensity versus radiation frequency with two-peak beam energy distribution function for various values of $\delta$ with harmonic field.

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