NUMERICAL EVALUATION OF BULK HTSC STAGGERED ARRAY UNDULATOR BY BEAN MODEL

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Abstract

In order to achieve short period undulator, we have studied the new type of undulator with bulk high critical temperature superconductor magnets in a staggered array configuration. We have developed the numerical calculation method for magnetic field in bulk high critical temperature superconductor staggered array undulator with the loop current model based on Bean model of type II superconductor [1]. The calculations well reproduced the experimental result at liquid nitrogen temperature. We have made the performance estimation at the lower temperature, and conclude that the undulator have a feasibility of a short period undulator.

INTRODUCTION

Short period undulators bring in several advantages, i.e. short lengths FEL with low electron beam energy, and high gain with same undulator length. To achieve short period undulator with the same K value, we have to generate strong undulator field. There are two main way to obtain strong undulator field. One is the undulator with low temperature superconductor wires. Another is the In-vacuum undulator. However, the superconducting wires have to be cooled down near liquid helium temperature (4.2 K) with potentially large thermal load from the electron beam or radiation. The permanent magnets have been used for a long time, thus, the drastic improvement cannot be expected.

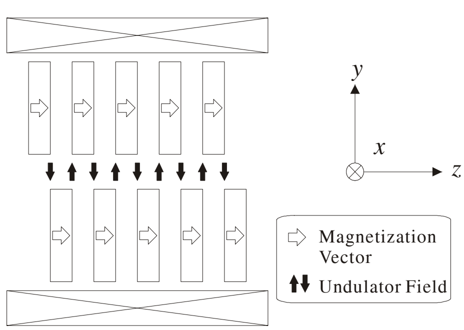

Figure 1: Schematic of new type undulator. White arrows indicate magnetization vectors of HTSC magnets. All magnetization vectors parallel to the direction of +*z*. Black arrows indicate undulator fields.

Bulk HTSC (high critical temperature superconductor) magnets are promising. Recent research progress reported that a YBaCuO bulk (26.5 mm dia.) traped the magnetic field of 17 Tesla at 29 K [2]. Moreover bulk HTSC magnets can be used at a temperature much above liquid He temperature. Therefore, present compact refrigerator systems make it possible for the bulk HTSC magnets to be used near the beam line. However, to use bulk HTSC magnets, we have to magnetize them below T_c (critical temperature). Magnetization method is main issue for using bulk HTSC magnets [3, 4].

We proposed staggered structure to magnetize bulk HTSC magnets [5]. The conventional Staggered Array Undulator (SAU) consists of ferromagnetic and non-magnetic pieces [6]. Bulk HTSC SAU consists of bulk HTSC magnets and non-magnetic pieces. A schematic of Bulk HTSC SAU is shown in Figure 1. It have bulk HTSC magnets with the magnetization direction to +*z*. The proof of principle experiment was performed in 3 period prototype at 77 K. The undulator field was successfully generated by bulk HTSC magnets which magnetized by an external solenoid, and the amplitude of undulator field was controlled by the solenoid current [7].

In this paper, we reported the loop current model based on Bean model, the comparison with the experiment aim for the validation of the model, and the estimated performance at lower temperature.

PRINCIPLE

The change of the solenoid field induce superconducting loop currents in the bulks which don't decay. The loop currents generate the undulator field. The change of the solenoid field ΔB_s is defined as

$$\Delta B_s = B_s^{end} - B_s^{start},$$

where B_s^{start} and B_s^{end} are the solenoid field just after the superconducting phase transition, and the solenoid field for the operation point of the undulator. Based on the Bean model, with a single bulk, the magnitude of the loop current is not determined by B_s^{end}, but ΔB_s. This suggests, that the *z*-component of the magnetic field in Bulk HTSC SAU can have the offsets independently with the undulator field. Specifically, if we choose B_s^{end}=0, the *z*-component from solenoid vanished.

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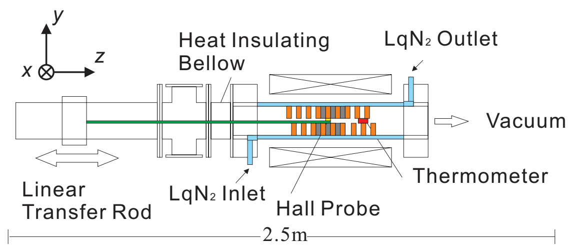

Figure 2: Experimental setup of 11 period prototype. The bulk HTSC magnets and the copper pieces (Fig. 3) are stacked in the vacuum chamber.

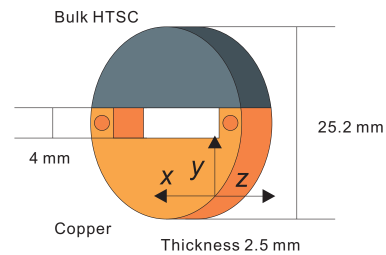

Figure 3: A bulk HTSC magnet and a copper piece (a half period).

EXPERIMENTAL SETUP

Figure 2 shows the experimental setup of 11 period prototype. The bulks were cooled down by the liquid N_2 flowing between the inner and the outer wall of vacuum chamber. The magnetic field in the undulator was measured by the hall probe on the linear transfer rod. The period λ_u=5 mm, the gap g=4 mm, the periodic number N=11, the radius of bulk R=12.6 mm, and the average critical current density J_c of the bulks is about 100 kA/mm^2.

Figure 3 shows a bulk HTSC magnet and a copper piece.

NUMERICAL MODEL

Figure 4 shows the numerical model of bulk. The bulk is represented by a series of concentric D-shape line loop currents. According to the Bean model of type II superconductors, the critical current density J_c defines the gradient of the magnetic field ($dB/dx=\mu_0 J_c$), where the critical current density J_c is field-independent. Therefore the critical current density J_c and the change of the external field ΔB_{ex} determine the depth of the current flowing region d_y.

From the Bean model, we assumed the following:

- The loop currents flow to compensate the z-component of the change of the external magnetic field ΔB_{ex} at the centre of the bulk.
- A loop has the current of strength of I_c or zero. The current I_c is constant, because the critical current density is field-independent.

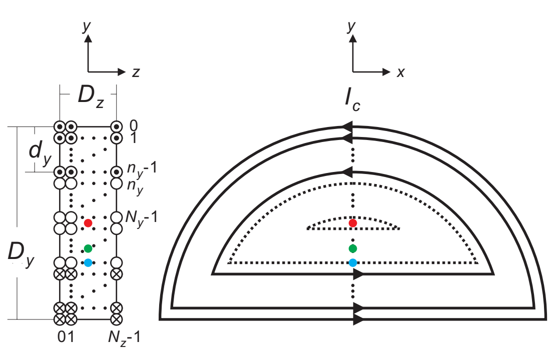

Figure 4: Numerical model of bulk HTSC magnet. The bulk represented by N_y times N_z line loop currents. The red point indicates the centre of bulk ($D_y/2$, $D_z/2$).

We define the penetration ratio of the magnetic field in bulk Λ_d as

$$\Lambda_d = \frac{d_y}{D_y/2}.$$

There are $2N$ bulks in N periods. To determine the depth of current flowing region of i^{th} bulk $d_{y,i}$, we have to consider the changes of the external field of the bulk i ($\Delta B_{ex,i}$). $\Delta B_{ex,i}$ contains the changes of the solenoid field ΔB_s and the magnetic field generated by bulk j ($j=1,2,\cdots,2N; j\neq i$) at the centre of i^{th} bulk ($B_j(\mathbf{r}_i)$). Therefore, these variables satisfy the relation

$$\alpha\mu_0 J_c d_{y,i} = -\Delta B_s - \sum_{j\neq i}^{2N} B_j\left(\mathbf{r}_j, d_{y,j}\right),$$

where α is the geometric factor derived from bulk shape and size. Using this equation, $d_{y,i}$ is calculated numerically with relaxation (iteration) method. On the other hand, if we use $d_{y,i}$ as parameter (like Λ_d=10%), this equation can be used to calculate the required changes of the solenoid field ΔB_s.

We assumed a field-independent J_c in the calculation. However, to determine the validity of J_c taken in the calculation, it is important to know the magnetic field applied to the bulks. We defined the magnetic field at the centre of bulk which located in the centre of the undulator as the restricting magnetic field B_{rs}.

Table 1 shows the parameters used for the calculations.

Table 1: Used Parameters for Calculations

Name	Definition	Values
λ_u	Period	10 mm, 3-15 mm
G	Gap	4 mm, 1-8 mm
N	Periodic number	11, 100

R	Radius of bulk	12.6 mm, 65 mm
D_y	Size of bulk (y)	R-g/2
D_z	Size of bulk (z)	$\lambda_u/2$
J_c	Critical current density	100, 3.5k, 10k A/mm^2
I_c	Current of a loop	$J_c D_y D_z/(2N_y N_z)$

RESULTS AND DISCUSSIONS

Comparisons with the experimental results

Figure 5 shows the result of the numerical simulation and the experiment. The parameters of simulation are period λ_u=5 mm, gap g=4 mm, periodic number N=11, the radius of bulk R=12.6 mm, critical current density J_c=100 kA/mm^2 (almost the same with the average J_c of the bulks used in experiment), and a change of the solenoid field ΔB_s =-13.7 mT. The field distribution of the magnetic field was well reproduced by the simulation without edge. To reproduce the peaks in edge magnetic field accurately, a more realistic model which includes the y-component of magnetization of the bulks and the field-dependency of J_c, is required.

From the comparisons with the experimental results, we found that our model well reproduced the experimental results in the region where the penetration ratio of the magnetic field in bulk Λ_d was less than 20%. Moreover, in this region, the place-to-place variations of the undulator fields cause by the piece-to-piece variations of the critical current densities of bulks were well suppressed [8]. Therefore, we took Λ_d=10% in the performance estimation.

Calculations at the lower temperature

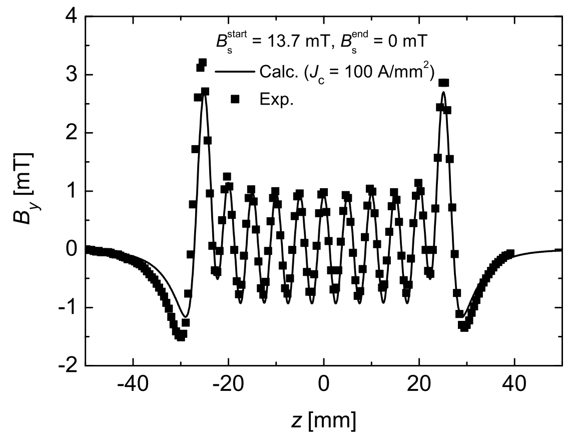

Figure 5: The comparison between the calculation and the experiment. The field distribution of the experiment was well reproduced by the calculation without the edge.

Figure 6 shows the calculation result. The period λ_u=10 mm, the gap g=4 mm, the periodic number N=100, the radius of bulk R=65 mm, the critical current density J_c=3.5 kA/mm^2. We took B_s^{start}=8 T and B_s^{end}=0 T for the penetration ratio of bulks near the center of the undulator to become Λ_d=10%. In this calculation, the restricting magnetic field B_{rs} was about 4 T. The critical current density (J_c) of 3.5 kA/mm^2 under the restricting magnetic field (B_{rs}) of 4 T can be achieved by cooling the bulks down to 20-30 K. The solid line and the dashed line indicate the y-component and z-component of the magnetic field. The undulator field B_0 is about 1 T. The z-component B_z is almost zero (B_z~0.1 T) near the centre. There are large B_z in the both edges of the undulator. Compensation methods to reduce the edge B_z is required.

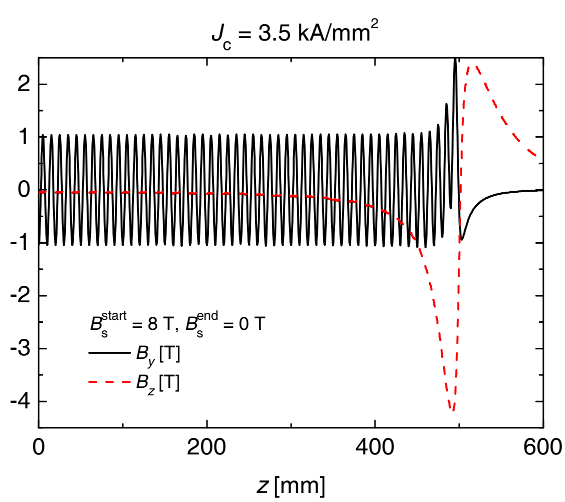

Figure 6: The distribution of the y and z-component of the magnetic field (B_y and B_z) along z axis in Bulk HTSC SAU.

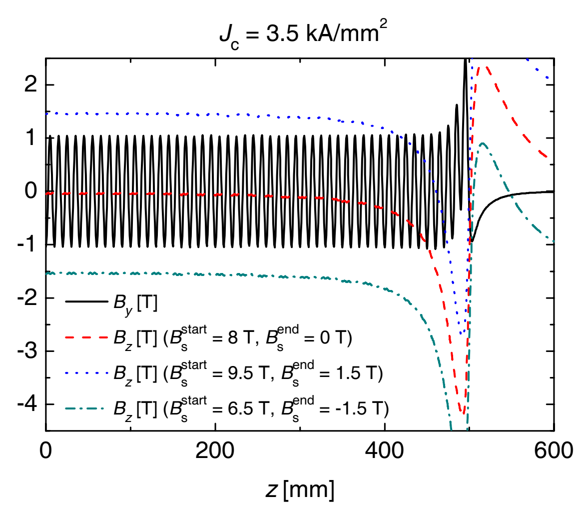

Figure 7: Basically the same with Fig. 8. The solenoid field at the end of the magnetization process was B_s^{end}=+1.5, 0, -1.5 T. B_z can changed without B_y change.

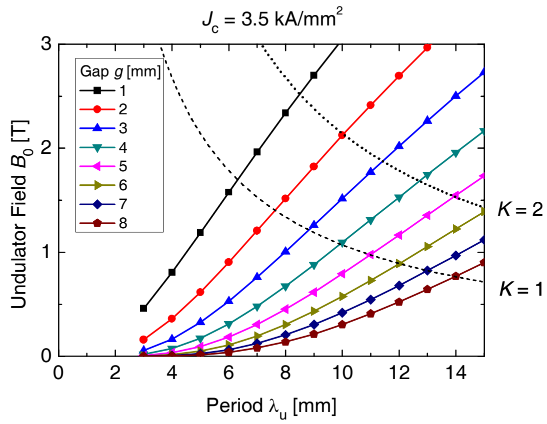

Figure 8: The undulator field dependence on the period and the gap of Bulk HTSC SAU. We assume the critical current density of 3.5 kA/mm^2.

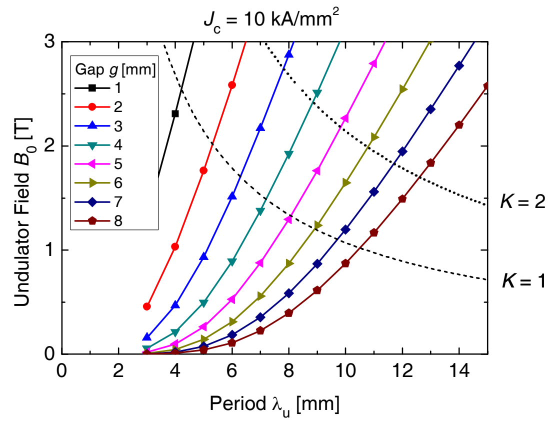

Figure 9: The same with Fig. 8. We assume the critical current density of 10 kA/mm^2.

Figure 7 shows the calculation result with the solenoid field at the end of the magnetization process B_s^{end}=+1.5, 0, -1.5 T. This result suggests, that by magnetization process, B_z can changed without B_y change. We can remove the offset of B_z which remains even after the compensation of the edge field.

Figure 8 shows the calculated performance of Bulk HTSC SAU with the critical current density J_c=3.5 kA/mm^2. The calculation conditions are the same with Fig. 6 without the period and the gap. The period λ_u=3-15 mm and the gap g=1-8 mm. The dashed black line and the dotted black line indicates the required undulator field to get K=1 and K=2, respectively. In this calculation, the restricting magnetic field B_{rs} was about 4 T (λ_u=10 mm, not depend largely on gap), and the required change of the solenoid field ΔB_s was about 8 T (λ_u=10 mm). The critical current density (J_c) of 3.5 kA/mm^2 under the restricting magnetic field (B_{rs}) of 4 T can also be achieved by cooling the bulks down to 20-30 K. This result suggests, that the condition K=1 can be achieved, for example, with λ_u=10 mm and g=4 mm, or λ_u=8 mm and g=2 mm.

Figure 9 shows the calculated performance of Bulk HTSC SAU with the critical current density J_c=10 kA/mm^2. The condition of calculations is the same with Fig. 8. In this calculation, the restricting magnetic field B_{rs} was about 10 T (λ_u=5 mm) or 13 T (λ_u=8 mm), and the required change of the solenoid field ΔB_s was about 20 T. The critical current density (J_c) of 10 kA/mm^2 under the restricting magnetic field (B_{rs}) of 10 T is not achievable even at the temperature of 4.2 K. However, J_c=15 kA/mm^2 under 3 T was achieved at the temperature of 4.2 K. The required change of the solenoid field of 20 T is the more difficult problem. This is equivalent to a demand of a superconducting solenoid of the strength of at least 10 T.

CONCLUSION

We performed the magnetic field calculation in Bulk HTSC SAU with the loop current model based on Bean model. As a result, we got the undulator field, though the compensation method to reduce the z-component of the magnetic field in the Bulk HTSC SAU will be required. The performance estimation at the critical current density of 3.5 kA/mm^2 suggests Bulk HTSC SAU can achieve K=1, for example, at λ_u=10 mm and g=4 mm, or λ_u=8 mm and g=2 mm at the temperature of 20-30 K. The results with the more challenging critical current density of 10 kA/mm^2 suggests Bulk HTSC SAU can achieve K=1, for example, at λ_u=8 mm and g=5 mm, or λ_u=5 mm and g=1.5 mm at the temperature of 4.2 K. From these result, we conclude Bulk HTSC SAU is a good candidate for a short period undulator in the next generation.

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