

THE EFFECT OF UNDULATOR HARMONICS FIELD ON FREE-ELECTRON LASER HARMONIC GENERATION *

Qika Jia, National Synchrotron Radiation Laboratory,
University of Science and Technology of China, Hefei, Anhui 230029, China

Abstract

The harmonics field effect of planar undulator on Free-Electron Laser (FEL) harmonic generation has been analyzed. For both the linear and the nonlinear harmonic generation, the harmonic generation fraction can be characterized by the coupling coefficients. The modification of coupling coefficients is given when third harmonics field component exist, thus the enhancement of the harmonic radiation can be predicted. With the third harmonics magnet field being 30 percent of the fundament, for both the small signal gain and the nonlinear harmonic generation, the intensity of third-harmonic radiation can maximally be doubled.

INTRODUCTION

Using the higher harmonic is a way of Free Electron Laser (FEL) developing towards the shorter wavelength ranges [1-4]. For a planar undulator with a sinusoidal periodic magnetic field, the electrons also radiate at odd harmonics on-axis due to their non-uniform axial motion. In actual planar undulators, the magnetic field is non-sinusoidal and with harmonics field components. The harmonic radiation can be enhanced by aptly increase the harmonic field component [5]. Therefore some methods for this purpose were proposed, such as putting high permeability shims inside the undualtor [6], optimizing magnetic blocks size in a standard Harbch undulator [7]. In this paper, we analysis the effect of undulator harmonics field on FEL harmonic generation, the case of third harmonics magnetic field is considered specially.

ANALYSIS

In a planar undulator with an ideal sinusoidal periodic magnetic field, the electrons oscillate at odd harmonics frequency in the transverse direction, thus lead to the odd harmonics radiations in the direction forward [8]. For a FEL utilizing such undulator, the n th harmonic optical field equation and the phase equation in one-dimensional mode are [9]

$$\frac{d}{dz} \tilde{a}_{sn} = \frac{r_e n_e a_u [J, J]_n \lambda_{sn}}{\gamma} \langle e^{-in\phi} \rangle \quad (1)$$

$$\frac{d^2 \phi}{dz^2} = -\frac{2a_u k_u}{\gamma^2} \text{Re} \sum_n [J, J]_n k_{sn} \tilde{a}_{sn} e^{in\phi} \quad (2)$$

*Work supported by the the National Nature Science Foundation of China under Grant No. 10975137

jiaqk@ustc.edu.cn

where $\tilde{a}_{sn} = a_{sn} e^{-i\phi_{sn}}$, $a_{sn} = eE_{sn}/(mc^2 k_{sn})$ and $a_u = eB_u/(mc^2 k_u)$ are dimensionless vector potential of the rms n th harmonic radiation field E_{sn} and undulator field B_u , respectively; $k_{sn} = 2\pi/\lambda_{sn}$, $k_u = 2\pi/\lambda_u$ are the corresponding wave number; ϕ_{sn} is the phase of the radiation field; r_e is the classical electron radius; n_e and γ is the density and energy of electrons; ϕ is the ponder motive phase of electron $\phi = (k_s + k_u)z - \omega_s t$. the angular bracket represents the average over the electron's initial phases and initial phase velocities ϕ_0 . $[J, J]_n$ is the coupling coefficient

$$[J, J]_n = (-1)^{\frac{n-1}{2}} \left[J_{\frac{n-1}{2}} \left(\frac{na_u^2}{2(1+a_u^2)} \right) - J_{\frac{n+1}{2}} \left(\frac{na_u^2}{2(1+a_u^2)} \right) \right] \quad (3)$$

J is integer order Bessel function. From Eq.(1-2) for n th harmonic optical field, the small signal gain in low gain FEL and the nonlinear harmonic generation in high gain FEL can be given by[9]

$$g_n = -n \left(\frac{[J, J]_n}{[J, J]_1} \right)^2 (4\pi N \rho)^3 \left\langle \frac{\partial}{\partial x} \sin c^2 \frac{x}{2} \right\rangle_{\phi_0} \quad (4)$$

$$\frac{P_n}{\rho P_e} \approx \left(\frac{n^{n-1} [J, J]_n}{n! [J, J]_1} \right)^2 \left(\frac{P_1}{\rho P_e} \right)^n \quad (5)$$

where, ρ is the Pierce parameter, $x = n\phi_0 L$, $L = N\lambda_u$ is the length of undulator, P_e is the power of the electron beam $P_1 = P_{ef} \exp[z/L_g]/9$ is the fundamental power; P_{ef} is the effective start-up shot noise power, it is equal to the fraction of the spontaneous undulator radiation in one power gain length[9]. Thus the harmonic generation are characterized by the coupling coefficients for both linear case and non-linear case. The coupling coefficients therefore the harmonic generation increase with undulator deflection parameter but very slowly after $a_u > 2$.

One can expect enhancement of the laser harmonics by adding a harmonic field to the fundamental sinusoidal undulator field.

In actual planar undulators, the magnetic field is non-sinusoidal, when expanded in Fourier series the field includes odd spatial harmonics due to the symmetry of the magnetic structure. Therefore the magnetic fields and corresponding dimensionless vector potential can be expressed by

$$B_u = \sum_m B_{um} \sin(mk_u z) \quad \tilde{a}_u = \sum_m \tilde{a}_{um} \cos(mk_u z) \quad (6)$$

where $k_u = 2\pi/\lambda_u$ is the wave number of fundamental magnetic fields; B_{um} and \tilde{a}_{um} is the peak of m th harmonics magnetic fields and corresponding dimensionless vector potential, m is all or part odd numbers depending on the

magnetic structure. Generally all harmonics components are much smaller than the fundamental component:

$$B_{um} \ll B_{u1}, \quad a_{um} = \frac{B_{um}}{mB_{u1}} a_{u1} \ll a_{u1} \quad (7)$$

Using relation $\beta_{\perp}^2 = \frac{\tilde{a}_u^2}{\gamma^2}$, the electrons velocity with the harmonics undulator fields is

$$\begin{aligned} \beta_{||} &= 1 - \frac{1}{2} \left(\frac{1}{\gamma^2} + \beta_{\perp}^2 \right) \\ &= \bar{\beta}_{||} - \frac{1}{\gamma^2} \left\{ \sum_m a_{um}^2 \cos(2mk_u z) + \right. \end{aligned} \quad (8)$$

$$\left. \sum_{m \neq l} a_{um} a_{ul} [\cos((m+l)k_u z) + \cos((m-l)k_u z)] \right\}$$

where $\bar{\beta}_{||} = 1 - \frac{1}{2\gamma^2} (1 + \sum_m a_{um}^2)$ is the average velocity,

and *rms* value of the field is used. Accordingly the resonance condition is

$$\lambda_{sn} = \frac{\lambda_u}{2n\gamma^2} \left(1 + \sum_m a_{um}^2 \right) \quad (9)$$

The longitudinal motion of electron is

$$\begin{aligned} z &= \bar{z} - \left\{ \sum_m \frac{\xi_m}{k_s} \sin(2k_u \bar{z}) + \sum_{m \neq l} \left[\frac{\xi_{m+l}}{k_s} \sin((m+l)k_u \bar{z}) + \right. \right. \\ &\quad \left. \left. \frac{\xi_{m-l}}{k_s} \sin((m-l)k_u \bar{z}) \right] \right\} \end{aligned} \quad (10)$$

where $\bar{z} = \bar{\beta}_{||} ct$,

$$\begin{aligned} \xi_m &= \frac{a_{um}^2}{2m(1 + \sum a_{ui}^2)} = \frac{r_m^2}{m} \xi_1, \\ \xi_{m \pm l} &= \frac{a_{um} a_{ul}}{(m \pm l)(1 + \sum a_{ui}^2)} = \frac{2r_m r_l}{m \pm l} \xi_1 \end{aligned} \quad (11)$$

$$r_m = \frac{a_{um}}{a_{u1}} = \frac{B_{um}}{mB_{u1}} \ll 1, \quad \xi_1 = \frac{a_{u1}^2}{2(1 + \sum a_{ui}^2)} < \frac{a_{u1}^2}{2(1 + a_{u1}^2)} < \frac{1}{2}$$

For the case that all magnetic harmonic components are much smaller than the fundamental component, all ξ_m ,

$\xi_{m \pm l}$ terms that don't contain the fundament are much smaller than one and can be neglected. Then Eq.(7) became as

$$\begin{aligned} z &\approx \bar{z} - \left\{ \frac{\xi_1}{k_s} \sin(2k_u \bar{z}) + \sum_{m \neq l} \left[\frac{\xi_{m+l}}{k_s} \sin((m+l)k_u \bar{z}) + \right. \right. \\ &\quad \left. \left. \frac{\xi_{m-l}}{k_s} \sin((m-l)k_u \bar{z}) \right] \right\} \end{aligned} \quad (12)$$

Including the harmonic magnetic fields (Eq.(6)) and with the optical field

$$\tilde{a}_s = \sum_n a_{sn} \sin[n(k_s z - \omega_s t) + \phi_{sn}] \quad (13)$$

the phase equation (Eq.(2)) now is

$$\phi'' = \frac{2k_u}{\gamma^2} \sum_{n,l} k_{sn} a_{sn} a_{ul} \{ \cos[(nk_s + lk_u)z - n\omega_s t + \phi_n] + \quad (14)$$

$$\cos[(nk_s - lk_u)z - n\omega_s t + \phi_n] \}$$

Substituting Eq.(12) to it, we get

$$\phi'' = \frac{2k_u}{\gamma^2} \sum_n k_{sn} a_{sn} a_{ul} f_n \operatorname{Re} e^{-i(n\phi + \phi_{sn})} \quad (15)$$

where

$$\begin{aligned} f_n &= \operatorname{Re} \sum_l \frac{a_{ul}}{a_{u1}} [e^{i(n-l)k_u \bar{z}} + e^{i(n+l)k_u \bar{z}}] e^{in\xi \sin(2k_u \bar{z})} \\ &\quad \prod_{m \neq l} e^{in\{\xi_{m+l} \sin[(m+l)k_u \bar{z}] + \xi_{m-l} \sin[(m-l)k_u \bar{z}]\}} \end{aligned} \quad (16)$$

In obtaining Eq.(16) the condition $k_s \gg k_u$ is used.

Similarly when magnetic harmonic field existed the *n*th harmonic optical equation becomes

$$\frac{d}{dz} \tilde{a}_{sn} = \frac{r_e n_e a_{u1} \lambda_{sn}}{\gamma} f_n \langle e^{-in\phi} \rangle \quad (17)$$

Compare Eq.(15,17) with Eq.(1-2), it can be seen that when magnetic harmonic fields are included the coupling coefficient is modified as

$$[J, J]_n \rightarrow f_n$$

In the exponential of the modified coupling coefficient (Eq.(16),) many terms are small and oscillate fast, a average over undulator period will eliminate these small contribution terms.

Among all the harmonics, the third harmonic is the most important. Next we consider the case that only third harmonic field exist, and all other harmonics are neglected. In this case, Eq.(12) becomes

$$z = \bar{z} - \frac{\xi_1}{k_{s1}} \sin(2k_u \bar{z}) - \frac{\xi_2}{k_{s1}} \sin(4k_u \bar{z}) \quad (18)$$

where

$$\begin{aligned} \xi_1 &= \frac{k_{s1} a_{u1}^2 (1 + \frac{a_{u3}}{a_{u1}})}{4k_u \gamma^2} = \frac{a_{u1} (a_{u1} + a_{u3})}{2(1 + a_{u1}^2 + a_{u3}^2)}, \\ \xi_2 &= \frac{k_{s1} a_{u3} a_{u1}}{8k_u \gamma^2} = \frac{a_{u1} a_{u3}}{4(1 + a_{u1}^2 + a_{u3}^2)} \end{aligned} \quad (19)$$

and have

$$\frac{\xi_2}{\xi_1} = \frac{a_{u3} / a_{u1}}{2(1 + a_{u3} / a_{u1})} = \frac{B_{u3} / B_{u1}}{6 + 2B_{u3} / B_{u1}} \ll 1$$

Then the modified coupling coefficient (Eq.(16)) is

$$\begin{aligned} f_n &= \sum_{l=1,3} \frac{a_{ul}}{a_{u1}} [e^{i(n-l)k_u \bar{z}} + e^{i(n+l)k_u \bar{z}}] \\ &\quad \sum_{h_1} \sum_{h_2} J_{h_1}(n\xi_1) J_{h_2}(n\xi_2) e^{i(h_1 + 2h_2)2k_u \bar{z}} \end{aligned} \quad (20)$$

here $n=1,3$. After a average over undulator period, the dominant product terms in the sum of above equation are those with $h_1 + 2h_2 = -(n \pm l) / 2$:

$$f_n = \sum_l \frac{a_{ul}}{a_{u1}} \left\{ \sum_{\substack{h_1, h_2, \\ h_1+2h_2=\frac{n+l}{2}}} J_{h_1}(n\zeta_1) J_{h_2}(n\zeta_2) + \sum_{\substack{h_1, h_2, \\ h_1+2h_2=\frac{n-l}{2}}} J_{h_1}(n\zeta_1) J_{h_2}(n\zeta_2) \right\} \quad n, l=1, 3 \quad (21)$$

For the small arguments, only Bessel functions of zero order will contribute. Because $\zeta_2 \ll \zeta_1 < 1/2$, Eq. (21) can be further simplified by taking $h_2=0$. Then at last we give the modified coupling coefficient as

$$f_1 = J_0(\zeta_2) \{ J_0(\zeta_1) - J_1(\zeta_1) \} + \frac{a_{u3}}{a_{u1}} [J_2(\zeta_1) + J_1(\zeta_1)] \quad (22)$$

$$f_3 = J_0(3\zeta_2) \{ J_2(3\zeta_1) - J_1(3\zeta_1) \} + \frac{a_{u3}}{a_{u1}} [J_0(3\zeta_1) - J_3(3\zeta_1)] \quad (23)$$

According above formulas, the modified harmonic coupling coefficients as function of undulator parameter are numerical calculated for different harmonic magnetic field fraction. The results reveal that the harmonic coupling coefficients so the harmonic emission are enhanced when B_{u3} have an opposite sign to B_{u1} , and are suppressed when the magnetic fields has the same sign (Fig.1). The results also show that the fundament has been less affected by harmonic magnetic field (Fig. 2).

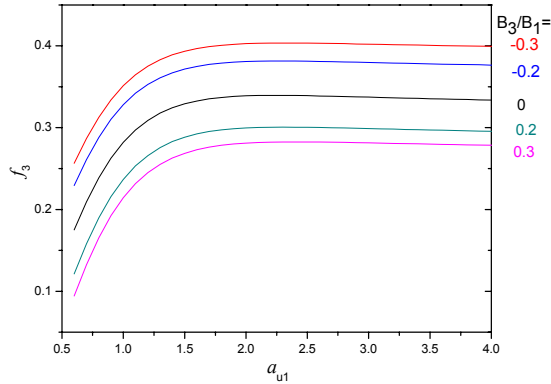


Figure1: Modified third harmonic coupling coefficient due to the harmonic magnetic field

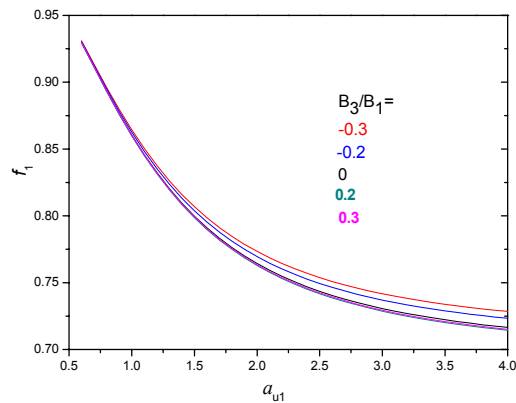


Figure 2: Modified fundamental coupling coefficient due to the harmonic magnetic field

For both the small signal gain and the nonlinear harmonic generation in high gain, the harmonic FEL radiation is proportional to the square of the coupling coefficient (Eq.(4) and (5)). Therefore comparing with the case without harmonic magnetic field presents, the enhancement of the 3rd harmonic radiation is

$$R_3 = \left(\frac{f_3 / f_1}{[J, J]_3 / [J, J]_1} \right)^2 \quad (24)$$

The dependence of third harmonic generation on the ratio of B_{u3} to B_{u1} is shown in Fig.3. The enhancement of third harmonic generation is shown in Fig.4. While in the calculation of Eq.(24), the fundamental magnetic field in the coupling coefficients and in the modified coupling coefficients are taken with a little difference to keep the resonant wavelengths same, it is a_u in the former, and a_{u1} in the later:

$$a_u^2 = a_{u1}^2 \left[1 + \left(\frac{a_{u3}}{a_{u1}} \right)^2 \right] \quad (25)$$

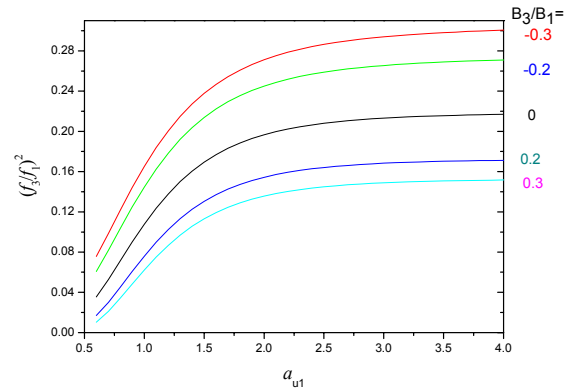


Figure 3: the effect of B_3 on FEL harmonic generation

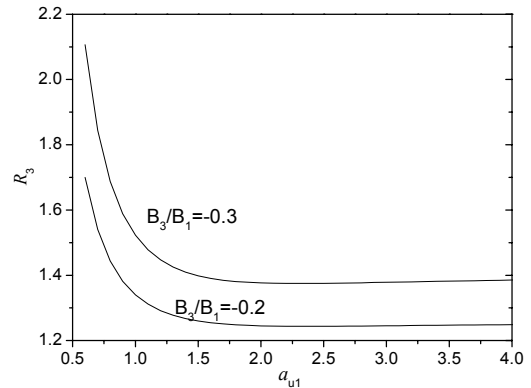


Figure 4: The enhancement of the FEL harmonic radiation with B_3 .

We can see that harmonics radiation enhancement increase with the magnetic field ratio of the harmonics to the fundamental, and for a given harmonic magnetic field

fraction, the enhancement is larger when the magnetic field is weaker. With the third harmonics magnetic field 30 percent of the fundamental, this field ratio translates into a vector potential ratio of 10 percent, the intensity increase of third-harmonic radiation is about 40 percent, and is more larger in smaller undulator deflecting parameter case, maximally doubled for undulator deflecting parameter $K(=\sqrt{2}a_u) = 1$.

SUMMARY

In summary, we have analysed the effects of undulator harmonics field on the harmonic coupling coefficients and FEL harmonic generation. For the case where third harmonic field present, analytical and easy to calculate expressions of the modified fundamental and harmonic coupling coefficients are given, and used to predict the effects of undulator harmonics field on the small signal gain in low gain FEL and the non-linear harmonic generation in high gain FEL. The numerical results demonstrate that the third harmonic emission can be distinctly enhanced by the undulator third harmonic field that has an opposite sign to the fundamental field, while the fundamental emission has been less affected. With a third harmonic field 30 percent of the fundamental field, the third-harmonic emission can be enhanced about 40 percent, and can maximally be doubled.

Owing to that the FEL simulation codes we had can't include the harmonic magnetic fields, here only analytic results are given. For the next work, we'll cast about for the numerical simulation study.

REFERENCES

- [1] W.B.Colson, "The nonlinear wave equation for higher harmonics in free-electron lasers", IEEE, J. Quantum Electron, vol.QE-17,pp.1417-1427, 1981
- [2] R. Bonifacio, L. De Salvo and P. Pierini "Large harmonic bunching in a high-gain free-electron laser", Nucl. Instr. and Meth. A vol.293, pp627-629,1990
- [3] H.P.Freund, S.G.Biedron, and S.V.Milton, "Nonlinear harmonic generation and proposed experimental verification in SASE FELs", Nucl. Instrum. Methods Phys.Res. Sect. A vol.445,te pp53-58,2000
- [4] Z.Huang , K.-J.Kim, "Three-dimensional analysis of harmonic generation in high-gain free-electron lasers", Phys.Rev.E vol.62,pp.7295-7308,2000; "Nonlinear harmonic generation of coherent amplification and self-amplified spontaneous emission", Nucl. Instrum. Methods Phys.Res. Sect. A vol.475, pp.112-117,2001
- [5] M.J. Schmitt and C.J. Elliott, IEEE J. Quantum Electron.QE-23 (1987) 1552.
- [6] M. Asakaw *et al.* Nucl. Instr. and Meth. in Phys. Res. A 358 (1995) 399-402, A 375 (1996) 416-319
- [7] Qi-ka Jia , "Undulator Harmonic field enhancement analysis", Proceedings of IPAC10, WEPD033/3165-3167
- [8] Qi-ka Jia , "Harmonic motion of electron trajectory in planar undulator" , PAC09-WE5RFP088
- [9] Qi-ka Jia , "An analysis of nonlinear harmonic generation in high gain free electron laser" IEEE J.Quantum Electron. 43, 833 (2007)