MOMENTUM MODULATIONS PRODUCED BY LASER-BEAM INTERACTION AT A PHOTOCATHODE

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Abstract

We study both analytically and numerically the effects of a laser pulse on the longitudinal phase space of an electron beam in the stage of extraction from the cathode. We show how the interaction can produce modulations in the longitudinal momentum distribution.

INTRODUCTION

The interaction between a laser and an electron beam can give rise to a number of phenomena, the most important among them are the Thomson scattering and the relativistic ponderomotive scattering. In a number of laboratories it was observed recently that the transverse distribution of optical transition radiation (OTR) emission from accelerated electron beams deviates substantially from the expected beam current distribution [1, 2, 3]. This effect of coherent OTR is related to the process of longitudinal microbunching [4, 5, 6]. Now there is an understanding that these observations are evidence of a longitudinal collective interaction process [2]. A possible explanation of such process was proposed in [7] as a collective Coulomb interaction that stabilizes and improves already existing microbunching. A source of the initial density modulation could possibly be a laser field [8] and some experiments are foreseen to check this proposal [9].

In this paper we want to analyze the possibility of microbunching an electron beam by means of the interaction with a Ti::Sa laser during the extraction from the photocathode.

MODEL EQUATIONS

In our set up, the laser propagates along the z axis, the electric field being polarized along the x axis, so that it lays in the incidence plane:

$$\mathbf{E} = \mathbf{e}_x E_0 I(\varphi)\sin\varphi$$
$$\mathbf{B} = \mathbf{e}_y \frac{E_0}{c} I(\varphi)\sin\varphi$$

with $\varphi = \omega t - kz$, $I(\varphi)$ represents the modulation of the laser field longitudinal profile and E_0 is the peak laser field. Here the laser electric field has been supposed transversally uniform and the laser z axis forms an incidence angle θ with the electrons axis z_{el}. The electrons are supposed to be at $x_{\mathrm{el}} = y_{\mathrm{el}} = z_{\mathrm{el}} = 0$ at $t = t_0$, being t_0 the electron extraction time, with an arbitrary initial momentum.

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The momentum equations in this configuration turn out to be:

$$\frac{dp_x}{dt} = -eE_0 I(\varphi)(1-\beta_z)\sin\varphi \tag{1a}$$
$$\frac{dp_z}{dt} = -eE_0 I(\varphi)\beta_x\sin\varphi \tag{1b}$$
$$\frac{dp_y}{dt} = 0 \tag{1c}$$

leading to

$$p_y(t) = p_{y,0}. \tag{2}$$

Furthermore, taking advantages from the equation for γ, one can write another constant of motion:

$$p_z(t) - \gamma(t) = p_{z,0} - \gamma_0, \tag{3}$$

where γ_0 is the initial Lorentz factor. From (1) we get:

$$\begin{aligned}\frac{d\varphi}{dt} &= \omega(1-\beta_z) = \omega\left(1 - \frac{p_z}{\gamma}\right)\\ &= \omega\,\frac{\gamma_0 - p_{z,0}}{\gamma_0 - p_{z,0} + p_z} = \omega\,\frac{\gamma_0 - p_{z,0}}{\gamma}.\end{aligned} \tag{4}$$

Moreover, from the definition of the Lorentz factor, we obtain a relation between p_z and p_x:

$$p_z = \frac{m_0^2c^2 + p_{y,0}^2 - (m_0\gamma_0 c - p_{z,0})^2}{2(m_0c - p_{z,0})} + \frac{p_x^2}{2(m_0c - p_{z,0})} \tag{5}$$

that, inserted in (4), leads to:

$$\begin{aligned}&(\varphi - \varphi_0^{\mathrm{e}})\left[(m_0\gamma_0c - p_{z,0})^2 + m_0^2c^2 + p_{y,0}^2\right]\\ &+ \int_{\varphi_0^{\mathrm{e}}}^{\varphi} p_x^2(s)\,ds = 2\omega(m_0\gamma_0c - p_{z,0})^2(t - t_0),\end{aligned} \tag{6}$$

where t_0 is the electron injection time and φ_0^{e} the laser phase experienced by the electron at injection.

From the above equations it is possible to deduce:

$$x(t) - x_0 = \frac{1}{k(m_0\gamma_0c - p_{z,0})}\int_{\varphi_0^{\mathrm{e}}}^{\varphi} p_x(s)\,ds \tag{7a}$$
$$z(t) = \frac{1}{2km_0^2c^2}\int_{\varphi_0^{\mathrm{e}}}^{\varphi} p_x^2(s)\,ds. \tag{7b}$$

To close the problem, it is, therefore, necessary to integrate first the equation for p_x:

$$p_x(t) - p_{x,0} = -\frac{e}{\omega}\int_{\varphi_0^{\mathrm{e}}}^{\varphi} E_0 I(s)\sin s\,ds \tag{8}$$

and, then, the integrals involving p_z.

The integral (8) can be performed for few types of laser longitudinal modulation, as, for instance, a flat top of finite duration, a Gaussian and a quadratic cosine modulation.

REALISTIC LASER PROFILE

We will now solve the equations of motion for the electron bunch when the laser profile is described by the envelope function:

$$I(\varphi)=\begin{cases}\sin^2\left(\frac{\pi}{2kL_\mathrm{r}}\varphi\right) & 0\le\varphi\le kL_\mathrm{r},\\ 1 & kL_\mathrm{r}<\varphi\le k(L_\mathrm{r}+L_\mathrm{c}),\\ \sin^2\left(\frac{\pi}{2kL_\mathrm{r}}\varphi-\frac{\pi L_\mathrm{r}}{2L_\mathrm{c}}\right) & k(L_\mathrm{r}+L_\mathrm{c})<\varphi\le kL_\mathrm{tot},\\ 0 & \text{otherwise},\end{cases} \tag{9}$$

where k is the laser wave vector, L_r and L_c are, respectively, the lengths of the rising and of the constant portion of the laser and $L_\mathrm{tot}=2L_\mathrm{r}+L_\mathrm{c}$ is the total laser pulse length; notice that the rising and falling times are assumed to be equal. We also suppose that $\mathbf{p}_0=0$, *i.e.* the beam is cold. In order to simplify calculations, from now on we will assume that any length scale of the laser contains an exact number of laser wavelengths; we will show afterward that this choice does not affect significantly the overall results.

From eq. (8) and (5), the momentum components due to the effect of the laser can then be written as

$$\frac{p_x(t_0)}{mc}=\frac{a}{2}H(t_0) \qquad \frac{p_z(t_0)}{mc}=\frac{a^2}{8}H^2(t_0) \tag{10}$$

where $a=\frac{eE_0}{m_0c\omega}$. Setting $\lambda_\mathrm{r}=L_\mathrm{r}/\lambda$, $\lambda_\mathrm{c}=L_\mathrm{c}/\lambda$, $\lambda_\mathrm{tot}=L_\mathrm{tot}/\lambda$ and $\lambda_\mathrm{r},\lambda_\mathrm{c},\lambda_\mathrm{tot}\in\mathbb{I}^+$ and, making the realistic assumption that $\lambda_\mathrm{r}\gg1$ (whereas λ_c can also be zero), $H(t_0)$ takes the simple form:

$$H(t_0)=\begin{cases}2\cos\omega t_0\sin^2\left(\frac{\omega t_0}{4\lambda_\mathrm{r}}\right) & \omega t_0\in\mathbb{D}_1\\ 2\cos\omega t_0 & \omega t_0\in\mathbb{D}_2\\ -2\cos\omega t_0\sin^2\left(\frac{\pi\lambda_\mathrm{c}}{2\lambda_\mathrm{r}}-\frac{\omega t_0}{4\lambda_\mathrm{r}}\right) & \omega t_0\in\mathbb{D}_3\\ 0 & \text{otherwise}\end{cases}, \tag{11}$$

where $\mathbb{D}_1=[0,2\pi\lambda_\mathrm{r}]$, $\mathbb{D}_2=(2\pi\lambda_\mathrm{r},2\pi(\lambda_\mathrm{r}+\lambda_\mathrm{c})]$ and $\mathbb{D}_3=(2\pi(\lambda_\mathrm{r}+\lambda_\mathrm{c}),2\pi\lambda_\mathrm{tot}]$. Since the laser usually impinges on the cathode with a finite angle θ, we need to project the momentum into the z_el direction of the electrons, obtaining:

$$p_{z\mathrm{el}}(t_0)=-\cos\theta\,p_z(t_0)-\sin\theta\,p_x(t_0). \tag{12}$$

Notice that the effect of the laser in the x direction is proportional to $a\cos\omega t_0$ while in the z direction it is proportional to $a^2\cos^2\omega t_0$; if we have a setup such that $a\sin\theta\simeq a^2\cos\theta$, we can expect a mixing of the longitudinal and transverse contributions, as is shown in FIG. 1.

The amount of displacement from the initial position due to $p_{z\mathrm{el}}(t_0)$, on the typical time scales of photocathode laser duration, is very small, provided we assume realistic values for the parameters $a\ll1$ and $E_0\le E_\mathrm{acc}$, where E_acc is the accelerating electric field. For this reason, we can consider $p_{z\mathrm{el}}(t_0)$ as an initial condition on the electrons momentum, setting $p_{z\mathrm{el}}^{(0)}\equiv p_{z\mathrm{el}}(t_0)$.

Whatever the laser profile, its effects on the electrons can be summarized as follows. The $\mathbf{E}\times\mathbf{B}$ term and the longitudinal gradient of the laser induce a momentum along the k direction which is constant on all the bunch; due to the assumptions made on a and E_0, its component along z_el is readily canceled by E_acc while the transverse component is likely compensated by the transport line. Most importantly, a modulation on the electron momentum is produced, which depends on the injection time t_0 and survives the subsequent acceleration.

Finally, let us comment on the assumption of cold beam: as shown before, the modulation in longitudinal momentum scales as a second degree polynomial of a. For a value of $a\approx10^{-2}$ and θ a few degrees, the modulations show an amplitude of a few hundreds eV/c while the typical thermal momenta of electrons are of the order of few eV/c: this means the thermal momenta can at most "blur" the induced modulations.

LONGITUDINAL MOMENTUM DISTRIBUTION

It is possible to use the results of the preceding section to calculate the electrons dynamic under the effect of an accelerating field and find out how the beam gets bunched in space by the effect of the laser. However, we prefer to focus on longitudinal momentum since its distribution does not change with acceleration and drifts. Moreover, acceleration can add to the moments depicted in FIG. 1 a linear function (as long as the laser pulse is short) which could be removed, at least in principle. FIG. 1 shows the momentum modulation and its distribution due to the action of a laser with an incidence angle of 3° and a longitudinal profile as in eq. (9). There appear a few main peaks and a number of secondary peaks. Upon inspection, it becomes clear that the electrons are modulated following the classical harmonic oscillator distribution for each laser period:

$$P(p)=\frac{1}{\pi p_n\sqrt{1-\frac{p^2}{p_n^2}}} \tag{13}$$

where p_n is the maximum extent of the oscillation. p_n is usually a slowly varying function of time (i.e. $p_n'(\varphi)\propto I'(\varphi)\ll\omega$). Due to this variation, the total distribution results in a sum of distributions like (13) with p_n varying accordingly to $I(\varphi)$:

$$P(p)=\sum_{n=0}^{N-1}\frac{\Theta\Big[E_0I\left(\frac{\pi n}{N}\right)-p\Big]\Theta\Big[E_0I\left(\frac{\pi(2n+1)}{2N}\right)+p\Big]}{\pi N\sqrt{\left(E_0I\left(\frac{\pi n}{N}\right)-p\right)\left(E_0I\left(\frac{\pi(2n+1)}{2N}\right)+p\right)}}, \tag{14}$$

where N is the number of laser periods contained in I and $\Theta(x)$ is the Heaviside theta function.

The presence of the secondary peaks does not allow to give the momentum distribution a manageable expression. Moreover, since they represent each laser oscillation

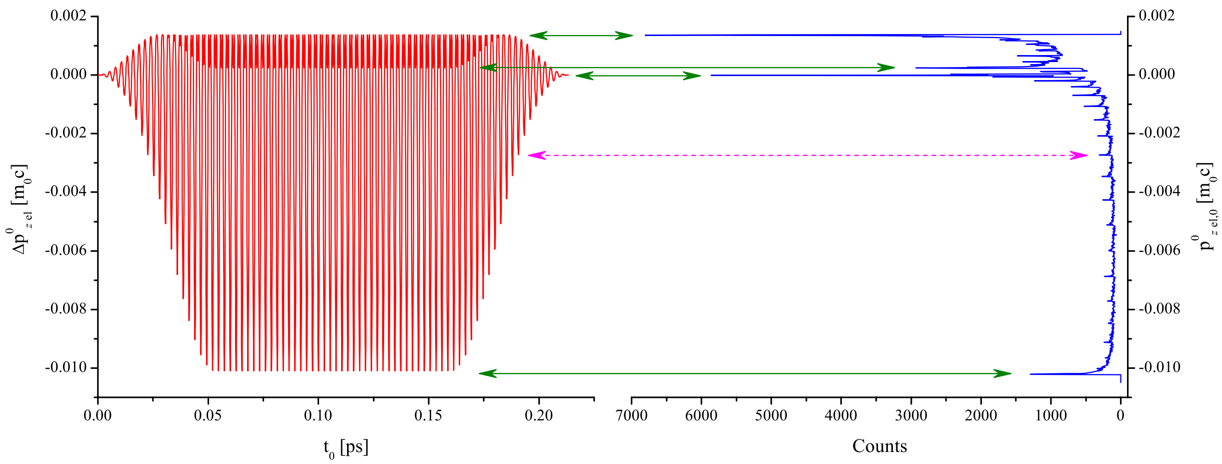

Figure 1: A plot of the initial longitudinal momentum modulation as a function of the extraction time t_0 (*left*) and the corresponding longitudinal momentum distribution (*right*). Green arrows show the correspondence between the main peaks and the stationary points of $I(\varphi)$, while the magenta line shows the presence of a minor peak due to a single laser oscillation (*i.e.* $\mathbf{E}'(\varphi) = 0$). The relevant parameters are: $a = 0.1$, $\omega = 0.75\pi \times 10^{15}$Hz, $L_\mathrm{r} = 20\lambda$, $L_\mathrm{c} = 40\lambda$, $\theta = \pi/60 = 3°$.

when $I'(\varphi) \neq 0$, their position is linked to the laser initial phase and their number to the exact number of the oscillations. Therefore, we would like to average out their presence, contenting ourselves to describe the baseline on which they stand. This task is easily accomplished in the limits $I'(\varphi) \ll \omega$ and $\lambda_\mathrm{r} \gg 1$ since we can replace the summation in (14) with an integration. If I has the form (9), it is then possible to describe the averaged distribution $\bar{P}$ of $p^{(0)}_{zel}$, taking particular care in identifying which are the dominant frequencies in (12) and whether they significantly mix or not due to the different weights. The result is quite cumbersome and will not be reported here in details. A comparison of a numerical sampling of eq. (12)

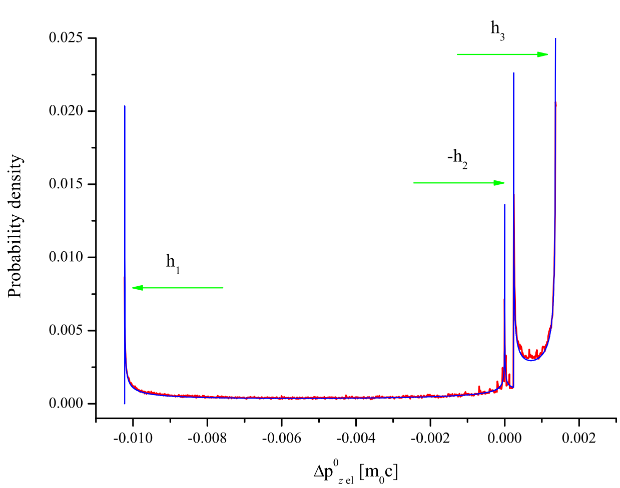

Figure 2: A confrontation of $\bar{P}$ (*blue*) and a stochastic sampling of the longitudinal momentum function (12) (*red*). The parameters are the same as in FIG. 1 except for L_c which is 200λ.

with the averaged distribution $\bar{P}$ is shown in FIG. 2; notice the presence of the secondary peaks around the main ones in the numerical sampling. Notice also that the distribution is formally divergent at the peaks: this implies that the effective height of each peak depends on the sampling of the function (12) and is not a significant quantity.

The real distribution can be well approximated by the expression:

$$\begin{aligned}\bar{P}(p) \approx & \frac{Q_1}{\sqrt{h_3 - p}\sqrt{h_1 + p}}\Theta\left[h_3 - p\right]\Theta\left[h_1 + p\right] \\ & + \frac{Q_2}{\sqrt{h_3 - p}\sqrt{h_2 + p}}\Theta\left[h_3 - p\right]\Theta\left[h_2 + p\right] \quad (15) \\ & + \frac{Q_3}{\sqrt{|p|}}\Theta\left[h_3 - p\right]\Theta\left[h_1 + p\right],\end{aligned}$$

where h_i are the positions of the main peaks, as shown in FIG. 2, and Q_i the relative weights due to the different charge content. The values of all such parameters can be argued from the full form of $\bar{P}$.

As a final consideration, we would like to stress that the laser frequency only contributes through the definition of a. The meaning of such dependence is that the laser frequency only measures the frequency with which the electrons "sample" the envelope; as long as I varies on scales much longer than λ, and the longitudinal electron density is high enough, the main features of the momentum distributions are not affected by ω. Moreover, if a particle is accelerated by an RF cavity or a DC gun, it experiences a higher laser frequency due to the Doppler effect; anyway, as long as its rigidity can be considered constant, the momentum distribution is unaffected by the acceleration. Finally, let us comment on the assumption on the initial transverse positions of the electrons, i.e. $x_\mathrm{el} = y_\mathrm{el} = 0$: if an electron starts with a displacement from this position, it will only experience a different initial laser phase (and, eventually, a different field strength if a transverse laser profile is added); this will affect the momentum distribution only quantitatively, not qualitatively.

NUMERICAL SIMULATIONS

In order to test the analytical model and the assumptions made, we run some numerical simulations using a modified version of the code RETAR [10] in which a realistic laser transverse profile with bending wavefronts [11] has been added; the longitudinal shape is Gaussian. The electron bunch has a diameter of 100 μm and the initial particle density is uniform both longitudinally and transversally. The initial energy spread is set to 3 eV/c and the emittance is 1 μm. The interaction with the laser takes place while the bunch is accelerated in a uniform electric field. Since we needed a high number of particles, the electron bunch length is smaller than the laser duration: therefore, no neat peaks are expected, since there are not domains in which $I'(\varphi)$ is zero. Only an enhancement of particles density on the edges of the Gaussian profile is foreseen.

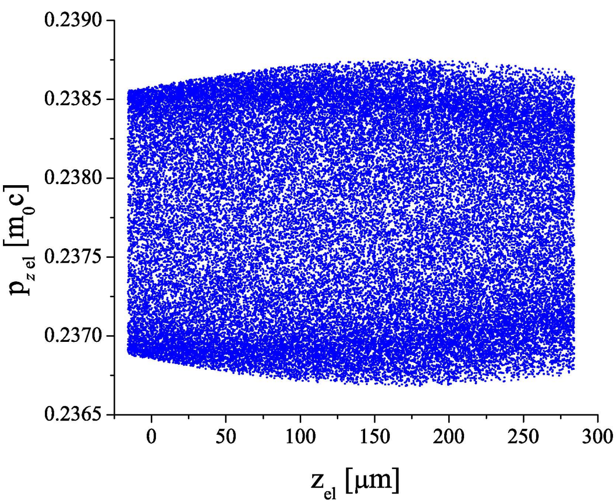

Figure 3: Longitudinal phase space of the electron bunch after the interaction with a Gaussian laser pulse and the acceleration in a DC gun. The relevant parameters are $\theta = 82°$ and $a = 2 \times 10^{-3}$.

In FIG. 3 a longitudinal phase space of the electrons is shown; there appears the expected density increase. The momentum density profile, for three different values of the angle θ, is shown in FIG. 4. As can be seen, the presence of the initial energy spread, emittance and the acceleration do not interfere significantly with the expected momentum distribution which depends mainly on the laser parameter a and the geometry of the interaction.

CONCLUSIONS

We studied the effect of a laser pulse on the electrons extracted from a photocathode both theoretically and numerically, showing that the laser fields can induce a modulation on the electrons longitudinal momentum due to the longitudinal laser profile itself. Such modulations are likely to survive the bunch acceleration in a linac and could possibly be converted in density modulations if the bunch goes through dispersive sections such as bending magnets or doglegs.

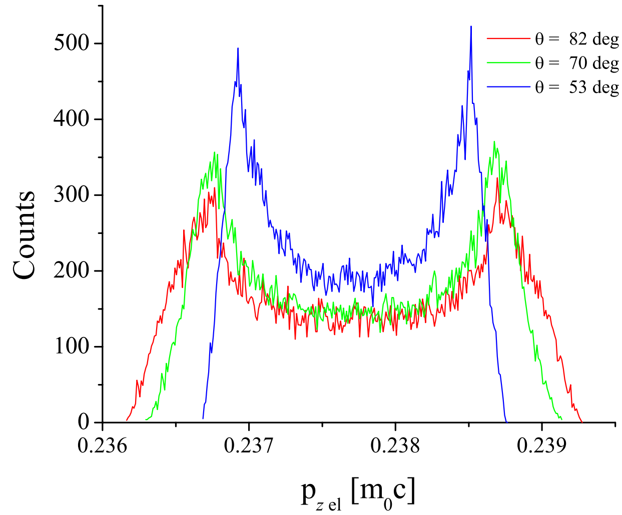

Figure 4: Longitudinal momentum density for three different angles of incidence. The laser parameter is $a = 2 \times 10^{-3}$

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