THE IMPERFECTNESS OF ELECTRON BUNCH INITIAL LONGITUDINAL PHASE SPACE ON A SEEDED FREE ELECTRON LASER PERFORMANCE*

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Abstract

A single-pass high-gain x-ray free electron laser (FEL) calls for a high quality electron bunch. In particular, for a seeded FEL amplifier and for a harmonic generation FEL, the electron bunch initial energy profile uniformity and peak current uniformity are crucial for generating an FEL with a narrow bandwidth. After the acceleration, compression, and transportation, the electron bunch energy profile entering the undulator can acquire temporal non-uniformity both in energy and local density. We study the effects of the electron bunch initial energy profile non-uniformity and local density variation on the FEL performance. Intrinsically, for a harmonic generation FEL, the harmonic generation starts with an electron bunch having energy modulation as well as density bunching at the previous stage FEL wavelength and its harmonics. Its effect on the harmonic generation FEL in the radiator is then studied.

Introduction

Free Electron Laser (FEL) is perceived as one of the candidates for the fourth generation light source. Success in commissioning the world's first x-ray (1.5-15Å) FEL – the LINAC Coherent Light Source (LCLS) – at SLAC National Accelerator Laboratory opens the gate for new science [1]. Further improving the FEL spectrum bandwidth is urged by various potential users. One of the possibilities to generate narrow bandwidth FEL is to invoke a coherent seed laser to start the FEL process, which is generally referred to as a seeded FEL. With a coherent seed laser, the radiator can set to have the resonant wavelength the same as the seed laser to simply form a FEL amplifier or an Optical Klystron (OK) [2]. An OK has two undulators with a magnetic buncher in between. For an OK, indeed the radiator can have the resonant frequency as one of the harmonics of the seed laser. In such an operation mode, a Harmonic Generation Free Electron Laser (HGFEL) can be configured [3, 4]. Due to the fact that the buncher between the two undulators will rotate the phase space on the seed wavelength scale, the electron bunch entering the radiator will have multi-frequency components in its energy spectrum. We investigate its impact on the radiator FEL performance, in particular the FEL bandwidth from this multi-frequency energy spectrum. In general, the electron bunch generated from the photoinjector has a very small energy spread and small emittance. During the acceleration, bunch compression, and transportation, the electron bunch will experience the RF curvature, the second order effect in the chicane, and collective effects, which will all lead to a nonuniform energy profile [5]. In addition, the electron bunch is subject to microbunching instability [6]. Thus, the electron bunch entering the undulator can have an energy modulation with multiple frequencies. Such energy modulation will impact the FEL performance and affect the FEL bandwidth. Studies have been conducted for an initial energy modulation [7, 8]. In this paper, we consider both the energy profile non-uniformity as well as local density non-uniformity on the free electron laser (FEL) performance for a FEL amplifier as well as for a harmonic generation FEL.

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Vlasov-Maxwell Analysis for an Initial Value Problem

For a FEL amplifier, the FEL process starts from a coherent seed; while for an optical klystron [2] and (high-gain) harmonic generation FEL [4, 9, 10], the FEL radiation in the radiator starts from coherent emission from a microbunched electron bunch. Nevertheless, the coherent emission once generated will be decomposed into the FEL guided modes and will be amplified due to the same FEL process. The FEL amplification process by an electron bunch with multi-frequency energy spectrum is the same and applicable to all these difference FEL configurations. Hence in the following, let us formulate the FEL start-up and evolution process when the electron bunch has energy non-uniformity.

To analyze the start-up of a seeded FEL amplifier we use the coupled set of Vlasov and Maxwell equations which describe the evolution of the electrons and the radiation fields [11]. This approach is used as well for the Self-Amplified Spontaneous Emission (SASE) FEL [12]. We will work with a one-dimensional system in this section.

Vlasov-Maxwell Equations We follow the analysis and notation of Refs. [12, 11]. Dimensionless variables are introduced as $Z = k_w z$, $\theta = (k_0 + k_w)z - \omega_0 t$, where $k_0 = 2\pi/\lambda_0$, $\omega_0 = k_0 c$, and $k_w = 2\pi/\lambda_w$ with λ_0 being the radiation wavelength, λ_w being the undulator period, and c being the speed of light in vacuum. We also introduce $p = 2(\gamma - \gamma_0)/\gamma_0$ as the measure of energy deviation, with γ the Lorentz factor of an electron in the electron bunch, and γ_0 the resonant energy defined by $\lambda_0 =$

$\lambda_w(1+K^2/2)/(2\gamma_0^2)$, for a planar undulator, where the undulator deflecting parameter $K \approx 93.4 B_w \lambda_w$ with B_w the peak magnetic field in Tesla and λ_w the undulator period in meter. The electron distribution function $\psi(\theta,p,Z)$ is normalized, *i.e.*, $\int \psi(\theta,p,Z)d\theta dp = 1$, with $\psi_0(\theta,p,Z)$ describing the slow varying unperturbed component. The FEL electric field is written as $E(t,z) = A(\theta,Z)e^{i(\theta-Z)}$ with $A(\theta,Z)$ being the slow varying envelope function.

The one-dimensional linearized Vlasov-Maxwell equations are,

$$\frac{\partial\psi}{\partial Z} + p\frac{\partial\psi}{\partial\theta} - \frac{2D_2}{\gamma_0^2}\left(Ae^{i\theta} + A^* e^{-i\theta}\right)\frac{\partial\psi_0}{\partial p} = 0, \quad (1)$$

and,

$$\left(\frac{\partial}{\partial Z} + \frac{\partial}{\partial\theta}\right) A(\theta,Z) = \frac{D_1}{\gamma_0} e^{-i\theta}\int dp\psi(\theta,p,Z), \quad (2)$$

where in SI units, $D_1 = ea_w n_0 [JJ]/(2\sqrt{2}k_w\varepsilon_0)$ and $D_2 = ea_w[JJ]/(\sqrt{2}k_w mc^2)$, with e and m being the charge and mass of the electron; $\varepsilon_0 \approx 8.85\times 10^{-12}$ F/m being the vacuum permittivity; n_0 being the electron bunch density in units of 1/m^3; and $[JJ] = J_0[a_w^2/2(1+a_w^2)] - J_1[a_w^2/2(1+a_w^2)]$ where the dimensionless rms undulator parameter $a_w \equiv K/\sqrt{2}$ and $J_0(x)$ and $J_1(x)$ are the zeroth and first order Bessel functions.

Initial Energy Imperfectness–General Solution

To model an energy imperfectness in the electron bunch coming into the undulator, we assume that the initial energy distribution function is

$$\psi_0 = \delta[p + g(\theta_0)] = \delta[p + g(\theta - pZ)], \quad (3)$$

where $g(\theta_0)$ is a general function.

The general solution is [8]

$$f(\theta,s) = f(-\infty,s) + \int_{-\infty}^{\theta} d\theta' e^{-s(\theta-\theta') + \int_{\theta'}^{\theta}\frac{i(2\rho)^3}{[s-ig(\theta'')]^2}d\theta''} \quad (4)$$
$$\times\left[A(\theta',0)\left\{+\frac{D_1}{\gamma_0}\tilde{B}(\theta',s)\right\} + \frac{D_1}{\gamma_0}\sum_j \frac{e^{-i\theta_j}\delta(\theta'-\theta_j)}{s-ig(\theta_j)}\right],$$

which relates to $A(\theta,Z)$ via Laplace transform, *i.e.*,

$$f(\theta,s) = \int_0^\infty dZ e^{-sZ} A(\theta,Z). \quad (5)$$

Likewise, we have introduced

$$\tilde{B}(\theta,s) = \int_0^\infty dZ e^{-sZ} B(\theta,Z), \quad (6)$$

for the pre-microbunched component, which is related to the bunching factor, b, as

$$\begin{aligned} b &\equiv \left|\frac{1}{2\pi}\int_{-\pi}^{\pi} d\theta \int dp e^{-i\theta}\psi_0(\theta,p)\right| \\ &\equiv \left|\frac{1}{2\pi}\int_{-\pi}^{\pi} d\theta B(\theta)\right|. \end{aligned} \quad (7)$$

Notice that, in the square brackets in Eq. (4), the first term $A(\theta,0)$ characterizes the initial seed for a seeded FEL, the second term models a pre-microbunched electron bunch, while the third term models the SASE FEL. In the following, let us focus on a seeded FEL, so that the second term and third term in the square brackets will be ignored in the derivation.

Initial Energy Modulation–an Example

In this section, the general function $g(\theta)$ as in Eq. (3) characterizing the nonuniform energy profile is represented as a Fourier series as in the following. Indeed, for electron bunch experienced microbunching instability, or in the harmonic generation FEL as explained above and detailed in the following Section , there can be an energy modulation along the electron bunch as

$$\gamma = \gamma_0 + \sum_{m=1}^{\infty} \varepsilon_m \sin[\omega_m(t-t_0)], \quad (8)$$

where ω_m characterizes the mth component of the energy modulation. The initial energy distribution function is then

$$\psi_0 = \delta\left[p + \sum_{m=1}^{\infty}\eta_m \sin(\omega_{\eta_m}\theta_0)\right], \quad (9)$$

where $\eta_m \equiv 2\varepsilon_m/\gamma_0$ and $\omega_{\eta_m} \equiv \omega_m/\omega_0$. For such a sinusoidal modulation, we have

$$\int_{\theta'}^{\theta}\frac{i(2\rho)^3}{\left[s - i\sum\limits_{m=1}^{\infty}\eta_m\sin(\omega_{\eta_m}\theta'')\right]^2}d\theta'' \approx \frac{i(2\rho)^3(\theta-\theta')}{s^2}$$
$$+\sum_{m=1}^{\infty}\frac{2\eta_m(2\rho)^3\left[\cos(\omega_{\eta_m}\theta) - \cos(\omega_{\eta_m}\theta')\right]}{\omega_{\eta_m}s^3}, \quad (10)$$

to the leading order in η_m.

The FEL Solution For a seeded FEL, let us throw away the initial value term, the pre-microbunched term as well as the SASE term, and keep only the seed in Eq. (4).

$$f(\theta,s) \approx \int_{-\infty}^{\theta} d\theta' A(\theta',0) \quad (11)$$
$$\times e^{-s(\theta-\theta') + \frac{i(2\rho)^3(\theta-\theta')}{s^2} + \sum\limits_{m=1}^{\infty}\frac{2\eta_m(2\rho)^3[\cos(\omega_{\eta_m}\theta)-\cos(\omega_{\eta_m}\theta')]}{\omega_{\eta_m}s^3}}.$$

The inverse Laplace transform then gives us the FEL electric field slow-varying envelope function as

$$A(\theta,Z) = \int_c \frac{ds}{2\pi i}e^{sZ}f(\theta,s)$$
$$\approx E_0\omega_0\frac{e^{i\pi/12 + \mathcal{W}^2\alpha_0(\theta - Z/3 + i^{5/3}/6)^2/[(1+\mathcal{W})\omega_0^2]}}{\sqrt{2\alpha_0(1+\mathcal{W})Z/\rho}}$$
$$\times e^{i\varpi}e^{i^{1/3}2\rho Z + \frac{3(\theta-Z/3)\rho}{2Z} - \frac{i^{1/3}9(\theta-Z/3)^2\rho}{2Z} - \sum\limits_{m=1}^{\infty}\frac{i2\eta_m\cos(\omega_{\eta_m}\theta)}{\omega_{\eta_m}}} \quad (12)$$

where

$$\varpi \equiv \sum_{m=1}^{\infty} \frac{2\eta_m}{\omega_{\eta_m}} e^{-\frac{\omega_0^2 \omega_{\eta_m}^2}{4\alpha_0(1+\mathcal{W})}} \cos\left[\frac{\mathcal{W}(\theta - Z/3 + i^{5/3}/6)\omega_{\eta_m}}{1+\mathcal{W}}\right], \tag{13}$$

and

$$\mathcal{W} \equiv \frac{9i^{1/3}\rho\omega_0^2}{2\alpha_0 Z}. \tag{14}$$

In getting the above solution, we assume an initial Gaussian seed,

$$\begin{aligned} & E(t, z=0) = E_0 e^{-i\omega_0 t - \alpha_0 t^2} = E_0 e^{i\theta - \theta^2\alpha_0/\omega_0^2} \\ \Longrightarrow \quad & A(\theta, 0) = E_0 e^{-\theta^2\alpha_0/\omega_0^2}, \end{aligned} \tag{15}$$

where $\alpha_0 = 1/(4\sigma_{t0}^2)$ with σ_{t0} being the initial seed rms pulse duration.

Impact on a Seeded FEL

The work developed in the previous sections is sufficient to study a seeded FEL amplifier when the electron bunch has nonuniform energy profile. Yet, for a harmonic generation FEL or an optical klystron configuration, there is no initial radiation seed, but rather the FEL will start from a premicrobunched electron bunch. In fact, this can be done by keeping the pre-bunched term (given in the curly brackets) and throw away the seed term and the SASE term in Eq. (4).

Electron energy profile into the radiator Since we are working with a cold electron bunch without intrinsic energy spread, the phase space distribution function at the exit of the modulator in a HG FEL will be

$$\delta(\delta\gamma - \Delta\gamma \sin\theta), \tag{16}$$

where $\delta\gamma \equiv (\gamma - \gamma_0)/\gamma_0$ with γ_0 as the electron centroid energy, and $\delta(x)$ is the Dirac delta-function. After the buncher, the phase space distribution is then

$$\delta\left[\delta\gamma - \Delta\gamma \sin\left(\theta - \frac{d\theta}{d\gamma}\delta\gamma - \theta_0\right)\right], \tag{17}$$

where $d\theta/d\gamma$ characterizes the buncher strength and θ_0 for an overall phase shift.

Based on the reversion of series method [13], $\delta\gamma$ is ready to be expressed as a Fourier series,

$$\delta\gamma = \sum_{m=1}^{\infty} a_m \sin[m(\theta - \theta_0)] \equiv \sum_{m=1}^{\infty} a_m \sin(m\Theta), \tag{18}$$

where the Fourier coefficient is calculated as [8]

$$\begin{aligned} a_m &= \frac{1}{\pi}\int_{-\pi}^{\pi} \sum_{n=1}^{\infty} \frac{\Delta\gamma^n}{n!} \left.\frac{d^{n-1}\sin^n\left(-\frac{d\theta}{d\gamma}x + \Theta\right)}{dx^{n-1}}\right|_{x=0} \\ &\times \sin(m\Theta) d\Theta \\ &= \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{\Delta\gamma^n}{n!}\frac{1}{(2i)^{n+1}} \\ &\times \sum_{k=0}^{n} c_n^k (-)^{n-k}\left[i(2k-n)\left(-\frac{d\theta}{d\gamma}\right)\right]^{n-1} \\ &\times \left[\frac{e^{i(2k+m-n)\pi} - 1}{i(2k+m-n)} - \frac{e^{i(2k-m-n)\pi} - 1}{i(2k-m-n)}\right]. \end{aligned} \tag{19}$$

As mentioned above, assuming that the radiator is resonant at frequency ω_0 which is the l^{th}-harmonic of the first undulator–the modulator–fundamental frequency, then we can rewrite Eq. (19) according to Eqs. (8) and (9), we have

$$\left\{ \begin{array}{lcl} \omega_m &=& \frac{m\omega_0}{l} \\ \omega_{\eta_m} &=& \frac{m}{l} \\ \varepsilon_m &=& a_m \\ \eta_m &=& \frac{2a_m}{\gamma_0} \end{array} \right. . \tag{20}$$

Combining Eqs. (20) and (12), one can estimate the impact on the FEL bandwidth from the energy modulation generated in the modulator.

Initial Density Modulation

The initial energy modulation is described as in Eq. (18), yet there is also initial density modulation. Integrating the energy part in the distribution function in Eq. (17), we have the bunching factor introduced [4], *i.e.*,

$$\begin{aligned} \mathcal{F}(\Theta) &\equiv \int \delta\left[\delta\gamma - \Delta\gamma\sin\left(\Theta - \frac{d\theta}{d\gamma}\Delta\gamma\right)\right] d\delta\gamma \\ &= 1 + 2\sum_{n=1}^{\infty} J_n\left(-n\frac{d\theta}{d\gamma}\Delta\gamma\right)\cos(n\Theta) \\ &\equiv \mathcal{F}_0 + \sum_{n=1}^{\infty}\mathcal{F}_n\cos(n\Theta). \end{aligned} \tag{21}$$

Notice that, according to the definition in Eq. (7), the amplitude of the density modulation is twice of the bunching factor b_n at each harmonics, *i.e.*,

$$\begin{aligned} |\mathcal{F}_n| &= 2b_n \equiv 2\left|\frac{1}{2\pi}\int_{-\pi}^{\pi} d\Theta e^{-in\Theta}\right. \\ &\times \left.\int \delta\left[\delta\gamma - \Delta\gamma\sin\left(\Theta - \frac{d\theta}{d\gamma}\Delta\gamma\right)\right] d\delta\gamma\right|. \end{aligned} \tag{22}$$

In the HG FEL configuration, the radiator is resonant to a certain harmonic of the modulator radiation frequency. Assuming that the radiator is resonant at the m-th harmonic, then the quantity $B(\theta)$ introduced in Eq. (7) is

$$\begin{aligned} & B(\theta) \equiv \\ & \int e^{-im\Theta_{\text{mod}}}\delta\left[\delta\gamma - \Delta\gamma\sin\left(\Theta_{\text{mod}} - \frac{d\theta_{\text{mod}}}{d\gamma}\Delta\gamma\right)\right] d\delta\gamma \\ & = \frac{\mathcal{F}_m}{2} + \text{oscillating terms.} \end{aligned} \tag{23}$$

Therefore, $\tilde{B}(\theta, s)$ introduced in Eq. (6), is then

$$\tilde{B}(\theta, s) = \int_0^{\infty} e^{-sZ}\frac{\mathcal{F}_m}{2}dz = \frac{\mathcal{F}_m}{2s}, \tag{24}$$

with all the oscillating terms thrown away.

The FEL Solution For a pre-microbunched FEL, let us throw away the initial value term, the seed as well as the SASE term, and keep only the pre-microbunched term in Eq. (4).

$$f(\theta, s) \approx \frac{D_1}{\gamma_0} \int_{-\infty}^{\theta} d\theta' \tilde{B}(\theta', s) \tag{25}$$
$$\times e^{-s(\theta-\theta') + \frac{i(2\rho)^3(\theta-\theta')}{s^2} + \sum_{m=1}^{\infty} \frac{2\eta_m (2\rho)^3 [\cos(\omega_{\eta_m}\theta) - \cos(\omega_{\eta_m}\theta')]}{\omega_{\eta_m} s^3}}.$$

The inverse Laplace transform then gives us the FEL electric field slow-varying envelope function as

$$A(\theta, Z) = \int_c \frac{ds}{2\pi i} e^{sZ} f(\theta, s)$$
$$\approx \frac{D_1 \mathcal{F}_m}{2\gamma_0} \int_0^{\infty} d\xi \mathcal{H}(\theta, \xi, Z, s, \eta), \tag{26}$$

with the function $\mathcal{H}(\theta, \xi, Z, s, \eta)$ approximated as

$$\mathcal{H}(\theta, \xi, Z, s, \eta) \cong \frac{e^{-i\pi/12}}{2\sqrt{2\pi Z\rho}} e^{\mathcal{M}}, \tag{27}$$

with

$$\mathcal{M} = i^{1/3} 2\rho Z - i^{1/3} 9\rho \left(\xi - \frac{Z}{3}\right)^2 \frac{1 - \frac{i^{-1/3}}{4Z\rho}}{2Z} \tag{28}$$
$$-\sum_{m=1}^{\infty} i2\eta_m \left\{\cos(\omega_{\eta_m}\theta) - \cos[\omega_{\eta_m}(\theta - \xi)]\right\} \frac{1 - \frac{3i^{-1/3}}{4Z\rho}}{\omega_{\eta_m}}.$$

Discussion

As a conclusion, in this paper, we study the effect on a seeded FEL amplifier performance due to an initial energy non-uniformity when the electron bunch enters the undulator. Such non-uniformity can come from the RF curvature, the collective effect induced microbunching instability, and also generic energy modulation in a HG FEL. We then discuss the influence on the FEL due to the generic energy modulation in a HG FEL as well as the impact from an initial density local non-uniformity.

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