BEAM TEMPERATURE EFFECTS ON THE GROWTH RATE OF A TWO-STREAM FREE ELECTRON LASER

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Abstract

The effects of temperature on the growth rate of a two-stream free electron laser with planar wiggler magnetic pump have been investigated. The dispersion equation has been derived through the use of continuity, momentum transfer, and Maxwell's equations. In the analysis, only the longitudinal component of the pressure tensor is considered in the electron equation of motion. The characteristics of the dispersion relation along with the growth rate are analyzed numerically. The results show that the growth rate in this system is relatively higher than the conventional FEL; finally we compare our results with other cases, like without beam temperature, and conventional FEL.

INTRODUCTION

The free electron laser (FEL) is a device that generates coherent electromagnetic radiation using relativistic electron beams as an energy source. This is a tunable, high power radiation source. Since, the radiation wavelength varies with electron energy and wiggler wavelength, it can be continuously tuned in frequency [1]. The two principle type of scattering process which occur in FEL experiments are wave-particle (Compton) and wave-wave (Raman) scattering [2]. In the last type the beam-plasma frequency is sufficiently high so that the coupling between the electromagnetic wave and the two beam wave, i.e. negative and positive energy modes can be considered independently. Two stream FEL amplifiers and multiple-beam FEL have been proposed and investigated during the last decades [3-10], Kulish, Jacob and Bekefi, Mehdian *et.al*, and also Mc Niel, have been presented an investigation of TSFEL. Two parallel electron beams with different relativistic velocities support unstable, exponentially growing space-charge waves over a wide frequency range.

Organizing the manuscript is as follow: a leaner theory of the two-stream FEL with planer wiggler magnetic pump is presented and investigated. In section II, fundamental equations presented and employ the fluid theory we obtain the dispersion relation. In section III, numerical study and discussion is presented, and conclusion is given in last section.

LINEAR THEORY AND STABILITYANALYSIS

In this system, consider a transversely homogeneous, nonneutralized relativistic two electron beams with different relativistic velocity in the z direction, moving through a spatially uniform, externally imposed static magnetic field is taken to be

Where $k_w = 2\pi/\lambda_w$ is the wave number and λ_w is the

$$B_\perp = B_w \sin(k_w z) \qquad (1)$$

wiggler wave length, B_w is constant which measure the strength of the wiggler field, the choice of the linearly polarized wiggler induced longitudinal electron oscillations. To make the longitudinal excursions small compared to the radiation wavelength, and thus minimize the possibility of particle untrapping the wiggler amplitude B_w must be chosen so that ($\overline{\omega}_c < \sqrt{2}$) [6].

The orbit equation for an electron is given by

$$\frac{d\vec{p}_\alpha}{dt} = -e\left(\vec{E} + \frac{\vec{v}_\alpha \times \vec{B}}{c}\right) - \frac{1}{n}\frac{\partial \pi_\alpha}{\partial z} e_z \qquad (2)$$

Subscript α=1, 2, refer to the different quantity for two streams, c, is the speed of the light, π is the longitudinal part of the stress tensor. For simplicity, we have only retained the longitudinal component of the stress tensor in the equation of motion to provide the approximate thermal contribution.

The electron continuity equation,

$$\frac{\partial n_\alpha}{\partial t} + \vec{\nabla}.(n_\alpha \vec{v}_\alpha) = 0 \qquad (3)$$

The poison equation,

$$\vec{\nabla}.\vec{E} = -4\pi \sum_{\alpha=1}^{2} e n_\alpha \qquad (4)$$

And the wave equation for the transverse component of the vector potential $A_\perp$ of the RF fields is given by,

$$\left(\frac{\partial^2 A_\perp}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 A_\perp}{\partial t^2}\right) = -\frac{4\pi}{c}\sum_{\alpha=1}^{2} J_{\alpha\perp} \qquad (5)$$

For analyze the properties of two-stream free electron laser within the linearized fluid theory the electron fluid variables will be written in terms of an unperturbed part plus perturbation part in the following way:

$$n_\alpha = n_{0\alpha} + n_{1\alpha},\ J_{\alpha\perp} = J_{0\alpha\perp} + J_{1\alpha\perp} \tag{6}$$

It is assumed that the beam is sufficiently tenuous, and the system is assumed to be uniform in the transverse directions, so, the transverse canonical momentum of a particle is a constant of the motion, i.e.

$$P_{1\perp} = \frac{e}{c} A_{1\perp} \tag{7}$$

$$J_{1\alpha\perp} = -e(n_{0\alpha} v_{1\alpha\perp} + n_{1\alpha} v_{0\alpha\perp}) \tag{8}$$

Where, $P_{1\perp}$ is the total transverse momentum. Combining equations (8) and (5) and using the conservation of transverse canonical momentum and neglecting small terms in the expression for $J_{\alpha\perp}$, yields,

$$\left(c^2 \frac{\partial^2}{\partial z^2} - \frac{\partial^2}{\partial t^2} - \sum_{\alpha=1}^{2} \frac{\omega_{p\alpha}^2}{\gamma_{0\alpha}}\right)\left(\frac{eA_{1\perp}}{m_0 c}\right) \cong -\frac{\omega_c}{k_w c}\cos(k_w z) \sum_{\alpha=1}^{2} \frac{\omega^2_{p\alpha}}{\gamma_{0\alpha}} \frac{n_{1\alpha}}{n_{0\alpha}} \tag{9}$$

Where $\omega_c = \frac{eB_w}{m_0 c}$ and $\omega_{p\alpha} = \left(\frac{4\pi n_{0\alpha} e^2}{m_0}\right)^{1/2}$ are non-relativistic cyclotron frequency and nonrelativistic plasma frequency respectively.

A linearized particle conservation equation for each stream and Poisons equation respectively are,

$$\frac{\partial n_{1\alpha}}{\partial t} + v_{0\alpha} \frac{\partial n_{1\alpha}}{\partial z} + \frac{n_{0\alpha}}{m_0 \gamma_{0\alpha}^3} \frac{\partial p_{1z\alpha}}{\partial z} = 0 \tag{10}$$

$$\frac{\partial E_{1z}}{\partial z} = -4\pi e \sum n_{1\alpha} \tag{11}$$

And linearized axial component of momentum conservation equation is,

$$\left(\frac{\partial}{\partial t} + v_{0\alpha}\frac{\partial}{\partial z}\right) p_{1z\alpha} = -eE_{1z} + \frac{e\omega_c}{c\gamma_{0\alpha} k_w}\left(\cos(k_w z)\frac{\partial A_{1\perp}}{\partial z} - k_w A_{1\perp}\sin(k_w z)\right) - \sum_{\alpha=1}^{2} \frac{3K_B T_\alpha}{n_{0\alpha}} \frac{\partial n_{1\alpha}}{\partial z} \tag{12}$$

Since the equilibrium state is periodicity, Floquet's theorem requires that all perturbs quantities such as, $A_\perp, n_\alpha, p_{z\alpha}, E_z$ vary as,

$$Z = \sum_{m=-\infty}^{m=+\infty} Z(m) e^{i(k_m z - \omega t)}$$

$$k_m = k + m k_w, \qquad m = 0, \pm 1, \pm 2, \ldots \tag{14}$$

Where Z(m) represent the series expansion coefficients of the quantity Z, k_m *and* ω are, respectively, wave numbers and frequency of mixed wave (radiation and space charge). However, we are now concerned with the lowest order coupling of waves, and therefore only the first three terms, $m = 0, \pm 1$,will be retined.

Equation (9) through (12) forms a closed set of equations which describe the interaction of the relativistic electron beams with the external magnetic field. The complicated structure of the dispersion relation in Eq. (15) can be reduced by assuming $(\varepsilon(\omega, k - k_w))^{-1} \cong 0$ and $(\varepsilon(\omega, k + k_w))^{-1} \neq 0$, because resonance condition cannot occur simultaneously, so, the dispersion relation can be written as,

$$\begin{aligned}
&\left[\gamma_{01}^3(\omega - kv_{01})^2 - \omega_{p1}^2 - 3k^2 v_{th1}^2 + \frac{\omega_{p1}^2 \omega_c^2 k \mathrm{K}_1}{4k_w^2 \gamma_{01}^2 \varepsilon}\right] \\
&\times\left[\gamma_{02}^3(\omega - kv_{02})^2 - \omega_{p2}^2 - 3k^2 v_{th2}^2 + \frac{\omega_{p2}^2 \omega_c^2 k \mathrm{K}_2}{4k_w^2 \gamma_{02}^2 \varepsilon}\right] - \\
&\left[-\omega_{p1}^2 - 3\frac{n_{01}}{n_{02}} v_{th2}^2 k^2 + \frac{\omega_c^2 \omega_{p2}^2 n_{01} k \mathrm{K}_2}{4k_w^2 \gamma_{01}\gamma_{02} n_{02}\varepsilon}\right] \times \\
&\left[\omega_{p2}^2 + 3\frac{n_{02}}{n_{01}} v_{th1}^2 k^2 - \frac{\omega_c^2 \omega_{p1}^2 n_{02} k \mathrm{K}_1}{4k_w^2 \gamma_{01}\gamma_{02} n_{02}\varepsilon}\right] = 0
\end{aligned} \tag{15}$$

Where $\frac{1}{\varepsilon} = \frac{1}{\varepsilon(k - k_w)} + \frac{1}{\varepsilon(k + k_w)}$, and

$v_{th\alpha} = \left(\frac{2K_B T_\alpha}{m}\right)^{1/2}$ is the thermal velocity.

$$\varepsilon(\omega, k + mk_w) = \omega^2 - (k + mk_w)^2 c^2 - \sum_{\alpha=1}^{2} \frac{\omega^2_{p\alpha}}{\gamma_{0\alpha}}, \quad m = 0, \pm 1 \tag{16}$$

$$\mathrm{K}_\alpha = k + k_w\left(1 - \frac{1}{\gamma_{0\alpha}^2}\right) \tag{17}$$

Final form of the dispersion relation is:

$$\left[\varepsilon\left(\gamma_{01}^3(\omega - kv_{01})^2 - \omega_{p1}^2 - 3k^2 v_{th1}^2\right) + \frac{\omega_{p1}^2 \omega_c^2 k \mathrm{K}_1}{4k_w^2 \gamma_{01}^2}\right]$$

$$\times[\varepsilon(\gamma_{02}^{3}(\omega-kv_{02})^2-\omega_{p2}^{2}-3k^2v_{th2}^{2})+\frac{\omega_{p2}^{2}\omega_c^{2}k\mathrm{K}_2}{4k_w^{2}\gamma_{02}^{2}}]-$$

$$[\varepsilon(-\omega_{\mathrm{p1}}^{2}-3\frac{\mathrm{n}_{01}}{\mathrm{n}_{02}}\mathrm{v}_{\mathrm{th2}}^{2}\mathrm{k}^2)+\frac{\omega_c^{2}\omega_{p2}^{2}n_{01}k\mathrm{K}_2}{4k_w^{2}\gamma_{01}\gamma_{02}n_{02}}]\times$$

$$[\varepsilon(\omega_{\mathrm{p2}}^{2}+3\frac{\mathrm{n}_{02}}{\mathrm{n}_{01}}\mathrm{v}_{\mathrm{th1}}^{2}\mathrm{k}^2)-\frac{\omega_c^{2}\omega_{p1}^{2}n_{02}k\mathrm{K}_1}{4k_w^{2}\gamma_{01}\gamma_{02}n_{02}}]=0 \qquad (18)$$

Where

$$\varepsilon(\omega,k+k_w)=\omega^2-(\mathrm{k}+\mathrm{k_w})^2\mathrm{c}^2-\sum_{\alpha=1}^{2}\frac{\omega^2{}_{\mathrm{p}\alpha}}{\gamma_{0\alpha}} \qquad (19)$$

NUMERICAL STUDY AND DISCUSSION

A numerical study of the spatial growth rate in a two stream FEL with planar wiggler has been made. Dispersion relation is the eight order polynomial, and it shows that the plasma oscillations are coupled to the electromagnetic waves through the transversely magnetic field. All quantity become dimensionless with c, speed of light and k_w . In Fig. 1, dielectric function of the uncoupled electromagnetic wave frequency plotted versus wavenumber, there is two roots for each value of frequency for$\varpi > .2$. The variation of spatial growth rate (Im k) versus frequency for the lowest mode n=0 is shown in Fig. 2, Lorentz relativistic factor γ_0 was taken to be 5.6 and 5.65 for the first and second beams, respectively, and other quantity were taken to be[6], $B_w=2kG, k_w=4cm, \omega_{p1}=9.4\times10^9, \omega_{p2}=1.9\times10^{11}\sec^{-1}$,in the case of considering beam temperature, give rise to an enhancement in the growth rate and also the unstable regime become broader rather than cold beam, it means that the tunable frequency of two-stream FEL in this case is decreased.

The plots of spatial growth rate as a function of frequency for different value of $\overline{\omega_c}$ are given in Fig. 3, $a)\bar{v}_{th1}=.8\times10^{-1}, \bar{v}_{th2}=.87\times10^{-1}, \overline{\omega}_c=.84(--), \overline{\omega}_c=.94(-), b)\bar{v}_{th1}=\bar{v}_{th2}=0, \overline{\omega}_c=.84(..), \overline{\omega}_c=.94(-.)$,

the growth rate increase as strength of the wiggler field increase, and unstable spectrum instability become broader as wiggler field strength increase, because relativistic Doppler shift frequency just depend on energy beams and wiggler wave length i.e. $\omega\cong2\gamma_{0\alpha}^{2}k_wc$. For different value of initial beam energy,
$a)\bar{v}_{th1}=.8\times10^{-1}, \bar{v}_{th2}=.87\times10^{-1}, \gamma_{01}=5.6$
$\gamma_{02}=5.65(..), \gamma_{01}=5.7, \gamma_{02}=5.75(-.)$
$b)\bar{v}_{th1}=\bar{v}_{th2}=0, \gamma_{01}=5.6, \gamma_{02}=5.65(--), \gamma_{01}=5.7, \gamma_{02}=5.75(-)$, spatial growth rate as a function of frequency is shown in Fig. 4, the growth rate and the width of the unstable spectrum decrease with beam energies.

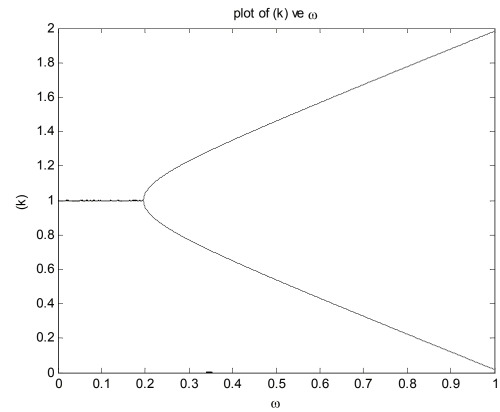

Figure 1: plot frequency versus k, for Dielectric coefficient $\gamma_1=5.6, \gamma_2=5.65, \overline{\omega}_{p1}=0.2, \overline{\omega}_{p2}=0.42$,

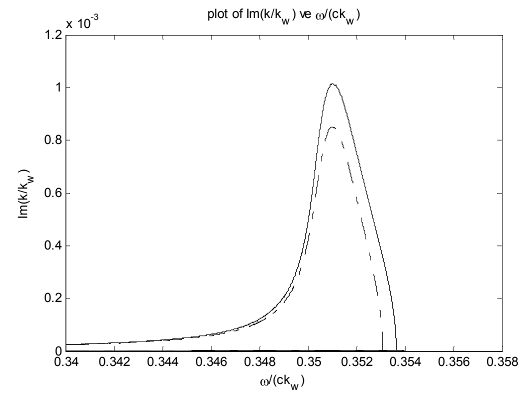

Figure 2: Growth rate for electrostatic and electromagnetic wave for the case, $\bar{v}_{th\alpha}=0(-.), \bar{v}_{th1}=.8\times10^{-1}$, $\bar{v}_{th2}=.87\times10^{-1}(-)$, $\gamma_{01}=5.6, \gamma_{02}=5.65, \overline{\omega}_c=.84$

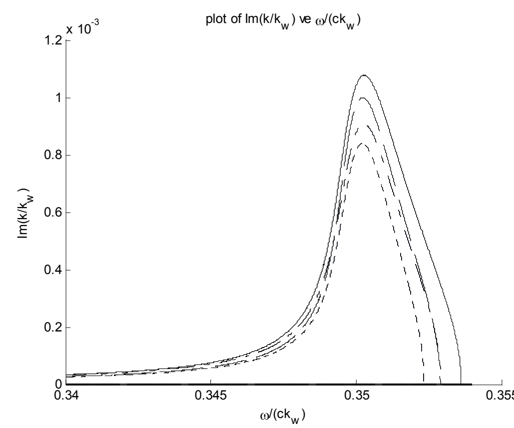

Figure 3: Spatial growth rate for electrostatic and electrom-agnetic wave for the case,

$a)\bar{v}_{th1}=.8\times10^{-1}, \bar{v}_{th2}=.87\times10^{-1}, \overline{\omega}_c=.84(--)$, $\overline{\omega}_c=.94(-)$

$b)\bar{v}_{th1}=\bar{v}_{th2}=0, \overline{\omega}_c=.84(..), \overline{\omega}_c=.94(-.), \gamma_{01}$ $\gamma_{01}=5.6, \gamma_{02}=5.65$

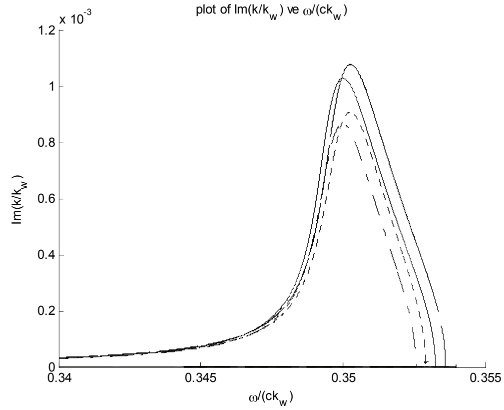

Figure 4: Spatial growth rate for electrostatic and electrom-agnetic wave for the case,
$a)\bar{v}_{th1} = .8 \times 10^{-1}, \bar{v}_{th2} = .87 \times 10^{-1}$, $\gamma_{01} = 5.6$,
$\gamma_{02} = 5.65(..)$, $\gamma_{01} = 5.7, \gamma_{02} = 5.75(-.)$
$b)\bar{v}_{th1} = \bar{v}_{th2} = 0, \gamma_{01} = 5.6, \gamma_{02} = 5.65(--)$
$\gamma_{01} = 5.7, \gamma_{02} = 5.75(-)$

CONCLUSION

In this paper we investigated the two-stream free electron laser with planar wiggler pump. Dispersion relation has been obtained with employ linear fluid theory and also spatial growth rate has been analyzed through the numerical results. It is found that: 1) growth rate is enhanced for warm beams and operating frequency become wider rather than the cold beams, 2) and growth rate increase when strength magnetic wiggler field increased, 3) growth rate and operation frequency decrease as initial energy increased.

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