USING THE LONGITUDINAL SPACE CHARGE INSTABILITY FOR GENERATION OF VUV AND X-RAY RADIATION

E.A. Schneidmiller and M.V. Yurkov
Deutsches Elektronen-Synchrotron (DESY), Hamburg, Germany

Abstract

Longitudinal space charge (LSC) driven microbunching instability in electron beam formation systems of X-ray FELs is a recently discovered effect hampering beam instrumentation and FEL operation. The instability was observed in different facilities in infrared and visible wavelength ranges. In this paper we propose to use such an instability for generation of vacuum ultraviolet (VUV) and X-ray radiation. A typical longitudinal space charge amplifier (LSCA) consists of few amplification cascades (drift space plus chicane) with a short undulator behind the last cascade. If the amplifier starts up from the shot noise, the amplified density modulation has a wide band, on the order of unity. The bandwidth of the radiation within the central cone is given by inverse number of undulator periods. A wavelength compression could be an attractive option for LSCA since the process is broadband, and a high compression stability is not required. LSCA can be used as a cheap addition to the existing or planned short-wavelength FELs. In particular, it can produce the second color for a pump-probe experiment. It is also possible to generate attosecond pulses in the VUV and X-ray regimes. Some user experiments can profit from a relatively large bandwidth of the radiation, and this is easy to obtain in LSCA scheme. Finally, since the amplification mechanism is broadband and robust, LSCA can be an interesting alternative to self-amplified spontaneous emission free electron laser (SASE FEL) in the case of using laser-plasma accelerators as drivers of light sources.

INTRODUCTION

Longitudinal space charge (LSC) driven microbunching instability [1, 2] in electron linacs with bunch compressors (used as drivers of short wavelength FELs) was a subject of intense theoretical and experimental studies during last years [3, 4, 5, 6, 7, 8, 9]. Such instability develops in infrared and visible wavelength ranges and can hamper electron beam diagnostics and FEL operation. Here we propose to use this effect for generation of VUV and X-ray radiation (see also [10] for more details).

GENERIC LSC AMPLIFIER

Scheme of an LSCA

Let us consider a scheme presented at Fig. 1. An amplification cascade consists of a focusing channel and a dispersive element (usually a chicane) with an optimized momentum compaction R_{56}. In a channel the energy modulations are accumulated, that are proportional to density modulations and space charge impedance of the drift space. In the chicane these energy modulations are converted into induced density modulations that are much larger that initial ones [1], the ratio defines a gain per cascade. In this paper we will mainly consider the case when the amplification starts up from the shot noise in the electron beam - although, in principle, the coherent density modulations can be amplified in the same way. A number of cascades is defined by the condition that the total gain, given by the product of partial gains in each cascade, is sufficient for saturation (density modulation on the order of unity) after the start up from shot noise. As we will see, in most cases two to four cascades would be sufficient. The amplified density modulation has a large relative bandwidth, typically in the range 50-100 %. Behind the last cascade a radiator undulator is installed, which produces a powerful radiation with a relatively narrow line (inverse number of periods) within the central cone. This radiation is transversely coherent, and the longitudinal coherence length is given by the product of the number of undulator periods by the radiation wavelength. When LSCA saturates in the last cascade, a typical enhancement of the radiation intensity over that of spontaneous emission is given by a number of electrons per wavelength.

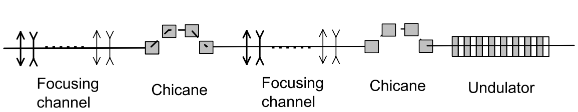

Figure 1: Conceptual scheme of an LSC amplifier.

Formula for a Gain per Cascade

Let us now present simple formulas for calculations of the gain and optimization of parameters of an LSCA. We assume that at the entrance to the amplifier there is only shot noise in the electron beam. Let us consider the linear amplification of spectral components of the noise within the amplifier band. The formula for amplitude gain per cascade was obtained in studies of microbunching instabilities in linacs with bunch compressors [1]:

$$G_n = Ck|R_{56}|\frac{I}{\gamma I_{\mathrm{A}}}\frac{4\pi|Z(k)|L_d}{Z_0}\exp\left(-\frac{C^2k^2R_{56}^2\sigma_\gamma^2}{2\gamma^2}\right) \quad (1)$$

Here $k = 1/\lambda\!\!\!^{-} = 2\pi/\lambda$ is the modulation wavenumber, Z is the impedance of a drift space (per unit length), Z_0 is the free-space impedance, L_d is the length of the drift space, I is the beam current, I_{A} is the Alfven current, R_{56} is the compaction factor of a chicane, C is the compression factor, γ is relativistic factor, and σ_γ is rms uncorrelated energy spread (in units of rest energy). It is assumed here that energy modulations are accumulated upstream of the chicane but not inside: there the self-interaction is suppressed due to R_{51} and R_{52} effects [11, 12, 13]. We also assume in the following that a length of the drift space is much larger than that of the chicane, while the R_{56} of the drift space is much smaller. Also note that the formula (1) was obtained under the condition of high gain, $G \gg 1$. In this case both the sign of R_{56} and the phase of impedance are not important. The value of the R_{56} is to be optimized for a highest gain at a desired wavelength.

In this paper we will consider LSC induced energy modulations in a drift space or in an undulator (at a wavelength that is much longer than the resonant one). In the latter case the relativistic factor γ in formulas for impedance should be substituted by the longitudinal relativistic factor γ_z [14].

We request that a density modulation does not change significantly in the drift space. This may happen due to plasma oscillations or due to a spread of longitudinal velocities for a finite beam emittance. Thus, a drift space length is limited by the condition: $L_d \le min(L_1, L_2)$. Here L_1 is the reduced wavelength $\lambda\!\!\!^{-}_p$ of plasma oscillations:

$$L_1 \simeq \lambda\!\!\!^{-}_p = \gamma_z \left(\frac{I}{\gamma I_{\rm A}} \frac{4\pi|Z|k}{Z_0} \right)^{-1/2} . \tag{2}$$

The second limitation follows from the condition that the longitudinal velocity spread due to a finite beam emittance does not spoil the modulations during the passage of the drift:

$$L_2 \simeq \frac{\lambda\!\!\!^{-}}{\sigma_\theta^2} = \frac{\beta \lambda\!\!\!^{-}}{\epsilon} , \tag{3}$$

where σ_θ is the angular spread in the beam, β is the beta function, $\epsilon = \epsilon_n/\gamma$ is beam emittance, ϵ_n is the normalized emittance.

Formulas for an Optimized LSC Amplifier

Let us first consider the case without wavelength compression, $C = 1$. We start optimization assuming that the beam parameters are fixed: current I, normalized emittance ϵ_n, beam energy γ and energy spread σ_γ in units of the rest energy, and longitudinal gamma-factor γ_z. We can select a central wavelength, optimize R_{56} of the dispersion section for the chosen wavelength, choose beta-function and optimize a length of the drift space. Our goal is to get a highest gain at a shortest wavelength.

The impedance increases with k, achieves maximum at

$$\lambda\!\!\!^{-} \simeq \lambda\!\!\!^{-}_{opt} \simeq \frac{\sigma_\perp}{\gamma_z} = \frac{\sqrt{\epsilon \beta}}{\gamma_z} , \tag{4}$$

and then decays in the asymptote of a pancake beam. Let us consider the wavelength about $2\pi \lambda\!\!\!^{-}_{opt}$ as an optimum choice, since the impedance is the largest, and transverse correlations of the LSC field are still on the order of the beam size. The impedance at this wavelength can be approximated by

$$\frac{4\pi|Z|}{Z_0} \simeq \frac{1}{\lambda\!\!\!^{-} \gamma_z^2} \simeq \frac{1}{\sigma_\perp \gamma_z} . \tag{5}$$

The optimal R_{56} of the dispersion section is

$$R_{56} \simeq \lambda\!\!\!^{-} \frac{\gamma}{\sigma_\gamma} . \tag{6}$$

Substituting (5) and (6) into (1), we get an estimate of the amplitude gain per cascade for the wavelength given by (4):

$$G_n \simeq \frac{I}{\sigma_\gamma I_{\rm A}} \frac{L_d}{\lambda\!\!\!^{-} \gamma_z^2} . \tag{7}$$

Thus, the gain per cascade is approximately equal to the longitudinal brightness of the electron beam multiplied by a number of LSC formation lengths.

Let us now consider the limitations on the drift length. The first limit (2) can be rewritten with the help of (5) as:

$$L_1 \simeq \lambda\!\!\!^{-}_p \simeq \lambda\!\!\!^{-} \gamma_z^2 \sqrt{\frac{\gamma I_{\rm A}}{I}} . \tag{8}$$

In the case $L_1 < L_2$ the drift space length can be chosen to be $L_d \simeq L_1$. In this case the expression for the gain (7) reduces to:

$$G_n \simeq \frac{1}{\sigma_\gamma} \sqrt{\frac{\gamma I}{I_{\rm A}}} . \tag{9}$$

This estimate for the gain was originally obtained in [1]. In the considered limit we have a relatively large beta-function. It is advisable to reduce it (if technically possible), since the wavelength (4) and length of the drift space (8) are also reduced but the gain (9) stays the same. This happens until the spread of longitudinal velocities starts playing a role, i.e. when $L_1 \simeq L_2$. The corresponding beta-function is

$$\beta_{cr} \simeq \epsilon \gamma_z^2 \sqrt{\frac{\gamma I_{\rm A}}{I}} . \tag{10}$$

If one further reduces beta-function, $\beta < \beta_{cr}$, the maximal drift length is given by $L_d \simeq L_2$. In this case from (3), (4), and (7) we find that the gain is proportional to the beam brightness in 6-D phase space:

$$G_n \simeq \frac{I}{\sigma_\gamma I_{\rm A}} \frac{\beta}{\epsilon \gamma_z^2} \simeq \frac{I}{\sigma_\gamma I_{\rm A}} \left(\frac{\lambda\!\!\!^{-}}{\epsilon} \right)^2 . \tag{11}$$

Although the gain can still be high for $\lambda\!\!\!^{-} \gg \epsilon$, it quickly decreases when one goes to shorter wavelengths - contrary to the case (9). Thus, the condition $L_d \simeq L_1 \simeq L_2$ (and $\beta \simeq \beta_{cr}$) allows one to get the highest gain at the shortest wavelength. At this point we have:

$$\lambda\!\!\!^{-} \simeq \epsilon \left(\frac{\gamma I_{\rm A}}{I} \right)^{1/4} , \quad L_d \simeq \epsilon \gamma_z^2 \left(\frac{\gamma I_{\rm A}}{I} \right)^{3/4} .$$

The amplitude gain per cascade is given by (9), the beta function is given by (10), and the R_{56} is given by (6).

To estimate the total gain (and number of cascades) required to reach saturation in LSCA, one has to estimate typical density modulation for the start-up from shot noise. The power spectral gain of the amplifier depends on the number of cascades n. For an optimal wavelength (4) as a central wavelength (neglecting the dependence of the impedance on k), and for the optimized R_{56} from (6), one easily obtains from (1) that the total power gain is proportional to $\hat{k}^{2n} \exp(-n\hat{k}^2)$, where $\hat{k} = k/k_{opt}$. Thus, the relative bandwidth of the amplifier is in the range 50-100 %, depending on the number of cascades. Then an effective shot noise density modulation can be estimated as [11]:

$$\rho_{sh} \simeq \frac{1}{\sqrt{N_\lambda}} , \tag{12}$$

where $N_\lambda = I\lambda/(ec)$ is a number of electrons per central wavelength of the amplified spectrum, $\lambda = 2\pi \lambda\!\!\!^{-}$. At saturation the density modulation ρ_{sat} is on the order of unity, so that the total amplitude gain ρ_{sat}/ρ_{sh} is:

$$G_{tot} = G_1 G_2 ... G_n \simeq \sqrt{N_\lambda} , \tag{13}$$

where G_n is the gain in the n-th cascade. The power gain of the saturated amplifier (showing enhancement of power in a radiator with respect to spontaneous emission) is: $G_{tot}^{(p)} = G_{tot}^2 \simeq N_\lambda$.

Wavelength Compression

As one can see from the formulas of the optimized LSCA, a typical operating wavelength of an optimized LSCA is significantly longer than a wavelength that can be reached in SASE FELs (they can lase at $\lambda\!\!\!^{-} \simeq \epsilon$). In order to go to shorter wavelengths for given electron beam parameters in LSCA, one would have to use wavelength compression. The broadband nature of the amplifier makes this option especially attractive. For coherent

FEL-type modulations and an undulator as a radiator, the tolerance for the chirp stability is tight what limits practically achievable compression factors. For an LSCA, however, one can go for much stronger compression. Alternatively, for a given compression factor one can significantly loosen the tolerances. Note also that nonlinearities of the longitudinal phase space do not play a significant role in the case of LSCA.

Undulator

At the entrance of the undulator we have chaotically modulated electron beam with a typical amplitude of the order of unity at saturation. The temporal correlations have the scale of a wavelength, and the spectrum is broad. The undulator radiation within the central cone $\sqrt{\lambda/L_w}$ (here L_w is the undulator length) has a relative bandwidth $N_w^{-1} \ll 1$, where N_w is the number of undulator periods. In the case when Fresnel number is small, $N_f = \sigma_\perp^2/(\lambdabar L_w) \ll 1$, the radiation power within the central cone is equal to the power of spontaneous emission multiplied by the power gain N_λ of the LSCA at saturation:

$$W \simeq W_{sp}\, N_\lambda \; . \tag{14}$$

In this limit the power within the central cone does not depend on the number of undulator periods. One can easily see that the Fresnel number is always small if the condition (4) is satisfied, transverse size of the beam in the undulator is the same as that in amplification cascades, and there is no wavelength compression. In this case the transverse coherence is guaranteed. We do not discuss here nonlinear harmonic generation in LSCA, since it would be very speculative without numerical simulations. We can only mention here that this should be possible in a saturated LSCA.

POSSIBLE APPLICATIONS OF LSCA

LSCA as a Cheap Addition to Existing or Planned X-ray FELs

Undulator beamlines of the existing and planned X-ray FELs often consist of long drift spaces and long undulators. Insertion of a few chicanes and a short undulator at the end may allow for a parasitic production of relatively long wavelength radiation (as compared with the FEL wavelength) by the same electron bunch. This would extend in an inexpensive way the wavelength range of a facility. Moreover, since both radiation pulses are perfectly synchronized, they can be used in pump-probe experiments.

As a first example let us consider the undulator beamline SASE1 at the European XFEL [15]. There is a long drift space (about 300 m) in front of SASE1 undulator, and 200 m long drift behind the undulator. The undulator itself has the total length of 200 m (magnetic length 165 m plus 35 meters of intersections). Let us consider the electron beam with the following parameters: energy 17.5 GeV, normalized slice emittance 0.4 mm mrad, peak current 3-4 kA, slice energy spread 1.5 MeV. The tunable-gap undulator is assumed to be tuned to the resonance with the wavelength 0.05 nm, so that $\gamma_z = 1.9 \times 10^4$. The optimal (for FEL operation) beta-function in the undulator for these beam parameters is about 15 m, and it is about 30-40 m in the drifts. The core of the bunch with high current saturates at the FEL wavelength in the undulator, so that this part of the bunch is spoiled (has a large energy spread). We consider parts of the bunch with the current about 1 kA assuming that there is no FEL saturation there. We propose to install three compact chicanes just in front of the undulator, just behind it, and at the end of the second drift. Thus, we have three amplification cascades of LSCA that operates parasitically. The last chicane is followed by a short undulator. From the formulas of the previous Section we find that the optimal wavelength for LSC instability is $\lambda \simeq 4$ nm. The optimal R_{56} is about 8 μm for all cascades. Beta-function in all cascades is much larger than β_{cr}, moreover the lengths of all cascades are shorter than reduced wavelength of plasma oscillations, i.e. $L_d < L_1$. Therefore, we use formula (7) to calculate gain in every cascade. We find that the total gain is given by the following product of partial gains: $G_{tot} \simeq 8 \times 13 \times 5 \simeq 500$. This is larger than the gain required to reach saturation, about 300 according to (13). We choose an undulator with 50 periods and a period length 10 cm. Radiation power within the central cone exceeds that of spontaneous emission by 5 orders of magnitude and is in sub-GW level with the bandwidth about 2 %, radiation is transversely coherent. The tunability can be easily achieved in the range of 2-10 nm by changing the R_{56} and the undulator gap. The soft X-ray pulses are synchronized with hard X-ray pulses produced by the core of the same bunch, so that these two colors can be used in pump-probe experiments. Alternatively, they can be separated and used by different experiments[1].

Note that in this example we considered a parasitic use of the beamline and of an unspoiled part of the electron bunch. With a dedicated use of the high-current part of the bunch one can essentially reduce the total length of the amplifier. Let us consider the same electron bunch as before, but now we assume that the core of the bunch with the current 3 kA is not spoiled by FEL interaction (for instance, some bunches are kicked in front of the undulator by the fast kicker). We consider an operation of LSCA in the drift behind the undulator, requiring beta-function to be about 10 m (somewhat larger that β_{cr}), thus the optimal wavelength is 2 nm. Choosing length of the drift in an amplification cascade to be 30 m (it is much smaller than $\lambdabar_p$), and the $R_{56} \simeq 4$ μm, we find with the help of (7) that the gain per cascade is about 5. To reach saturation one would need four cascades, so that the total length of the amplifier would be about 120 m. A gigawatt-level radiation power would then be produced within the central cone of a short undulator, tunability between 1 nm and 5 nm is easy to obtain.

Generation of Attosecond Pulses

There are many proposals to produce attosecond pulses from FELs [16, 17, 18, 19, 20, 21, 22]. In principle, by using strongly nonlinear manipulations with the longitudinal phase space, one can reduce X-ray pulse duration down to several cycles [22]. Here we note that the broadband nature of the LSC instability suggests that few-cycle pulses can be naturally produced in LSCA.

As an example, let us consider the electron beam with the following parameters: the energy 1.5 GeV, bunch charge 200 pC, peak current 1 kA, normalized emittance 0.5 μm, rms energy spread 100 keV ($\sigma_\gamma = 0.2$). We choose beta-function to be 1.5 m, i.e. much larger than the value one gets from (10). From (4) we get $\lambdabar \simeq 5$ nm, i.e. the wavelength λ is about 30 nm. The optimal R_{56} for this wavelength is about 75 μm, the chicane can be as short as 50 cm. We choose the length of a drift space $L_d = 1.5$ m. One can find from (9) that the amplitude gain per cascade is

[1] As an option one can consider the bending system (with properly adjusted R_{56}) between SASE1 and the downstream soft X-ray undulator SASE3 [15] as an alternative to the last chicane. Then the short undulator is placed in SASE3 beamline thus extending its wavelength range.

about 10, so we need 3 cascades for saturation according to (13).

Now let us introduce wavelength compression into the scheme. This can be done in a way similar to that considered in [22]. In front of the last chicane we install a two-period undulator (period length 10 cm) which is resonant with the wavelength of 750 nm of a laser producing pulses with energy of 1 mJ and duration of 5 fs (FWHM). The maximum of energy modulation can be about 15 MeV and should be adjusted for a desired compression. Note that one can in-couple the laser beam at the position of the previous chicane where the electron beam deviates from the axis by a few millimeters. Consider as an example the compression of a short slice, having the maximal energy slope, by a factor of 7.5, i.e. from 30 nm to 4 nm. According to (1), the optimal R_{56} for the last chicane is by the compression factor smaller than that of the previous chicanes, i.e. it is 10 μm. In this case the gain per cascade is the same, i.e. it is about 10 in our example for the compressed slice, what is sufficient for saturation. The current in this slice has increased by a factor of 7.5, and the width of the generated current spike is about 100 nm. The radiator undulator has 5-10 periods with the period length of 3 cm. The radiated power is estimated at sub-gigawatt level with the pulse duration about 100 as. One should also notice that total length of the system is rather small, less than 10 m.

LSCA Driven by a Laser-plasma Accelerator

The technology of laser-plasma accelerators progresses well [23], a GeV beam is already obtained [24]. The electron beam with the energy about 200 MeV was sent through the undulator, and spontaneous undulator radiation at 18 nm wavelength was obtained [25]. The VUV and X-ray FELs driven by these accelerators are proposed [26, 27]. However, it is not clear at the moment if tight requirements on electron beam parameters and their stability, overall accuracy of the system performance etc., could be achieved in the next years.

Contrary to FELs, the amplification mechanism of LSCA is very robust. For example, it can tolerate large energy chirps. In the case of an FEL the energy chirp parameter is $\lambdabar h/\rho^2$, where h is the energy chirp and ρ is the FEL parameter [29]. The energy chirp parameter should be small as compared to unity in order to not affect FEL gain. Contrary to that, mechanism of LSCA is broadband, so that "effective ρ" is on the order of one. In other words, in a drift space the influence of the chirp can be always neglected. Of course, if one would like to avoid compression (decompression) in chicanes of LSCA, one should require $hR_{56} \ll 1$. If the R_{56} is chosen according to (6), then the condition for the chirp can be formulated as $h\lambdabar \ll \sigma_\gamma/\gamma$.

In an FEL there are stringent requirements on straightness of the trajectory: the electron beam must overlap with radiation over a long distance. In the case of LSCA one should only require that the angles of the electron orbit should be smaller that $\lambdabar/\sigma_\perp$ what means for the optimal wavelength γ_z^{-1}.

One can speculate (since some important parameters of beams have never been measured) that LSCA could be an interesting alternative to FELs, at least as the first step towards building light sources based on laser-plasma accelerators. One of the most important unknown parameters is the slice energy spread (slice size is given by a typical wavelength amplified in LSCA), since the measured value is usually a projected energy spread, dominated by an energy chirp along the bunch. An interesting option would be to use an energy chirp, induced by LSC and wakefields over the whole bunch [28, 26], for the wavelength compression as discussed in Section 2. Taking into account the sign of the energy chirp, one should use, for instance doglegs instead of chicanes. One can also consider LSCA as a preamplifier (making sure that it does not saturate) with the final amplification in an FEL.

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