FREE ELECTRON LASER AND POSITRONIUM STIMULATED ANNIHILATION

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Abstract

In this contribution we combine concepts from different fields to show that the stimulated annihilation of positronium could be technologically achievable in the next future, providing a source of gamma rays to to be exploited for a wealth of applications. We analyze the feasibility of such a device by developing a preliminary design of an electron-positron recombination device for the generation of a "gamma ray laser".

INTRODUCTION

More than three decades ago in a few seminal papers Rivlin [1, 2, 3] specified the conditions underlying the process of stimulated annihilation of positronium. Even though there were no physical reasons forbidding this process, the prohibitive requirements on the production of the positronium and the stimulating laser exceeded the technologies available at that time.

Here we will show that the next generation of Accelerators [4], Laser [5]s, Free Electron Lasers [6] and charged beam production and handling [11, 12] seems to provide a possible environment for such a challenging enterprise.

The investigation started by Rivlin aimed at the possibility of realizing a laser using positronium atoms and was motivated essentially by the following points:

1. since electron-positron bound states are significantly lighter than ordinary atoms, they are more easily formed at relativistic energies;
2. the radiation emitted by such hydrogen-like systems turn to be up-shifted in the lab frame towards much larger frequencies;
3. the energy of the positronium can be tuned in order that the photon from an external laser works in the positronium center of mass as a gamma ray with exactly the energy required to induce the process of stimulated annihilation.

Regarding the point 2 we note that the energy $E = \hbar\omega_0$ of a laser photon in the lab frame is up-shifted to $E' = 2\gamma\hbar\omega_0$ in the frame of a positronium system moving with a relativistic factor γ. Assuming that this value is resonant with one of the transition channels of the system (stimulated radiative decay or annihilation), it is easily seen that the stimulated radiation has, in the laboratory frame, an energy equal to $E" = 4\gamma^2\hbar\omega_0$.

As to the point 3 we note that the stimulated two-photon annihilation is compatible with the laws of conservation of energy and momentum, only when the relative motion is such that the photon frequency in the e^-e^+ rest frame is exactly $\frac{mc^2}{\hbar}$. More precisely in the case of counter-propagating laser and positronium beams the following condition should be satisfied:

$$\frac{mc^2}{\hbar\omega_0} = \gamma + \sqrt{\gamma^2 - 1} \simeq 2\gamma \tag{1}$$

Which can also be cast in the form

$$\gamma \simeq \frac{\lambda\!\!\!^{-}_0}{2\lambda\!\!\!^{-}_c} \tag{2}$$

where $\lambda\!\!\!^{-}_c = \frac{\hbar}{mc}$ is the reduced Compton wave-length. The condition (2) is e. g. satisfied using a laser beam of $20\,\mathrm{nm}$ and an electron energy not exceeding $2.5\,\mathrm{GeV}$. Lasers within the above range of wave-lengths are within the future capabilities of a SASE FEL.

Several schemes have been considered for the production of positrons, usually based on e^-e^+ pair production by γ in high density material slabs [7]. Proposed layouts for incoherent γ production are based on a variety of mechanisms, ranging from Compton back-scattering of electron beams off the radiation from Optical Klystron FELs [8, 9], lasers [10] or virtual quanta in pulsed undulators [7]. The foreseen achievable γ intensity may vary over a wide interval, depending on beam parameters, ranging from $10^{11} \div 10^{12}\gamma/s$ in Refs. [8, 9] to $3.5\cdot 10^9 \div 1.4\cdot 10^{10}\gamma/\mathrm{s}$ in Ref. [7] ($0.35\gamma/e@10\mathrm{Hz}@1 - 4\cdot 10^9\mathrm{e/pulse}@46GeV$). Pair conversion material slab is accompanied by parasite processes (Compton, Bhabha and Møller scattering, photoelectric effect, bremsstrahlung) that greatly affect e^+ final (collection) efficiency. Ref. [7] quotes a collection efficiency of $8.5\cdot 10^{-3} e^-e^+\ \mathrm{pair/incident}\ \gamma$ against a primary (conversion) efficiency of $0.105\ e^-e^+\ \mathrm{pair/incident}\ \gamma$. According to these results, maximum achievable positronium rate must be scaled at least by two order of magnitude with respect to the available γ intensity.

The main purpose of our proposal is the production of high energy positronium atoms at high rate. To this aim, we consider the scheme of Fig. 1, where we have reported the so called Kayak-Paddle Cooler (KPC) for the production of positronium [11, 12].

This type of device has been conceived to cool the electrons/positrons for the production of positronium atoms, with a rate ranging between $10^{-8}\mathrm{s}^{-1}$ (para-positronium) and $10^{-10}\mathrm{s}^{-1}$ (ortho-positronium). In this scheme the

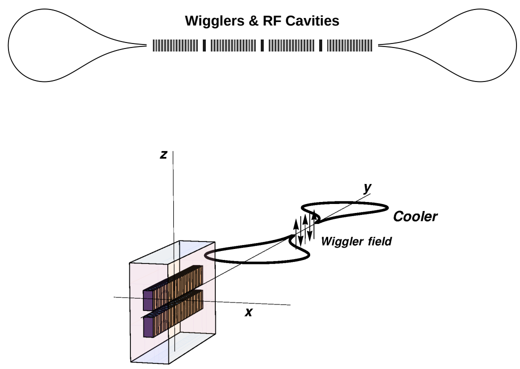

Figure 1: "Kayak Paddle Cooler" (KPC) scheme.

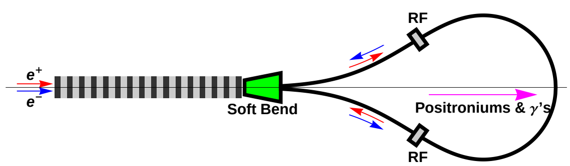

Figure 2: Positronium atoms formation section.

magnetic wigglers and the RF cavities are used to cool the beams of electrons and positrons providing the energetic and kinematic conditions suitable for the formation of positronium atoms. Postponing to the second part of the paper the discussion on the conditions to be met for the desired operation of KPC, we note that the positronium formation occurs after the electron and positron beams have been cooled enough for quantum effects to play an important role in the process dynamics. The e^-e^+ pairs form positronium states (see Fig. 2) in the last undulator, before the soft bend used to separate electrons and positrons failing to bind together, which are re-injected into the system and eventually pass once again through the cooling process.

In the following we consider some modifications to purpose of generating coherent gamma radiation. A first refinement we propose to the KPC scheme is the addition of two counter-propagating lasers along the e^-e^+ beam line. Using the Free Electron Laser (FEL) analogy, the EM field moving toward the charged beams plays the role of the undulator, while the co-propagating laser is to be considered as the stimulating field.

The energy of this last field is quasi resonant with the photon energy the beams radiate individually in the "undulator", namely

$$\begin{aligned} e_{\mathrm{f}} &= \hbar \frac{2\pi c}{\lambda_{\mathrm{f}}} \\ \lambda_{\mathrm{f}} &= \frac{\lambda_{\mathrm{u}}}{2\gamma^2}\left(1+\frac{K^2}{2}\right) \end{aligned} \tag{3}$$

The system parameters are furthermore arranged so that co-propagating photons energy equals, in the beams reference frame, the positronium ionization potential. In this way charged particles energy loss triggers, via a FEL-like mechanism, the formation of positronium. This process, even though not strictly necessary, should act as a further cooling element enhancing the stimulated radiative formation of the bound system.

In order to describe further evolution of the bound system, a clear distinction must be made between para-positronium and ortho-positronium.

In the last case, life time is $\tau \approx 1.4 \cdot 10^{-7}$s and annihilation yields three photons, whose energies can be parametrized in the beam reference frame as follows:

$$\begin{aligned} \hbar\omega &= \chi mc^2 \\ \hbar\omega_1 &= (1+\eta_1)\, mc^2 \\ \hbar\omega_2 &= (1+\eta_2)\, mc^2 \end{aligned} \tag{4}$$

With the above notation, introduced by Rivlin [1] to treat this specific problem,the energy and momentum conservation writes

$$\begin{aligned} \chi + \eta_1 + \eta_2 &= \frac{v^2}{c^2} - \epsilon \\ \chi\hat{\kappa} + (1+\eta_1)\,\hat{\kappa}_1 + (1+\eta_2)\,\hat{\kappa}_2 &= 2\frac{\overrightarrow{v}}{c} \end{aligned} \tag{5}$$

We have denoted by $mc^2 \simeq 0.511\,\mathrm{MeV}$, $e_{\mathrm{I}} = \epsilon mc^2 \simeq 6.77\,\mathrm{eV}$ the electron/positron rest and positronium's ionization energies, respectively, and by $\hat{\kappa}, \hat{\kappa}_1, \hat{\kappa}_2$ the unit wave vectors associated with the three emitted photons.

Let us make now the assumption that the energy of one of the photons, involved in the annihilation is very small. Accordingly, if we take $\chi \ll 1$, it follows as well by momentum conservation that $\eta_1, \eta_2 \ll 1$ and we can also infer that the hard photons are emitted in almost opposite direction at a very small angle with respect to the line of flight. By setting indeed $\hat{\kappa}_2 = -\hat{\kappa}_1 + \overrightarrow{\delta}$ we find, since $v^2 \ll c^2, \epsilon \simeq 10^{-5}$, that

$$\left|\overrightarrow{\delta}\right| = \left|\frac{2\dfrac{\overrightarrow{v}}{c} - \chi\hat{\kappa} - (\eta_1 - \eta_2)\,\hat{\kappa}_1}{1+\eta_2}\right| \ll 1 \tag{6}$$

The hard components are therefore emitted (in the positronium rest frame) in opposite directions in a cone of aperture $\left|\overrightarrow{\delta}\right|$.

We have stressed that the energy of the photons, co-propagating with the electrons, be equivalent to the positronium ionization energy in its rest frame. If we denote by $e^\star = \epsilon^\star mc^2$ the energy of the photon in the laboratory frame, we obtain that such a condition is ensured whenever the relativistic factor of the two beams is given by

$$\gamma_p \simeq \frac{\epsilon^\star}{\epsilon} \tag{7}$$

The energy of the spontaneously scattered photons is required to be quasi resonant with the incident photons, therefore

$$\lambda_{\mathrm{f}} = \frac{\lambda_{\mathrm{u}}}{2}\left(\frac{\epsilon}{\epsilon^{\star}}\right)^2\left(1+\frac{K^2}{2}\right) \tag{8}$$

Since the photon wavelength and energy are related, in practical units by

$$\lambda\,[\mathrm{cm}] = \frac{1.24\cdot 10^{-7}}{e\,[\mathrm{keV}]}$$

then use of equation (8) yields the following condition, linking the undulator and external laser parameters

$$\lambda_u\,[\mathrm{cm}]\left(1+\frac{K^2}{2}\right) \simeq 2.48\cdot 10^{-7}\frac{e^{\star}\,[\mathrm{keV}]}{e_{\mathrm{I}}\,[\mathrm{keV}]^2} \tag{9}$$

By assuming that the laser photons have energies around $30\,\mathrm{KeV}$, we find from equations (7) and (9) the result

$$\lambda_{\mathrm{u}}\,[\mathrm{cm}]\left(1+\frac{K^2}{2}\right) \simeq 0.1 \tag{10}$$

and an energy of about $\gamma_p \simeq 4\cdot 10^3$. The undulator could therefore be provided by a microwave beam or a pulsed magneto-static undulator as described in Ref. [7], while the "stimulating" laser by a FEL operating around 0.5Å. Both devices are not far from the present technological capabilities.

So far the system has acted as a laser cooler, catalyzing the formation of positronium states (see Fig. 2); the next step is to understand how stimulated annihilation may be induced.

The probability of spontaneous emission of a soft photon in the energy interval $\Delta\chi$ and solid angle $\Delta\Omega$ is

$$W_{\mathrm{sp}} = \frac{f\,(\chi)\,\Delta\chi}{A\tau}\frac{\Delta\Omega}{4\pi} \tag{11}$$

where $f\,(\chi)$ (see Fig 3) is the spectral distribution calculated in Ref. [13]:

$$\begin{aligned} f[\chi] \;&=\; 2\left[\frac{\chi(1-\chi)}{(2-\chi)^2} - \frac{2(1-\chi)^2\log(1-\chi)}{(2-\chi)^3}\right. \\ &\qquad\left.\frac{2-\chi}{\chi} + \frac{2(1-\chi)\log(1-\chi)}{\chi^2}\right] \end{aligned}$$

and

$$A = \int_0^1 f\,(\chi)\,d\chi = \pi^2 - 9 \approx 0.87$$

is just a normalization factor while τ is the life time. In the presence of a stimulating EM field of power density Π, the annihilation rate is proportional to the number of photons present in the mode associated to the transition, namely

$$W_{\mathrm{st}} = \frac{f\,(\chi_0)\,\Pi}{F\,(\nu)\,A\tau} = 2\pi^2\frac{f\,(\chi_0)\,\Pi}{A\tau\omega_0^3} \tag{12}$$

where

$$\chi_0 = \frac{\hbar\omega_0}{mc^2} \overset{\hbar=c=1}{=} \frac{\omega_0}{m}$$

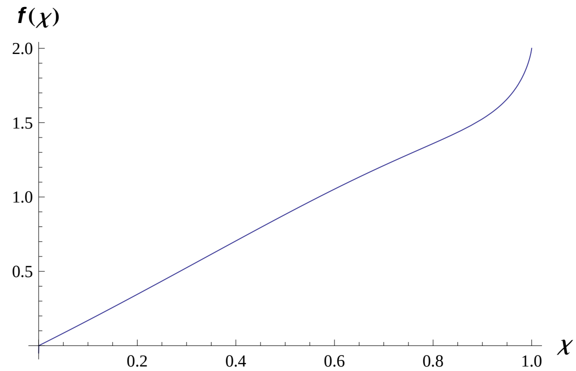

Figure 3: Spectral distribution for the spontaneous emission of a soft photon.

while function

$$F\,(\nu) = \frac{2h\nu^3}{c^2} = \frac{\hbar\omega^3}{2\pi^2c^2} \overset{\hbar=c=1}{=} \frac{\omega^3}{2\pi^2}$$

comes from Planck's formula for black-body's spectral radiance. The stimulated-to-spontaneous ratio is thus given by

$$R = \frac{W_{\mathrm{st}}}{W_{\mathrm{sp}}} = \frac{8\pi^3\Pi}{m\omega_0^3\Delta\chi\Delta\Omega} \tag{13}$$

while the probability is

$$P_{\mathrm{st}} = W_{\mathrm{st}}\cdot\tau = \frac{2\pi^3\Pi}{Am\omega_0^3} \tag{14}$$

The use of standard laser parameters allows the evaluation of the quantities given in equations (13,14), suggesting that R may well exceed 10^{20} while the probability of having a stimulated event is of the order of *10%*.

The number of stimulated photons produced in the process is therefore

$$n_{\mathrm{st}} = P_{st}\cdot N_{\mathrm{p}} \tag{15}$$

where N_{p} is the number of positronium atoms, which can be evaluated by the cross section for ortho-positronium formation [12]:

$$\sigma_{\uparrow\uparrow} \simeq \alpha\frac{\pi r_0^2}{v_{\mathrm{rel}}}c \tag{16}$$

where v_{rel} is the relative transverse velocity of electron and positrons. In the lab frame

$$v_{\mathrm{rel}} = \frac{K}{\gamma}c \tag{17}$$

thus obtaining

$$\sigma_{\uparrow\uparrow} \simeq \alpha\frac{\pi r_0^2}{K}\gamma \tag{18}$$

The luminosity for single bunch infra-beam collision is

$$\begin{aligned} L \;&\simeq\; \frac{N^2c}{4\pi\sigma_{\mathrm{z}}\lambda_{\mathrm{w}}\Sigma'_y} \\ \Sigma'_y \;&=\; \sqrt{\frac{(\gamma\epsilon_{\mathrm{y}})\,\beta_{\mathrm{y}}}{\gamma}} \end{aligned} \tag{19}$$

where N is the number of particles in the electron or positron bunch, σ_z is the longitudinal bunch length, λ_w is the wiggler period and ϵ_y is the beam emittance in the horizontal direction.

Let us now discuss the times characterizing the process, which are essentially

1. the cooling time;
2. the life time of ortho and para positronium atoms.

Regarding the first, we note that [11]

$$c\tau_{\rm cool} \simeq \frac{3\pi}{2}\left(\frac{\lambda_{\rm w}}{2\pi}\right)^2 \frac{\alpha}{\gamma^2 K \sigma_{\uparrow\uparrow}} \tag{20}$$

The use of typical parameters like $\lambda_{\rm w}\,[{\rm cm}] = 10\pi, K = 1, \gamma = \gamma_{\rm p}$, yields a cooling time of few tens of seconds. The life time of the ortho-positronium is $1.4 \cdot 10^{-7}{\rm s}$ and this requires an interaction length of few tens of meters for the last undulator section.

We have considered so far only the case of ortho-positronium ("↑↑" configuration). The life time of para-positronium ("↑↓" configuration), decaying in two photons is shorter by at least two orders of magnitudes ($1.23 \cdot 10^{-10}{\rm s}$) than its spin 1 counterpart and has a production cross section larger by a factor α^{-1}. In the lab frame we can expect decay in two photons each one with an energy of the order of *0.5* MeV, the kinematical conditions of the stimulating photon fields are essentially the same as in the case of ortho-positronium, and we should expect a number of coherent gamma rays, associated with this decay channel, of about $10^9{\rm s}^{-1}$.

The e-beam emittance given by [11, 12]

$$\begin{aligned} \gamma\epsilon_{\rm y} &\simeq \frac{1}{2}\frac{\lambda_{\rm c}\beta_{\rm y}K}{\lambda_{\rm w}} \\ \gamma\epsilon_{\rm x} &\simeq \frac{1}{2}\left(1+\frac{K^2}{2}\right)\frac{\lambda_{\rm c}\beta_{\rm x}K}{\lambda_{\rm w}} \end{aligned} \tag{21}$$

which are consistent with the condition according to which the electron and positron beams should be cooled to reach the Fermi gas configuration. Finally, the matching condition between gamma ray laser and particle beam phase space area requires that

$$\epsilon_{\rm x,y} \simeq \frac{\lambda_c}{2} \longrightarrow \frac{\beta_{\rm x,y}K}{\gamma\lambda_{\rm w}} \simeq 1 \tag{22}$$

CONCLUSIONS

We examine some modifications to the e^-e^+ "kayak paddle cooler" described in Refs. [11, 12] to enhance the γ's production rate via stimulated annihilation of positronium. The scheme is based on two EM fields counter-propagating along the e^-e^+ beam line, one playing the role of the undulator, the other stimulating the transition to the bound state. Whichever the scheme, it cannot be overemphasized the advantage that stimulated transition to bound state bring in terms of γ fluence.

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