INVESTIGATION OF THE R_{56} OF A PERMANENT MAGNET PHASE SHIFTER

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Abstract

In the European XFEL permanent magnet phase shifters will be used routinely between two undulator segments. Their main purpose is to control the ponderomotive phase of the electrons with respect to the emitted radiation. In addition the path length is dependent on the electron energy, which corresponds to a small R_{56}. In this paper we investigate the R_{56} of a permanent magnet phase shifter and propose to use it to fine tune R_{56} by adjusting the phase shifter gap.

INTRODUCTION

X-ray free electron laser facilities, especially the ones based on self-amplified spontaneous emission (SASE) configuration [1, 2], require tens of meters or even over 100 m long undulators to achieve saturation. Such a long undultor system can not be built as one continuous device: They must be segmented due to mechanical reasons as well as efficient beam control. As a result, additional instruments such as quadrupoles, beam position monitors, synchrotron radiation absorbers, vacuum pumps, etc. are needed in regular intervals. For example, the European x-ray free electron laser facility [3] will be constructed with 5 m long undulator segments and the intersection between two segments is 1.1 m.

Once the undulator gap is changed the resonance wavelength changes accordingly, therefore it is a convenient way to tune the radiation wavelength. However, the phase advance between electrons and photon field through the intersection in between two adjacent undulator segments varies too. Consequently a device called phase shifter is needed, which allows an additional and adjustable phase delay to properly match the phase between individual undulator segments.

On the other hand, phase shifters are also needed for some specialized purposes: For instance, it can be used for polarization control in a crossed undulator system [4, 5, 6, 7].

A phase shifter could either use a chicane consisting of three electron magnets (EMs) [8, 9] or permanent magnets [10]. Due to some specific concerns such as occupied space, field quality and dissipated heat, et.al., The European XFEL adopts the latter choice. Nevertheless, both types are in principle magnet chicanes, therefore they unavoidably induce some extent of R_{56} to the beam. The R_{56} of a phase shifter is, however, very small. Therefore not much attention has been paid to this aspect. Whereas in x-ray FELs the beam is micro bunched on the 1 Angstrom scale, therefore even a very small R_{56} by the phase shifters maybe enormously affect the the properties of the radiation.

The present paper addresses the R_{56} studies for a permanent magnetic phase shifter. The study uses the configuration used in the European XFEL. The expression of R_{56} is deduced and its impact to the phase space of electron beam is discussed. Further more, the potential use of a phase shifter as a R_{56} fine tuner is also investigated.

R_{56} OF A PHASE SHIFTER

The field of a permanent magnetic phase shifter can be described as a double sinus like curve, as shown in Fig. 1 [10]:
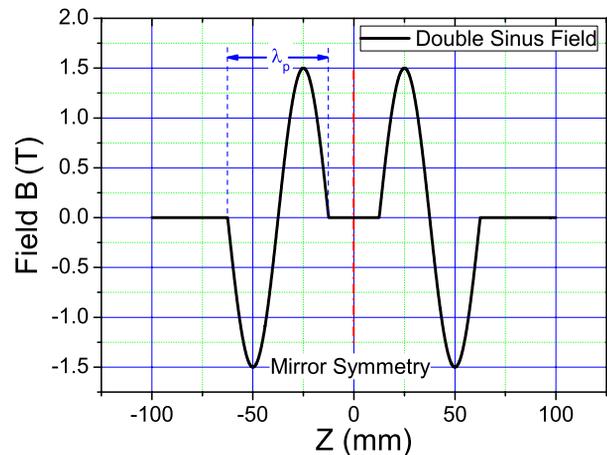

Figure 1: Description of the field of a permanent phase shifter: The black curve of double sinus shows the model, λ_p is its period.

The flying time T of electrons though a magnet field is given by:

$$
\begin{aligned}
cT = &\left(1 + \frac{1}{2\gamma^2}\right) L_D \\
&+ \frac{1}{2}\left(\frac{e}{m\gamma c}\right)^2 \int_{-L_D/2}^{L_D/2} dz \left(\int_{-L_D/2}^{z} B_y(z')dz'\right)^2,
\end{aligned} \tag{1}
$$

where L_D is the length of magnet field, c the velocity of light and $B_y(z)$ the magnet field distribution: The field extends from $z = -\frac{L_D}{2}$ to $z = \frac{L_D}{2}$. Outside it is zero. The magnet field direction is supposed along y direction and z is the coordinate along the beam path. e is the electron charge, m its mass and γ the electron's energy in unit of its rest mass (0.511 MeV).

Inducing the magnet vertor potential: $\nabla \times \vec{A} = \vec{B}$, so

$$B_y = \frac{\partial}{\partial z}A_x - \frac{\partial}{\partial x}A_z. \tag{2}$$

Assume B_y independent to x, which is the normal case, the second term in Eq. (2) equals to zero, and the field integration is just the potential A_x:

$$A_x = \int_{-L_D/2}^{z} B_y(z')dz'. \tag{3}$$

In addition, the first term of L_D in Eq. (1) can be dropped to calculate the time difference ΔT between electrons and photons, because in the same time T photon passes the length of L_D. Consequently, the phase advance can be calculated:

$$\begin{aligned}\phi &= \frac{2\pi\Delta Tc}{\lambda_R} \\ &= \frac{2\pi}{\lambda_R}\left[\frac{L_D}{2\gamma^2} + \frac{1}{2}\left(\frac{e}{m\gamma c}\right)^2 \int_{-L_D/2}^{L_D/2} A_x^2(z)dz\right],\end{aligned} \tag{4}$$

where λ_R is the radiation wavelength. To be noticed, in Ref. [10], the term of $\int_{-L_D/2}^{L_D/2} A_x^2(z)dz$ is called phase integral (PI). To simplify our analysis, PI is calculated by half part of the phase shifter, named PI_h. The whole phase shifter's PI is simply double of PI_h.

$$PI_h = \int_{-L_D/2}^{\lambda_p} A_x^2(z)dz, \tag{5}$$

where λ_p is one sinus period as shown in Fig. 1.

Further more, according to Fig. 1 the field distribution B_y follows sinus function. Similar to the undulator parameter K, a phase shifter parameter K_p can be defined:

$$K_p = \frac{\sqrt{2}e}{mc}\sqrt{\frac{1}{\lambda_p}\int_{-L_D/2}^{\lambda_p} A_x^2(z)dz}. \tag{6}$$

And the relationship between K_p and PI_h is:

$$PI_h = \frac{1}{2}\left(\frac{mc}{e}\right)^2\lambda_p K_p^2. \tag{7}$$

PI_h can further be expressed by the peak field B_{y0}:

$$PI_h = \frac{3}{8\pi^2}B_{y0}^2\lambda_p^3, \tag{8}$$

and the K_p accordingly be expressed by B_{y0} too:

$$K_p = 0.934\sqrt{3}B_{y0}[T]\lambda_p[cm], \tag{9}$$

note that it is $\sqrt{3}$ times of the normal undulator parameter K [10].

Substitute Eq. (8) and Eq. (9) into Eq. (4) the phase shift $\frac{\phi}{2}$ by the first half part of phase shifter is:

$$\begin{aligned}\frac{\phi}{2} &= \frac{\pi}{\lambda_R\gamma^2}\left[\lambda_p + \left(\frac{e}{mc}\right)^2 PI_h\right] \\ &= \frac{\lambda_p\pi}{\lambda_R\gamma^2}\left(1 + \frac{K_p^2}{2}\right).\end{aligned} \tag{10}$$

According to Eq. (10) ϕ depends on particle's energy γ. Its differential is:

$$\frac{1}{2}\frac{d\phi}{d\gamma} = -2\frac{\lambda_p\pi}{\lambda_R\gamma^3}\left(1 + \frac{K_p^2}{2}\right). \tag{11}$$

Assume a reference electron whose energy equals to the average energy, each electron's relative longitudinal position s is accordingly defined:

$$\phi = \frac{2\pi s}{\lambda_R}. \tag{12}$$

Substitute Eq. (12) into Eq. (11):

$$\delta s = -\frac{2\lambda_p}{\gamma^2}\left(1 + \frac{K_p^2}{2}\right)\frac{\delta\gamma}{\gamma}. \tag{13}$$

The definition of R_{56} is $\delta s = R_{56}\frac{\delta\gamma}{\gamma}$, so the R_{56} of the whole phase shifter is:

$$R_{56} = -\frac{2\lambda_p}{\gamma^2}\left(1 + \frac{K_p^2}{2}\right). \tag{14}$$

Eq. (14) is quite similar to the resonance condition of undulator. More over, normally $K_p^2 >> 1$, therefore according to Eq. (14), the R_{56} of a phase shifter is proportional to its peak field squared and third power of period λ_p.

Further more, because the iron yoke of a permanent phase shifter sets Neumann boundary conditions, the magnetic situation is fully equivalent to that found in Halbach-type hybrid undulators. The gap dependence can therefore be described with an exponential of the from

$$B_{y0} = a\exp(b(g/\lambda_p) + c(g/\lambda_p)^2). \tag{15}$$

Where g is the gap, a, b, c the dimensionless coefficients. In order to get estimates preliminary values they were taken from Ref. [11]: $a = 3.694, b = -5.068, c = 1.520$.

Combine Eq. (9), Eq. (14) and Eq. (15) the dependence of R_{56} on λ_p can be estimated. The results are illustrated in Fig. 2: It is seen that a phase shifter with λ_p of 10 cm could provide R_{56} of -100 nm. Such a phase shifter could be used as a R_{56} fine tuner, as discussed in the next section.

In addition, the maximum R_{56} of the phase shifter used in the European XFEL could be evaluated accordingly: $\lambda_p = 0.05m$, $B_{y0} = 1.5T$, $\gamma = 34246(17.5 \text{ GeV})$, consequently $K_p = 12$ and

$$R_{56} = -6.14\ nm.$$

POTENTIAL USE OF THE R_{56} OF A PHASE SHIFTER

Impact of the Bunch Factor b_1 to SASE1 of the European XFEL

As seen by the discussion in the last section, the induced R_{56} of a phase shifter is as small as several nanometers,
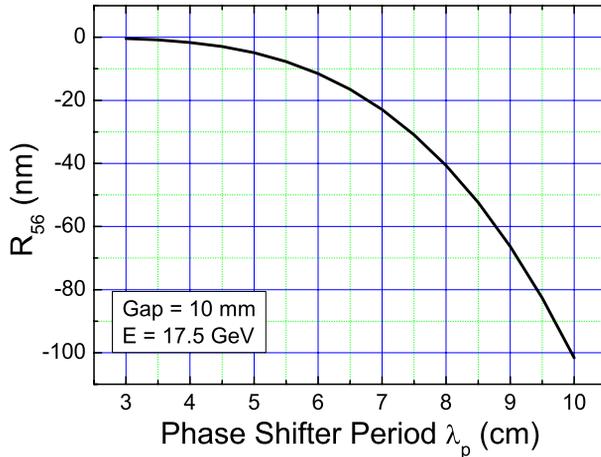
Figure 2: R_{56} versus λ_p of phase shifters. The gap is assumed 10 mm and the beam energy is assumed 17.5 GeV.

therefore it could be neglected in most cases. However, the shortest wavelength of the European XFEL is 0.1 nm, so the radiation quality sensitively corresponds to the properties of micro bunched beams. Therefore, a weak R_{56} could also have an impact on the micro bunched beam.

In order to investigate their impact a brief simulation was done for the SASE1 beam line in the European XFEL. Table. 1 lists the basic parameters of SASE1. The maximum value of R_{56} is -6.14 nm.

Table 1: Parameters used for simulation

Parameter	Value	Unit
Undulator period λ_U	35.6	mm
K_{Peak} at 10 mm gap	3.3	
$K = K_{RMS}$ at 10 mm gap	2.33	
Undulator segment length	5	m
Electron beam energy	17.5	GeV
Energy spread (RMS)	1.5	MeV
Radiation wavelength	0.1	nm
Bunch peak current	5	kA
Normalized emittance (RMS)	1.4	mm mrad
Average β-function	32	m
Maximum R_{56} of phase shifters	-6.14	nm

Due to the interaction between the electron beam and the photon field, the energy spread increases along undulator. It is shown in Fig. 3. In addition the beam transverse size is also illustrated.

Seven reference positions are selected to investigate the variations of bunch factor b_1 by phase shifters at different energy spread conditions. They are marked by the circles in Fig.3. The bunch factor b_1 is defined by:

$$b_1 = \left| < e^{-i\phi_n} > \right|, \tag{16}$$

where ϕ_n is the nth electron's ponderomotive phase, $<>$ means averaging over all electrons of the bunch.

Fig. 4 illustrates the bunch factor at each position: $b_{1,b}$ the number before passing the phase shifter and $b_{1,a}$ the
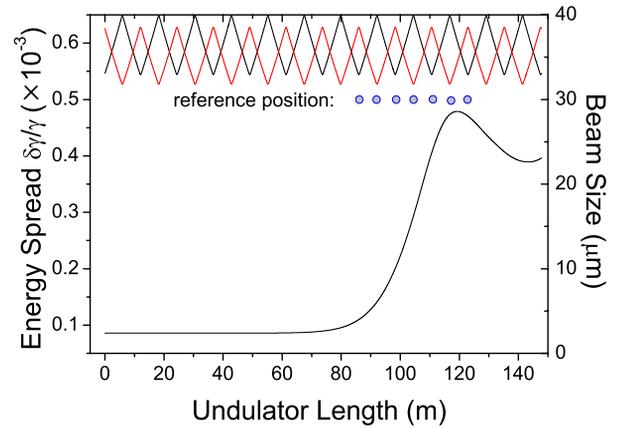
number after the phase shifter. The blue dotted line illustrates their difference, $b_{1,a} - b_{1,b}$.

According to Fig. 4 at the positions before 120 m, phase shifters slightly enhance the bunch factor b_1, on the contrary after 120 m phase shifter decreases it. This phenomenon is because b_1 saturates at the position around 120 m. The shape of phase space clock wise rotates during the whole FEL process. Before saturation is reached, b_1 is increased by clock wise rotation and after saturation this leads to debunching. A negative R_{56} provides an additional clock wise rotation, quite like the performance of dispersion chicane used in HGHG FEL [12]. Therefore it helps to increase b_1 before saturation is reached and accelerates debunching afterwards.

Nevertheless the variation of the bunching factor b_1 by a phase shifter is smaller than 0.05. Therefore it is beneficial to the FEL process, but its effect is limited.

Figure 3: Energy spread and beam transverse size versus undulator length for SASE1 of the European XFEL.
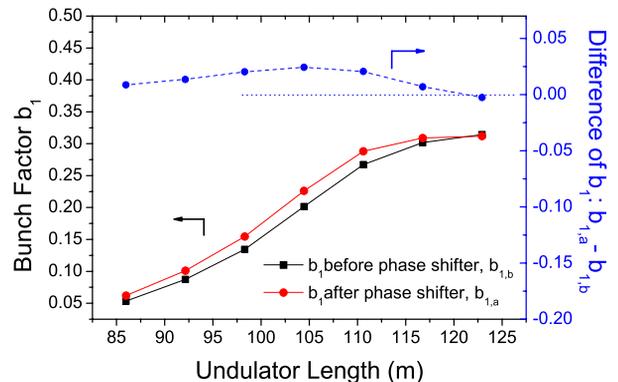

Figure 4: Bunch factor b_1 before and after the phase shifter: Black square line the bunch factor before phase shifters $b_{1,b}$, red dotted line the bunch factor after phase shifters $b_{1,a}$, the blue dotted line is their difference $b_{1,a} - b_{1,b}$.

Phase Shifter as R_{56} Fine Tuner

In some specific cases fine tuning of R_{56} down to several nanometers is needed. For example the Fig. 9 in Ref. [13] shows that a change of R_{56} by some tens of nanometers heavily influences the power and polarization of a crossed undulator placed behind the bunching undulator. R_{56} may be optimized dynamically for a given radiation wavelength. This accuracy is hard to obtain with other means.

The permanent phase shifter studied in this paper gives a possibility of tuning R_{56} with nanometer precision. Fig. 2 shows that a phase shifter with λ_p of 10 cm, which is realizable with standard technology, gives R_{56} of -100 nm. More over, R_{56} could be dynamically tuned by adjusting the gap, g. Fig. 5 illustrates R_{56} and its differential dR_{56}/dg versus gap. The period λ_p of the phase shifter is chosen to be 10 cm. In the range of 10 to 50 mm gap, R_{56} changes from -100 nm to -5 nm and the maximum differential is $10\ nm/mm$. Because the gap can be reliably adjusted with an accuracy of 10 μm, in principle R_{56} could be precisely tuned with a resolution of 0.1 nm, which is an extremely small number.
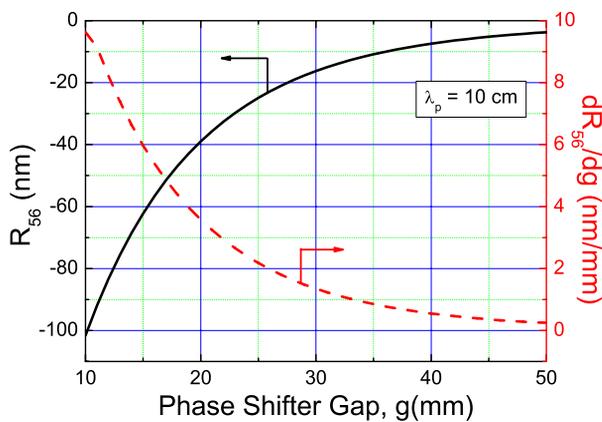

Figure 5: R_{56} (black solid curve) and its differential $\frac{dR_{56}}{dg}$ (red dash curve) versus gap. The period λ_p is 10 cm.

SUMMARY

The R_{56} induced by phase shifters based on permanent magnets is investigated. A parameter K_p is defined for easily calculation of R_{56}: R_{56} is proportional to K_p squared and K_p is proportional to the product of period λ_p and peak field B_{y0}. More over, permanent magnet phase shifters satisfy the magnetic situation found in Halbach-type hybrid undulator, which allows easy analytic estimation of the R_{56} as a function of λ_p and gap.

The impact on bunch factor b_1 by phase shifters are accordingly studied for the European XFEL. Simulations show that it could slightly fasten the bunching process before the saturation, therefore no harmful effects are expected.

In addition, a potential use of permanent phase shifter as a R_{56} fine tuner is proposed. A phase shifter with a period

length of 10 cm could provide a R_{56} from 0 to -100 nm as well as dynamically tuning of R_{56} with a resolution of 0.1 nm. The studies in Ref. [13] show that a minor variation of R_{56} could greatly change the power and polarization driven by a system comprised of a planar buncher and a pair crossed undulators, so a phase shifter could provide accurate control in that case. However, further detailed studies involving investigation of ISR and CSR effects should be done in the future work.

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