MODELS OF SPACE-CHARGE INDUCED OPTICAL MICROBUNCHING

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Abstract

Longitudinal space-charge forces can be a major source of micro-bunching instability. In this paper we briefly report about analytical, numerical and experimental methods for the description and characterization of space-charge induced optical microbunching. We discuss a three-dimensional theoretical model for the high frequency limit of space-charge interactions leading to density modulation at the optical scale. Particular emphasis is given to the effect of transverse thermal motion on the angular distribution of micro-bunching and to its connection to the physics of Landau damping in longitudinal plasma oscillations. We discuss a comparison of our model with high resolution numerical simulations with three-dimensional periodic boundary conditions. Finally we discuss a method for the experimental characterization of the transverse microbunching distribution from the coherent optical transition radiation emitted by the electrons.

INTRODUCTION

Micro-bunching instability is a critically important parasitic effect in high brightness relativistic electron sources, as it has the potential to seriously degrade the beam quality [1, 2, 3, 4, 5, 6, 7], thus compromising applications such as high gain free-electron lasers. Longitudinal space-charge can be a source of micro-bunching growth for wavelengths that are much shorter than the electron bunch length [8, 7, 9, 10].

In what follows, we will deal with the high-frequency regime of space-charge interactions, defined by the condition $\sigma_x >> \gamma\lambda/2\pi$, where σ_x is the root mean square transverse size of the electron beam, λ is the wavelength of interest and the Lorentz factor γ is the energy of the electron beam normalized to mc^2. It has been shown that in this limit, the Fourier components of the electric field generated by an uncorrelated electron distribution have a transverse distribution which is composed of several uncorrelated speckles [11]. Furthermore, in the high-frequency limit, the transverse dependence of the space-charge fields strongly affects the electron dynamic [11] and the problem needs to be addressed with a fully three-dimensional treatment. For this reason, we will refer to this limit as the three-dimensional limit. In this context, it is useful to define the bunching factor with an angular dependence [9]:

$$b = \frac{1}{N}\sum_{n=1}^{N} e^{-ik(z_n+\sin\theta(x_n\cos\phi+y_n\sin\phi))} \quad (1)$$

where N is the number of particles in the electron bunch, θ and ϕ are, respectively, the polar and azimuthal angles relative to the beam propagation axis z, z_n is the longitudinal position along the bunch of the n-th particle and x_n and y_n are the transverse positions.

In this paper, we show that in the high frequency limit, with the assumption that longitudinal motion is quasi-laminar, the problem of space-charge interactions leading to micro-bunching growth becomes formally equivalent to that of one-dimensional plasma oscillations in a warm electron plasma. We show that transverse emittance induces strong Landau damping at high transverse spatial frequencies, significantly narrowing the angular width of the micro-bunching gain with respect to the characteristic angular width of the space-charge fields. We compare the results of our analysis and those of high resolution molecular dynamics simulations with periodic boundary conditions in three dimensions. Finally we discuss a method for the experimental characterization of the transverse structure of the microbunched distribution.

A PLASMA OSCILLATION MODEL OF SPACE-CHARGE INDUCED MICROBUNCHING INSTABILITY

In this section, we discuss an analytical model of microbunching formation starting from shot-noise including both three-dimensional effects due to the electric self-field geometry and thermal effects due to finite beam emittance. The details of this analysis will be discussed elsewhere and we will limit ourselves to a brief outline of the derivation and an interpretation of the results.

We model the formation of microbunching as follows: the electron beam initially undergoes an external-force-free drift and space-charge generates an energy modulation starting from shot-noise. After the drift the electrons go through a series of optical elements which rearrange their longitudinal and transverse phase space coordinates according to a given transfer matrix R_{ij}. For simplicity we will only retain the R_{56} transport element, which causes microbunching growth due to longitudinal rearrangement.

We base our analysis on the Vlasov's equation. In our analysis we make the simplifying assumptions that the 0-th order charge distribution is uniform in space and constant in time. The first assumption is valid in the three-dimensional limit for wavelengths that are much smaller than the beam length. The second assumption is valid if the interaction happens close to a waist and the length of the drift is smaller than the minimum beta-function.

The electron beam particle distribution is described by a six-dimensional distribution function $f(\vec{x}_\perp, z, \vec{\beta}_\perp, p, \tau)$. Where $\vec{x}_\perp$ is the transverse position, z is the longitudinal position in the beam coordinate system, $\vec{\beta}_\perp$ is the transverse velocity normalized to the speed of light, $p = \Delta\gamma/\gamma$ is the relative energy deviation and $\tau = ct$ where c is the speed of light and t is the time, measured from the beginning of the interaction. The distribution function is normalized to the total number of particles N.

We expand the distribution function to first order in perturbation theory: $f = f_0 + f_1$, with $|f_1| << |f_0|$. With the previous assumptions we can specify the following form for the 0-th order distribution function: $f_0 = n_0 f_v(\vec{\beta}_\perp, p)$, where n_0 is the local average particle density.

With these underlying assumptions, the evolution of the perturbed distribution function can be easily be computed by solving the coupled Vlasov's and Poisson's equations by means of Laplace-Fourier methods. It can be shown that the bunching factor, after longitudinal rearrangement due to R_{56} is:

$$b_{R_{56}} = -\frac{1}{N}\sum_j e^{s_j L_d} \frac{1}{\left.\frac{\partial \epsilon_p}{\partial s}\right|_{s=s_j}} \frac{\omega_p^2 R_{56}\gamma^2}{c^2(1+(\gamma\theta)^2)}$$
$$\int \frac{e^{-ikpR_{56}} f_v}{s_j + ik\left(\theta\beta_x + \frac{p}{\gamma^2}\right)} dp d^2\vec{\beta} \int \frac{\left.\hat{f}_1\right|_{\tau=0}}{s_j + ik\left(\theta\beta_x + \frac{p}{\gamma^2}\right)} dp d^2\vec{\beta}. \quad (2)$$

where the sum is performed over all the zeros s_j of the beam's plasma dielectric function ϵ_p defined as:

$$\epsilon_p = 1 + \frac{\omega_p^2}{c^2(1+(\gamma\theta)^2)} \frac{\gamma^2}{ik} \int \frac{\frac{\partial f_v}{\partial p} + \theta \frac{\partial f_v}{\partial \beta_x}}{s + ik\left(\theta\beta_x + \frac{p}{\gamma^2}\right)} dp d^2\vec{\beta} \quad (3)$$

with $\omega_p^2 = \frac{e^2 n_0}{\epsilon_0 m \gamma^3}$ being the relativistic beam plasma frequency.

We make the further assumption that longitudinal motion is quasi-laminar, i.e.:$f_v = \frac{1}{(2\pi)^{3/2}\sigma_\beta^2\sigma_p} e^{-\frac{p^2}{2\sigma_p^2} - \frac{\beta^2}{2\sigma_\beta^2}}$ with $k\sigma_p/\gamma^2 << \omega_p/c$. With this final assumption, the plasma dielectric function can be expressed as a function of one dimensionless parameter. We give the following definitions: $k_D = \omega_p/c\sigma_\beta$ is the Debye wave-number, which we employ to normalize the transverse wave-number as $K = k\theta/k_D$; the Laplace variable s is normalized to the plasma frequency as $\Omega = -cs/i\omega_p$; finally, we normalize the transverse velocity to the thermal velocity spread: $B = \beta_x/\sigma_\beta$, $F = \frac{1}{(2\pi)^{1/2}} e^{-\frac{B^2}{2}}$. The resulting scaled beam plasma dielectric function is then:

$$\epsilon_p = 1 - \frac{1}{K^2} \int_{\tilde{c}} \frac{\frac{\partial F}{\partial B}}{B - \frac{\Omega}{K}} dB. \quad (4)$$

where the integral is performed over the Landau contour, which runs, in the complex B plane, below the singularity at $B = \frac{\Omega}{K}$. The zeroes of (4) can be found numerically and are, in general, complex. The imaginary part of the scaled frequency Ω_j is always negative, resulting in an exponential decay (Landau damping) of the micro-bunching as a function of the drift length. Also, if $\Omega_R - i\Omega_I$ is a solution (with Ω_R, Ω_I positive real numbers), then $-\Omega_R - i\Omega_I$ is also a solution [12]. We will thus denote as $\Omega_\pm$ the two dominant roots of the dielectric function (i.e. the roots with the smallest damping constant). Figure 1 which shows the real and imaginary parts of the dominant root Ω_+ (see also [12]). The imaginary part of $-\Omega_+$ is a growing function of K. For small values of K the damping constant $-\Im\{\Omega_+\}$ is small and can be neglected for drifts that are significantly shorter than a plasma wavelength, as is usual in most experimental situations. However, for $K > 1$, the damping term is significantly bigger than 1, resulting in a strong suppression of the micro-bunching gain at angles bigger than k_D/k.

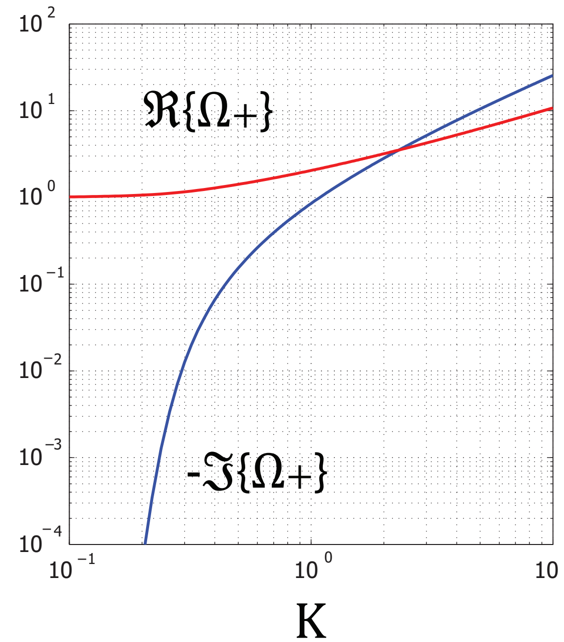

Figure 1: Real part (red line) and imaginary part (blue line) of the roots of the plasma dielectric function for a transversely warm beam, as a function of the scaled wave-number K [12].

It can be shown that, in the dominant pole approximation, assuming that the initial value of the perturbed distri-

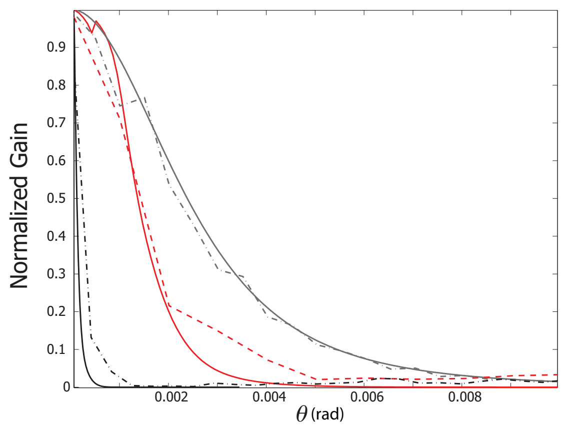

Figure 2: Theoretical results (solid lines) versus numerical simulations (dashed lines) for different values of σ_β corresponding to a normalized emittance of $\epsilon = 0$ (gray line), $\epsilon = 0.1$ mm-mrad (red line) and $\epsilon = 1$ mm-mrad (black line). The results of the simulations are averaged over 50 independent runs

bution function is only due to shot-noise, the microbunching gain, defined as $g = N < |b_{R_{56}}|^2 >$, is:

$$g = 2\left(\frac{\omega_p}{c(1+(\gamma\theta)^2)}\gamma^2 R_{56}\right)^2 e^{-(k\sigma_p R_{56})^2}\left(\left|\frac{K e^{-i\Omega_+\frac{\omega_p}{c}L_d}}{1-\frac{\Omega_+^2}{1+K^2}}\right|^2 \left(\left|\frac{K^2(1+K^2)}{\Omega_+^2}\right| - \sqrt{2\pi}K\frac{\Re\{e^{-\frac{\Omega_+^2}{2K^2}}\}}{\Im\{\Omega_+\}}\right) - \Re\left\{\left(\frac{K e^{-i\Omega_+\frac{\omega_p}{c}L_d}}{1-\frac{\Omega_+^2}{1+K^2}}\right)^2 \left(\frac{K^2(1+K^2)}{\Omega_+^2} - \sqrt{2\pi}K\frac{e^{-\frac{\Omega_+^2}{2K^2}}}{\Omega_+}\right)\right\}\right). \quad (5)$$

NUMERICAL MODELLING

The problem of numerical modelling of high frequency space-charge induced microbunching has been approached with a molecular dynamics method, in which the electron dynamic under the effect of space-charge fields is computed with periodic boundary conditions in three dimensions.

Details of this computational approach will be published elsewhere. Here we show a comparison of the results from the code with the analytical description in a specific example.

We choose the following beam parameters, corresponding to a typical electron beam produced by an RF photoinjector: a current of $I = 40$ A, an RMS envelope size of $\sigma_x = 85$ µm and an energy of 135 MeV ($\gamma = 270$). The length of the drift is $L_d = 4$ m. Figure 2 shows the angular dependence of the micro-bunching gain for several values of σ_β for a wavelength of $\lambda = 0.5$ µm, obtained from the theoretical analysis and from the simulation code. Note that, for the 1 mm-mrad case, the drift length is longer than the beta-function and the results of the theory are not accurate and should be interpreted with care.

An extension of this numerical approach, with periodicity only in the z-direction, is currently being pursued with the aid of the particle tracking code MaryLie-Impact [13].

METHODS FOR EXPERIMENTAL CHARACTERIZATION

The experimental characterization of the transverse structure of the microbunched distribution, in the high frequency domain, by means of coherent optical transition radiation (COTR) measurements is the subject of ongoing research.

An extensive analytical description of the properties of COTR emitted by microbunched electron beams can be found in [14]. Here we will briefly describe a method to characterize the single-shot microbunching distribution by simultaneous measurement of near and far field patterns and we will leave a more detailed description of the planned experiments for future publication.

The goal of these experimental methods, is that of reconstructing, in amplitude and phase, the longitudinal Fourier transform of the charge distribution, i.e. the quantity $B(x, y, k) = \int_{-\infty}^{+\infty} \rho(x, y, z)e^{(-ikz)}dz$, where ρ is the charge density and $k = 2\pi/\lambda$ with λ being the wavelength of interest.

The field radiated by COTR, in the near field zone, directly corresponds to the transverse space-charge field generated by the electrons. Furthermore, the field distribution in the far-field zone, corresponds to the Fourier transform of the near-field distribution.

Simultaneous measurement of both near and far field distributions, at a single frequency (i.e. after frequency filtering) allows the application of iterative phase retrieval algorithms on a strongly constrained set of data (namely the modulus of two signals which form a Fourier-transform pair), to recover the exact field distribution in amplitude and phase. The field distribution can then be deconvoluted with the space-charge field Green's function to recover the microbunching pattern.

Figure 3 shows a simulation of the method described above, with the original micro-bunching distribution and the distribution reconstructed from the amplitude of near and far field emitted by COTR.

CONCLUSIONS

In this paper, we described methods for the study of high-frequency space-charge induced microbunching instability. Using the mathematical methods developed for Landau damping in longitudinal plasma oscillations, we have given a theoretical description of the microbunching amplification process starting from shot-noise for a beam in a free drift. The results of the theory have been compared to those generated by a high resolution numerical code with

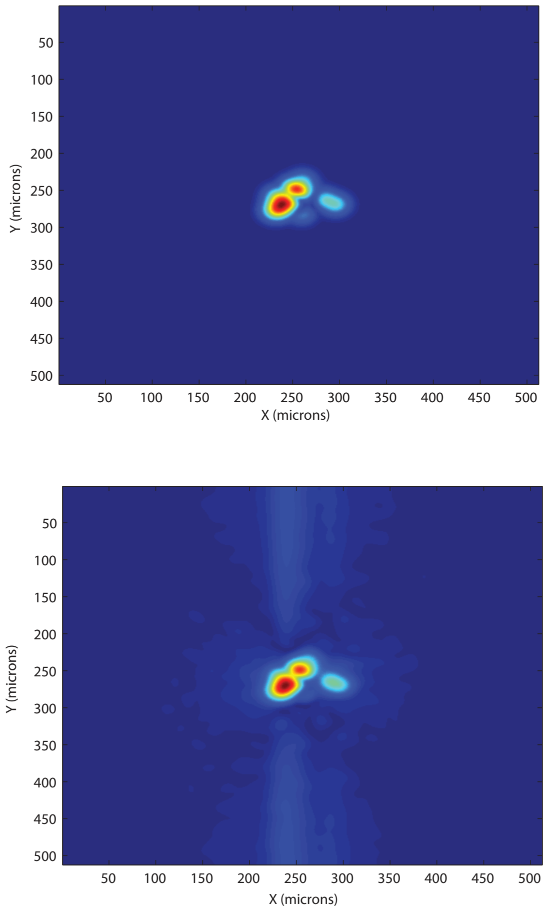

Figure 3: Amplitude of the original distribution of B(x,y) (left) and of the distribution reconstructed from near and far field amplitude with phase retrieval.

periodic boundary conditions in three dimensions and have been found to be in good agreement. Finally, we have described a method for the experimental characterization of the transverse structure of micro-bunching. This paper is intended to be a brief overview of these methods, which are still the subject of ongoing research. A more detailed description will be published in the future.

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