SMITH-PURCELL RADIATION WITH NEGATIVE-INDEX MATERIAL *

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Abstract

A scheme of Smith-Purcell free-electron laser with using a grating made of negative-index material is analyzed. Through theoretical analysis and computations, we show that the Smith-Purcell radiation is stronger in a specific range of radiation angle, from a grating of negative-index material, than positive-index material and perfect conductor. The dispersion relation for the surface mode of a grating of negative-index material is also worked out, which shows the possibility of realizing a Smith-Purcell free-electron laser with tens of keV electron beam.

INTRODUCTION

There is currently interest on the research of negative-index material, which shows many exotic and remarkable electromagnetic phenomena, such as reversed Cerenkov radiation and reversed Doppler shifts [1]. Recent successes in fabricating these artificial materials [2-4] have initiated an exploration into the use of them to investigate new physics and to develop new applications [5-7]. It has been demonstrated in theoretical analysis and simulation that enhanced diffraction can be achieved from a grating with negative-index material compared with a grating with positive-index material when a plane-wave is incident [8-9]. And this implies a possibility of realizing a high-performance Smith-Purcell free-electron laser. In this paper, we investigate the Smith-Purcell radiation and surface mode from a grating with negative-index material.

BASIC THEORY

We calculate the two-dimensional Smith-Purcell radiation from a grating with a homogeneous, isotropic, linear material. The grating is with a sinusoidal profile $x = g(z) = \dfrac{h}{2}\cos(\frac{2\pi}{d}z)$, where d is the period and h the amplitude. The region $x > g(z)$ is vacuous, whereas the medium occupies the region $x < g(z)$, characterized by relative permeability ε_r and permittivity μ_r. If the medium is of the negative-index material, the real part of both permeability and permittivity is negative. And the positive-index material requires that the real part of both permeability and permittivity are positive. The incident wave is supposed

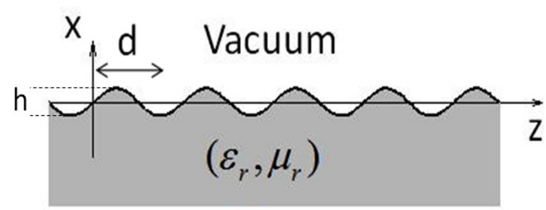

Figure 1: Schematic diagram of grating.

to be from a line charge with q coulombs per length passing above the grating with the distance x_0 and velocity v along z-axis. Only considering the TM mode, the y-directed component of magnetic field of the evanescent wave from the line charge is given by $\frac{q}{2}e^{\varsigma_0(x-x_0)}e^{j\frac{\omega}{v}z}$ (Hereafter the time part $e^{-j\omega t}$ is omitted), where $\varsigma_0 = \sqrt{\frac{\omega^2}{v^2} - \frac{\omega^2}{c^2}}$, ω the angular frequency, c the light velocity in vacuum [10]. The diffracted and refracted fields by the grating can be represented by Rayleigh expansions [11], so, the y-directed component of the total magnetic field outside the corrugations can be written as

$$H_y^{(1)} = \frac{q}{2}e^{\varsigma_0(x-x_0)}e^{j\frac{\omega}{v}z} + \sum_{p=-\infty}^{\infty} A_p e^{j\alpha_p x}e^{jk_p z} \quad x > \max g(z) \quad (1)$$

$$H_y^{(2)} = \sum_{p=-\infty}^{\infty} B_p e^{-j\beta_p x}e^{jk_p z}, x < \min g(z) \quad (2)$$

Here, A_p and B_p are scalar coefficients to be determined and

$$k_p = \frac{\omega}{v} + \frac{2p\pi}{d},$$

$$\alpha_p = \sqrt{\frac{\omega^2}{c^2} - k_p^2}, \beta_p = \sqrt{\frac{\varepsilon_r\mu_r\omega^2}{c_2} - k_p^2}.$$

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For the vacuous half-space the conditions $\mathrm{Re}(\alpha_p) > 0$ and $\mathrm{Im}(\alpha_p) > 0$ are required, whereas the medium half-space requires $\mathrm{Re}(\beta_p) > 0$ for positive-index material, $\mathrm{Re}(\beta_p) < 0$ for negative-index material, and $\mathrm{Im}(\beta_p) > 0$. The corresponding electric fields can be achieved from Maxwell equations. The continuity of the tangential components of the total electric field and magnetic field at the boundary $x = g(z)$ requires $H_y^{(1)} = H_y^{(2)}$ and $\vec{n}\cdot\nabla H_y^{(1)} = \varepsilon_r^{-1}\vec{n}\cdot\nabla H_y^{(2)}$, where $\vec{n}$ is a unit vector normal to the boundary. According to the Rayleigh hypothesis we assume that the expansions in Eqs.(1-2) can be used in the boundary conditions. Following the method of Maradudin [13], we multiply both sides of a boundary condition by $e^{jk_r z}$ and then integrate with respect to z over one period. After some algebraic works, we obtain a system of linear equations as

$$\sum_{p=-\infty}^{\infty} - A_p \cdot U_{r-p}^{p} + B_p \cdot V_{r-p}^{p} = \frac{qe^{-\varsigma_0 x_0}}{2} T_r \qquad (3)$$

$$\sum_{p=-\infty}^{\infty} A_p \cdot \frac{\frac{\omega^2}{c^2} - k_p k_r}{\alpha_p} \cdot U_{r-p}^{p} + B_p \cdot \frac{\frac{\varepsilon_r \mu_r \omega^2}{c^2} - k_p k_r}{\varepsilon_r \beta_p} \cdot V_{r-p}^{p} = -\frac{jqe^{-\varsigma_0 x_0}}{2} \cdot \frac{\frac{\omega^2}{c^2} - \frac{\omega}{v}\cdot k_r}{\varsigma_0} \cdot T_r \qquad (4)$$

where

$$U_{r-p}^{p} = \frac{1}{d}\int_0^d e^{j\alpha_p g(z)} e^{-j\frac{2\pi(r-p)z}{d}} dz$$

$$V_{r-p}^{p} = \frac{1}{d}\int_0^d e^{-j\beta_p g(z)} e^{-j\frac{2\pi(r-p)z}{d}} dz$$

$$T_r = \frac{1}{d}\int_0^d e^{\varsigma_0 g(z)} e^{-j\frac{2\pi r z}{d}} dz$$

If the grating is made of perfect conductor, the equations can be simplified as

$$\sum_{p=-\infty}^{\infty} A_p \cdot \frac{\frac{\omega^2}{c^2} - k_p k_r}{\alpha_p} \cdot U_{r-p}^{p} = -\frac{jqe^{-\varsigma_0 x_0}}{2} \cdot \frac{\frac{\omega^2}{c^2} - \frac{\omega}{v}\cdot k_r}{\varsigma_0} \cdot T_r \qquad (5)$$

As is known, when the integer p is negative, there exist radiation modes, so called Smith-Purcell radiation. The radiation frequency is dependent on the observation angle and electron velocity, and it can be known from the dispersion relation $k_p = \frac{\omega}{c}\cos\theta_p = \frac{\omega}{v} + \frac{2p\pi}{d}$, where θ_p is measured from electron moving direction. The diffraction coefficient A_p can be worked out through solving the equations numerically. In order to be independent of influence from charge q, we define the radiation factor as $R_p = A_p \big/ (\frac{1}{2} qe^{-\varsigma_0 x_0})$ [10].

Some computations are carried out and the radiated flux $|R_{-1}|^2$ in the 1st-order radiating wave as a function of the observation angle θ_{-1} for 35 keV electrons are given in Fig. 2 and Fig. 3. In Fig.2, comparing with the positive-index material ($\varepsilon_r = 5, \mu_r = 1$), it is shown that the radiated flux is higher in the region from $\pi/2$ to π for the negative-index material ($\varepsilon_r = -5, \mu_r = -1$), which means strong radiation can be obtained. In Fig. 3, we plot the cases of negative-index material and perfect conductor. Comparing with the perfect conductor, the negative-index material shows strong radiation in the region from $\pi/4$ to $3\pi/4$, while outside this region the radiation from the perfect conductor predominates.

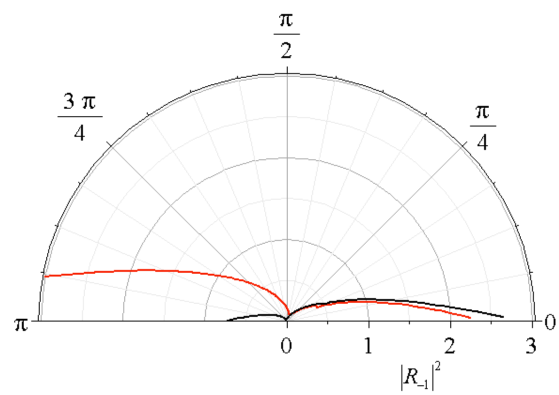

Figure 2: Radiated flux $|R_{-1}|^2$ as a function of the observation angle θ_{-1} for 35 keV electrons. Grating period d =1 mm, h/d =0.1. (Red line for negative-index material $\varepsilon_r = -5, \mu_r = -1$. Black line for positive-index material $\varepsilon_r = 5, \mu_r = 1$).

SURFACE WAVE

We have known that the surface modes of a grating play an important role in the operation of a Smith-Purcell free-electron laser [13-19]. The continuous electron beam interacts with the surface mode and is bunched periodically when beam current beyond so called start current, then the periodic bunches emits in the form of super-radiant Smith-Purcell radiation at a certain angle. Next, we explore the surface mode of a grating with negative-index material.

Considering the case of without incident wave, there are only evanescent wave near the corrugation boundary.

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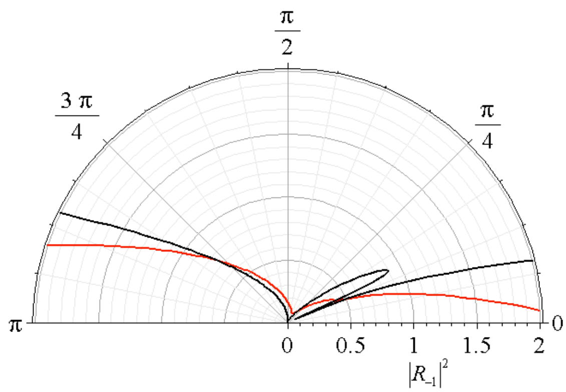

Figure 3: Radiated flux $|R_{-1}|^2$ as a function of the observation angle θ_{-1} for 35 keV electrons. Grating period d =1 mm, h/d=0.1. (Red line for negative-index material $\varepsilon_r = -5$, $\mu_r = -1$. Black line for perfect conductor).

Thus, the y-directed component of the total magnetic field outside the corrugations can be written as

$$H_y^{(1)} = \sum_{m=-\infty}^{\infty} C_m e^{-\alpha_m x} e^{jk_m z} \quad , x > \max g(z) \tag{6}$$

$$H_y^{(2)} = \sum_{m=-\infty}^{\infty} D_m e^{\beta_m x} e^{jk_m z} \quad , x < \min g(z) \tag{7}$$

Here, C_m and D_m are scalar coefficients and

$$k_m = k + \frac{2m\pi}{d} ,$$

$$\alpha_m = \sqrt{k_m^2 - \frac{\omega^2}{c^2}}, \beta_m = \sqrt{k_m^2 - \frac{\varepsilon_r \mu_r \omega^2}{c^2}} .$$

For the vacuous half-space the conditions $\mathrm{Re}(\alpha_m) > 0$ and $\mathrm{Im}(\alpha_m) < 0$ are required, whereas the medium half-space requires $\mathrm{Re}(\beta_m) > 0$, $\mathrm{Im}(\beta_m) > 0$ for negative-index material, and $\mathrm{Im}(\beta_m) < 0$ for positive-index material. Using the same boundary conditions mentioned above, it is straightforward to get a system of linear equations as

$$\sum_{m=-\infty}^{\infty} - C_m \cdot F_{n-m}^m + D_m \cdot G_{n-m}^m = 0 \tag{8}$$

$$\sum_{m=-\infty}^{\infty} C_m \cdot \frac{\frac{\omega^2}{c^2} - k_m k_n}{\alpha_m} \cdot F_{n-m}^m + D_m \cdot \frac{\frac{\varepsilon_r \mu_r \omega^2}{c^2} - k_m k_n}{\varepsilon_r \beta_m} \cdot G_{n-m}^m = 0 \tag{9}$$

where

$$F_{n-m}^m = \frac{1}{d} \int_0^d e^{-\alpha_m g(z)} e^{-j\frac{2\pi(n-m)z}{d}} dz$$

$$G_{n-m}^m = \frac{1}{d} \int_0^d e^{\beta_m g(z)} e^{-j\frac{2\pi(n-m)z}{d}} dz$$

The dispersion relation for surface mode is obtained by equating to zero the determinant of the coefficients in these equations. Through numerically calculating we get the dispersion curve for negative-index material ($\varepsilon_r = -9, \mu_r = -0.1$) and ($\varepsilon_r = -0.1, \mu_r = -11$)as is shown in Fig. 4. The 35 keV beam line is also plotted, and the intersection implies the operation point, where the surface wave is synchronous with electron beam. It is also shown that the 35 keV electron beam interacts with the backward-wave, thus, the external feedback system is not necessary. If the beam current is high enough for a device to oscillate, the electron beam would be periodically bunched, and those periodic bunches can emit at the second harmonic in the form of super-radiant Smith-Purcell radiation [16-19].

The permeability ε_r and permittivity μ_r are limited in a narrow region, ($-1 < \mu_r < 0, -1 > \varepsilon_r > 1/\mu_r$) and ($\mu_r < -1, -1 < \varepsilon_r < 1/\mu_r$), for the existence of surface wave. However, this limitation can be relaxed by placing a conductor boundary at the bottom of the grating. Such a grating scheme is under research.

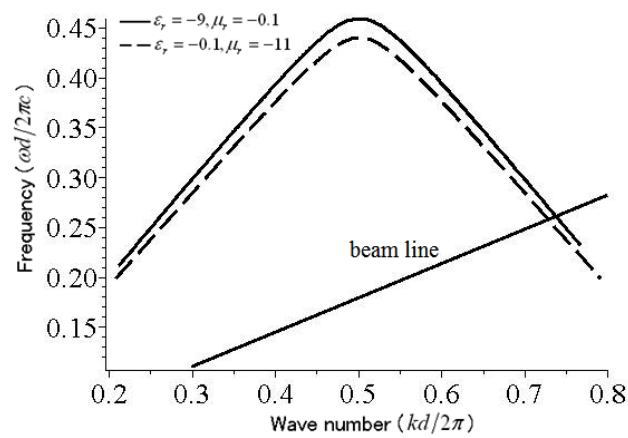

Figure 4: Dispersion relation of surface waves.

CONCLUSION

We proposed a grating system made of negative-index material to improve the Smith-Purcell free-electron laser. With theoretical analysis and numerical calculation, we demonstrated that the Smith-Purcell radiation from a negative-index grating is stronger than that from a conductor grating in a certain range of radiation angle. We also proved the existence of the surface wave, which plays an important role in a Smith-Purcell free-electron laser.

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