COMPARISON OF GROWTH RATES OF TWO-STREAM FREE ELECTRON LASERS (TSFEL) WITH PLANAR WIGGLER MAGNET AND AC ELECTRICAL WIGGLER PUMPS

N. Mahdizadeh[1], F. M. Aghamir[2]
[1]Department of Physics,Sabzevar Branch, IslamicAzadUniversity, Sabzevar, Iran
[2]Department of Physics, University of Tehran, Tehran, Iran

Abstract

A Comparison between growth rates of a Two-Stream Free-Electron Laser (TSFEL) with a planar magnet wiggler pump and ac electrical wiggler pump has been presented. With the aid of fluid theory, dispersion relations are derived and their characteristics have been numerically analyzed. Growth rate for two pumps with the similar data has been plotted. It found that in ac electrical wiggler pump instability occur in lower wavelength

INTRODUCTION

Use of two-stream rather than one stream has some advantages. For example, occurrence of Two-Stream instability cause to gain and growth rate enhancement [1-2]. A comparative study of cyclotron masers, ion-channel lasers and free electron laser has been done by K. R. Chen and et al [3]. A comparison of ac FEL and magnet wiggler FEL has been done by Yan and Dawson [4]. All of these works have done in conventional FEL, in this paper we present a comparison of ac FEL and magneticplanar wiggler pump in Two-Stream Free Electron Laser.

FUNDAMENTAL EQUATIONS

Two transversely homogenous non-neutralized relativistic electron beams with different velocity v_1 and v_2 propagate along the positive z direction. In magnetic wiggler pump two electron beams enter to the magnetic wiggler field, the wiggler field is given by

$$\vec{B} = B_w \sin(k_w z)\,\widehat{e_y} \quad (1)$$

Here $k_w = (2\pi)/\lambda_w$ is the wiggler wavenumber, λ_w is the wiggler wavelength, and $\widehat{e_y}$ denotes the unit vector. The wiggler field amplitude B_w is assumed to be constant. In an ac electrical pump two electron beams enter to the ac electric wiggler field, the wiggler field is given by

$$\vec{E} = E_p \sin(\omega_p t)\widehat{e_x}. \quad (2)$$

Here $\omega_p = \frac{2\pi}{T_p}$ is the wiggler frequency, and $\widehat{e_x}$ denotes the unit vector. The wiggler field amplitude E_p is assumed to be constant. This electric field may be an ac field in the super conducting cavity or a relativistic plasma density wave excited in the plasma medium.
The orbit equation for an electron can be expressed as

$$\frac{d\vec{p}_\alpha}{dt} = -e\left(\vec{E} + \frac{\vec{v}_\alpha \times \vec{B}}{c}\right) - \frac{1}{n_\alpha}\frac{\partial \pi_\alpha}{\partial z}\widehat{e_z}. \quad (3)$$

The subscript α=1, 2, refer to the different beams and $n_\alpha$is the beam density, and $\pi_\alpha$is the longitudinal part of the stress tensor. Longitudinal oscillations are expected to be affected by the thermal motion; therefore, we have only retained the longitudinal component of the stress tensor in the equation of motion to provide the contribution of the thermal effect. The continuity equation reads,

$$\frac{\partial n_\alpha}{\partial t} + \vec{\nabla}.(n_\alpha \vec{v}_\alpha) = 0. \quad (4)$$

The Poisson's equation is,

$$\vec{\nabla}.\vec{E} = -4\pi \sum_{\alpha=1}^{2} e n_\alpha \, . \quad (5)$$

The wave equation for the transverse component of the vector potential $A_x$of the RF fields is given by

$$\frac{\partial^2 A_x}{\partial z^2} - \frac{\partial^2 A_x}{c^2 \partial t^2} = -\frac{4\pi}{c}\sum_{\alpha=1}^{2} J_{\alpha x}\,, \quad (6)$$

$$\vec{B} = \vec{\nabla} \times \vec{A}. \quad (7)$$

Where $\vec{J}$ is the current density and can be represented as

$$\vec{J}_{1\alpha x} = -e(n_{0\alpha}\vec{v}_{1\alpha x} + n_{1\alpha}\vec{v}_{0\alpha x}). \quad (8)$$

It is assumed that beams are sufficiently tenuous, and the system is assumed to be uniform in the transverse direction; therefore, the transverse canonical momentum of a particle is a constant of the motion,

$$p_{1x} = \frac{e}{c}A_{1x}. \quad (9)$$

Where, p_{1x} refer to the perturbed total transverse momentum.
Since the equilibrium state is periodic, Floquet's theorem requires that all perturbed quantities such as$A_x, n_\alpha, p_{z\alpha}$ and $E_z$vary as

$$Z = \sum_{m=-\infty}^{m=+\infty} Z(m) e^{i(k_m z} - \omega t)\, . \quad (10)$$

$$k_m = k + mk_w, \qquad m = 0, \pm1, \pm2, ... \quad (11)$$

Where Z(m) represents the series expansion coefficients of the quantity of Z, herek_m and ωrefer to the wavenumber and frequency of mixed wave,respectively. The lowest order coupling of waves is of interest, therefore only the first three terms $m = 0, \pm1$, will be retained.

DISPERSION RELATIONS

Equations (3-6) form a closed set which describes the interaction of the relativistic electron beams with the external magnetic field and ac electrical field. Utilizing the above equations, the dispersion relation for magnetic wiggler pump can be written as

$$\left[\varepsilon(\gamma_{01}^3(\omega - (k + k_w)v_{01})^2 - \omega_{p1}^2 - 3(k + k_w)^2 v_{th1}^2) - \frac{\omega_{p1}^2 \omega_c^2 (k + k_w)^2}{4k_w^2 \gamma_{01}^2}\right]$$

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$$\times\left[\varepsilon\left(\gamma_{02}^3\left(\omega-(k+k_w)v_{02}\right)^2-\omega_{p2}^2-3(k+k_w)^2v_{th2}^2\right)-\frac{\omega_{p2}^2\omega_c^2(k+k_w)^2}{4k_w^2\gamma_{02}}\right]$$
$$-\left[\varepsilon\left(\omega_{p1}^2+\frac{n_{01}}{n_{02}}3(k+k_w)^2v_{th1}^2\right)+\frac{\omega_{p2}^2\omega_c^2n_{01}(k+k_w)^2}{4n_{02}k_w^2\gamma_{01}\gamma_{02}}\right]\times$$
$$\left[\varepsilon\left(\omega_{p2}^2+3\frac{n_{02}}{n_{01}}(k+k_w)^2v_{th2}^2\right)+\frac{\omega_{p1}^2\omega_c^2n_{02}(k+k_w)^2}{4k_w^2n_{01}\gamma_{01}\gamma_{02}}\right]=0. \quad (12)$$

Where
$$\varepsilon=\omega^2-c^2k^2-\sum_{\alpha=1}^{2}\frac{\omega_{p\alpha}^2}{\gamma_{0\alpha}}. \quad (13)$$

And the dispersion relation for ac electric wiggler pump can be written as

$$\{[\omega_{p1}^2-\gamma_{01}^3(\omega-(k+k_p)v_{01z})^2+3(k+k_p)^2v_{th1}^2]\,\epsilon+\frac{e^2E_p^2k(k+k_p)\omega_{p1}^2n_{01}}{4m^2\gamma_{01}^2\omega_p^2n_{02}}\}\times$$
$$\{[\omega_{p2}^2-\gamma_{02}^3(\omega-(k+k_p)v_{02z})^2+3(k+k_p)^2v_{th2}^2]\,\epsilon+\frac{e^2E_p^2k(k+k_p)\omega_{p2}^2n_{02}}{4m^2\gamma_{02}^2\omega_p^2n_{01}}\}-$$
$$\{[\omega_{p1}^2+3\frac{n_{01}}{n_{02}}(k+k_p)^2v_{th1}^2]\epsilon+\frac{e^2E_p^2k(k+k_p)\omega_{p2}^2n_{01}}{4m^2\gamma_{01}\gamma_{02}\omega_p^2n_{02}}\}\times$$
$$\{[\omega_{p2}^2+3\frac{n_{02}}{n_{01}}(k+k_p)^2v_{th2}^2]\epsilon+\frac{e^2E_p^2k(k+k_p)\omega_{p1}^2n_{02}}{4m^2\gamma_{01}\gamma_{02}\omega_p^2n_{01}}\}=0 \quad (14)$$

$$\epsilon=(\omega^2-c^2k^2-\sum_{\alpha=1}^{2}\frac{\omega_{p\alpha}^2}{\gamma_{0\alpha}}). \quad (15)$$

Two dispersion relations are the same, in magnetic wiggler pump we have $\frac{\omega_c^2}{k_w^2}$ but in ac electrical wiggler pump we have $\frac{e^2E_p^2}{m^2\omega_p^2}$. A numerical study for the instability spectrum and the corresponding growth rare are given in Figure. 1 and 2, for the magnetic wiggler pump and ac electrical wiggler pump respectively.

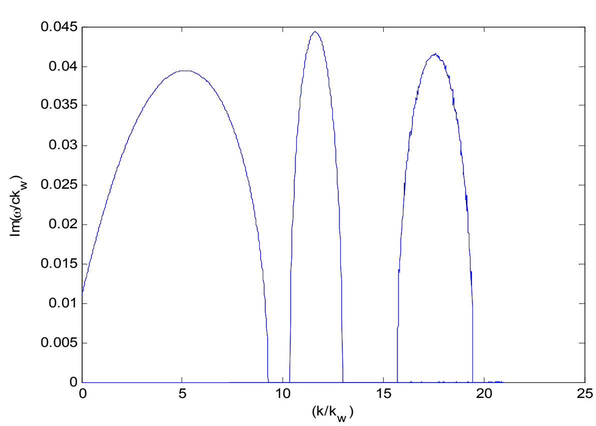

Figure 1: Temporal growth rate for TS and FEL instabilities with magnetic wiggler pump, for the case $,\bar{v}_{th1}=0.8\times10^{-1}$, $\bar{v}_{th2}=0.87\times10^{-1},\gamma_{01}=3.6$, $\gamma_{02}=3,\bar{\omega}_c=0.51$.

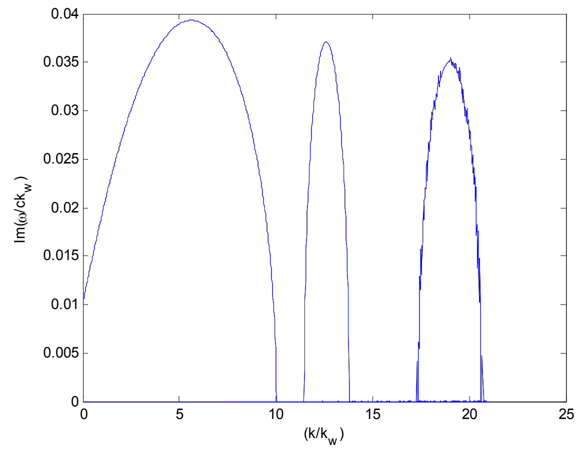

Figure 2: Temporal growth rate for TS and FEL instabilities with an ac electrical wiggler pump, for the case $,\bar{v}_{th1}=0.8\times10^{-1}$, $\bar{v}_{th2}=0.87\times10^{-1},\gamma_{01}=3.6,\gamma_{02}=3$, $\frac{e^2E_p^2}{m^2\omega_p^2}=0.51$.

As we see in the figures 1 and 2 in an ac electrical wiggler pump instability occur in shorter wavelength but peak growth rate of FEL instability is lower than magnetic wiggler pump one.

CONCLUTIONS

With the aid of fluid theory dispersion relations for TSFEL with an ac electrical wiggler pump and planar magnetic wiggler pump are derived.Two dispersion relations almost are the same, with this difference that in magnetic wiggler pump we have $\frac{\omega_c^2}{k_w^2}$ but in ac electrical wiggler pump we have $\frac{e^2E_p^2}{m^2\omega_p^2}$. Growth rates for two cases have been drowning. It has been found that for ac electrical wiggler pump FEL instability occur in lower wavelength, and peak growth rate is lower than magnetic wiggler pump.

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