NONLINEAR ANALYSES IN TWO-STREAM FREE-ELECTRON LASER WITH HELICAL WIGGLER PUMP

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Abstract

In this paper the nonlinear dynamic of Two-Stream Free Electron Laser (TSFEL) has been investigated. A set of coupled nonlinear differential equation in 1D approximation is employed, which governs the self-consistent evolution of an electromagnetic wave in the presence of two electron beams. Two relativistic electron beams with different velocities propagate through the wiggler field. Coupled nonlinear differential equations are solved numerically. Power versus axial distance has been plotted. It has been found that the FEL reaches the saturation regime in a longer axial distance in comparison the TSFEL.

INTRODUCTION

The use of two electron beams rather than one electron beam in free-electron laser have some advantages. For example, two-stream instability occurs and causes to gain enhancement and also increase growth rate [1-2]. In this paper we have investigated nonlinear dynamics of TSFEL. Nonlinear theory of FEL with helical wiggler and axial magnetic field has been studied by Freund et al[3]. Nonlinear saturation mechanism of FEL in presence of ion channel have investigated with Raghavi et al[4]. In TSFEL no calculation has been reported. The purpose of the present paper is to investigate of saturation mechanism of TSFEL with helical wiggler pump.

ELECTRON ORBIT EQUATIONS

Two transversely homogeneous non-neutralized relativistic electron beams with different velocity v_1 and v_2 propagate along the positive z direction enter to the magnetic wiggler field, the wiggler field is given by

$$\vec{B}(z) = B_w[\hat{e}_x \cos(k_w z) + \hat{e}_y \sin(k_w z)]. \quad (1)$$

Where B_w denotes the wiggler amplitude, $k_w(= 2\pi/\lambda_w)$ wiggler wave number, λ_w being wiggler period, $\hat{e}_x$, $\hat{e}_y$ are the unit vectors of a Cartesian cordinat system.

The time-varying vector potential radiation fields is written as follows

$$\delta\vec{A}(z,t) = \delta\hat{A}(z)[\hat{e}_x \cos\alpha_{+i}(z,t) - \hat{e}_y \sin\alpha_{+i}(z,t)] \quad (2)$$

$$\delta\emptyset(z,t) = \delta\hat{\emptyset}(z)\cos\ \alpha_i(z,t). \quad (3)$$

The subscript i=1, 2, refer to the different beams. $\delta\hat{A}(z)$ and $\delta\hat{\emptyset}(z)$ are the time independent amplitude of the vector and scalar potentials. $\alpha_{+i}(z,t)$ and $\alpha_i(z,t)$ are electromagnetic and space charge phases that defined as

$$\alpha_{+i}(z,t) = \int_0^z d\acute{z} k_+(\acute{z}) - \omega t \quad (4)$$

While,

$$\alpha_i(z,t) = \int_0^z d\acute{z} k(\acute{z}) - \omega t. \quad (5)$$

Where, ω is the wave frequency, $k_+(z)$ and $k(z)$ are the wave vectors. This is equivalent to the WKB formulation in which it is assumed implicitly that the amplitudes and wavenumbers vary slowly over a wavelength [3].

The electron orbit equations can be obtained by substitution of the static fields in Lorentz force equation

$$\frac{d\vec{p}_i}{dt} = -e\left[\delta\vec{E} + \frac{\vec{v}_i}{c}\times\left(\vec{B}_w + \delta\vec{B}\right)\right] \quad (6)$$

where $\delta\vec{E}$ and $\delta\vec{B}$ are the fluctuating electromagnetic fields which are derivable from the vector and scalar potentials (2) and (3).It is convenient to write equation in rotating frame with the wiggler field as

$$\begin{cases}\hat{e}_1 = \cos k_w z\, \hat{e}_x + \sin k_w z\, \hat{e}_y \\ \hat{e}_2 = -\sin k_w z\, \hat{e}_x + \cos k_w z\, \hat{e}_y\end{cases} \quad (7\text{-}1)$$

$$\hat{e}_3 = \hat{e}_z. \quad (7\text{-}2)$$

It has been assumed that the amplitude and phase are slowly varying functions of position ($\left|\frac{\partial\delta\hat{A}}{\partial z}\right| \ll \left|k_+\delta\hat{A}\right|$), and this occurs only in the vicinity of the wave particle resonance at $\omega \cong k_+ v_z$, with this assumption equation (6)yields three differential equation for three components of momentum like variable $\vec{u}_i = \vec{p}_i/mc$. Using new dimensionless variables: $\vec{\beta}_i = \frac{\vec{v}_i}{c}, \bar{z} = zk_w, \bar{t} = tck_w, \bar{\omega} = \frac{\omega}{ck_w}, \bar{k} = \frac{k}{k_w}$, these equation can be written as

$$\frac{du_{1i}}{d\bar{z}} = u_{2i} + \frac{d\delta a}{d\bar{z}}\cos\psi_i \quad (8\text{-}1)$$

$$\frac{du_{2i}}{d\bar{z}} = -u_{1i} - \frac{d\delta a}{d\bar{z}}\sin\psi_i - \widehat{\omega}_w \quad (8\text{-}2)$$

$$\frac{du_{3i}}{d\bar{z}} = \bar{k}_+\delta a\frac{u_{1i}\sin\psi_i + u_{2i}\cos\psi_i}{u_{3i}} - \frac{1}{\beta_{3i}}(\bar{k}\delta\varphi \sin\psi_{sc\,i} - \frac{d}{d\bar{z}}\delta\varphi\cos\psi_{sci}) + \widehat{\omega}_w\frac{u_{2i}}{u_{3i}}. \quad (8\text{-}3)$$

Where $\widehat{\omega}_w = \frac{eB_w}{mk_w c^2}$ is the wiggler parameter, $\delta a = \frac{e\delta\hat{A}(\bar{z})}{mc^2}$, $\delta\varphi = \frac{e\delta\hat{\emptyset}(\bar{z})}{mc^2}$.

Here $\psi(\equiv \alpha_+(\bar{z},\bar{t}) + \bar{z})$ and $\psi_{sc}(\equiv \alpha(\bar{z},\bar{t}))$are ponderomotive and space charge phase respectively, which given as

$$\frac{d\psi_i(\bar{z})}{d\bar{z}} = \bar{k}_+(\bar{z}) + 1 - \frac{\bar{\omega}}{\beta_{3i}} \quad (9\text{-}1)$$

$$\frac{d\psi_{sci}(\bar{z})}{d\bar{z}} = \bar{k}(\bar{z}) - \frac{\bar{\omega}}{\beta_{3i}}. \quad (9\text{-}2)$$

In above differential equations we have changed the integration parameter from $\bar{t}$ to $\bar{z}$, according to the relation $\frac{d}{d\bar{t}} = \beta_3\frac{d}{d\bar{z}}$.

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FIELD EQUATIONS

In the Coulomb gauge the Maxwell's equations can be written as [3]

$$\left(\frac{\partial^2}{\partial \bar{z}^2} - \frac{\partial^2}{\partial \bar{t}^2}\right)\delta a(\bar{z},\bar{t}) = -4\pi\delta J_\perp(\bar{z},\bar{t}) \qquad (10\text{-}1)$$

$$\frac{\partial^2}{\partial\bar{z}\partial\bar{t}}\delta\varphi(\bar{z},\bar{t}) = 4\pi\delta J_z(\bar{z},\bar{t})\ . \qquad (10\text{-}2)$$

Where $\delta\vec{J}(z,t)$ is the nonlinear current density and $\delta J_\perp(z,t)$ is the component of the latter perpendicular (along) the z direction. The current densities can be written as average over the entry time t_0 (defined as the time at which an electron crosses the $\bar{z}$=0 plane)

$$\delta J(\bar{z},\bar{t}) = -\frac{1}{4}\sum_{i=1}^{2}\omega_{bi}^2\int_{-\infty}^{+\infty} d\bar{t}_0\beta_i(\bar{t},\bar{t}_0)\frac{\delta[\bar{t}-\tau(\bar{z},\bar{t}_0)]}{|\beta_{zi}(\bar{t},\bar{t}_0)|}\ . \qquad (11)$$

Where $\omega_{bi}^2 = \frac{4\pi n_{bi}e^2}{mc^2}$, $\vec{\beta}(\bar{t},\bar{t}_0)$is the velocity of an electron at time $\bar{t}$ which crossed the entry plane at time $\bar{t}_0$, and

$$\tau(\bar{z},\bar{t}_0) = \bar{t}_0 + \int_0^z\frac{d\acute{z}}{\beta_{zi}(\acute{z},\bar{t}_0)}. \qquad (12)$$

It should be remarked that it is assumed implicitly in the current density that the electron beam is monoenergetic.

By substitution of microscopic fields and source current (2) and (3) into the Maxwell's equations (10-1) and (10-2) a set of coupled nonlinear differential equations for the slowly varying amplitudes and phases is obtained. The nonlinear equation (10-1) can be reduce to three first order differential equations for: $\delta a, \Gamma_+$ and k_+. Where Γ_+ defines the growth rate (i.e. the logarithmic derivative) of the field vector potential. These equations read

$$\frac{d\delta a}{d\bar{z}} \equiv \Gamma_+\delta a, \qquad (13\text{-}1)$$

$$\frac{d\Gamma_+}{d\bar{z}} = \left(-\bar{\omega}^2 + \bar{k}_+^2 - \Gamma_+{}^2\right) + \sum_{i=1}^{2}\frac{\omega_{bi}^2}{\delta a}\left\langle\frac{u_{1i}\cos\psi_i - u_2\sin\psi_i}{|u_{3i}|}\right\rangle \qquad (13\text{-}2)$$

$$\frac{d\bar{k}_+}{d\bar{z}} = -2\bar{k}_+\Gamma_+ - \sum_{i=1}^{2}\frac{\omega_{bi}^2\beta_{z0i}}{\delta a}\left\langle\frac{u_{1i}\sin\psi_i - u_{2i}\cos\psi_i}{|u_{3i}|}\right\rangle, \qquad (13\text{-}3)$$

$$\frac{d\delta\varphi}{d\bar{z}} = -2\sum_{i=1}^{2}\frac{\omega_{bi}^2}{\delta a\bar{\omega}}\langle sin\psi_{sci}\rangle \qquad (13\text{-}4)$$

$$k = -2\sum_{i=1}^{2}\frac{\omega_{bi}^2\beta_{ozi}}{\delta\varphi\bar{\omega}}\langle cos\psi_{sci}\rangle. \qquad (13\text{-}5)$$

The average of beam electrons on axial phase is as fallow

$$\langle(\cdots)\rangle = \frac{1}{2\pi}\int_{-\pi}^{\pi}d\psi_0\,(\cdots), \qquad (14)$$

$\psi_0(= -\bar{\omega}\bar{t}_0)$ is the initial phase.

Equations (8) and (9) together with Eq. (13) give a set of 10N+4 self-consistent first order differential equations , where N refer to the number of electrons. These equations can be solved numerically to find the wave evolution in the TSFEL.

NUMERICAL SOLUTIONS

The set of coupled nonlinear deferential equations for amplifier free-electron laser can be solved by forth order Runge-Kutta method.

The averages in the dynamical equations can be calculated by N th order Gaussian quadrature technique. We choose, N = 10, so the simulation can be done for 1000 electrons.

At first, it is assumed that a uniform and single energy electron beams with axial symmetry be injected into system. The electrons are picked within the range $-\pi \le \theta_0 \le \pi$.

The parameters are: the amplitude of wiggler field B_w = 2kG, wavelength of wiggler λ_w = 2 cm; electron beam with energy 250kev, 255kev; input signal frequency $\bar{\omega}$ = 15, 15.351 [1].

In Fig. 1, evolution of the radiation power in TSFEL as a function of axial position is presented.

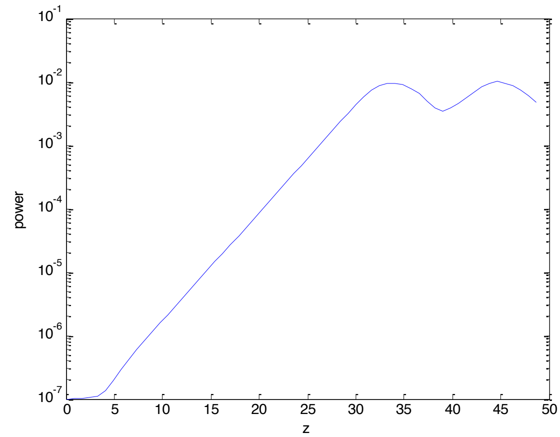

Figure 1: Evolution of the radiation power as a function of axial position.

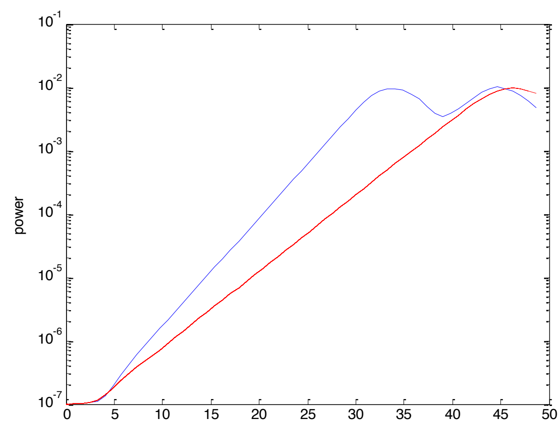

Figure 2: A comparison between evolutions of the radiation poweras a function of axial position for FEL (red) TSFEL (blue).

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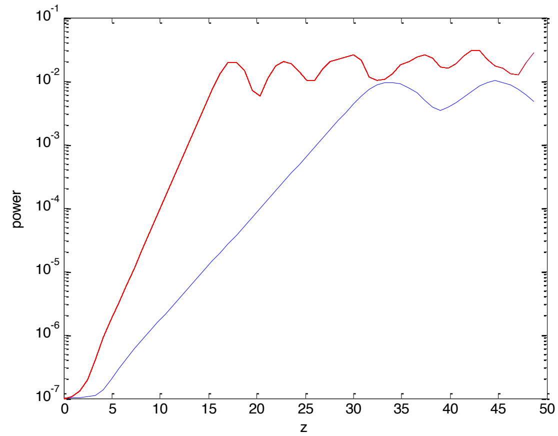

Figure 3: Evolution of the radiation power as a function of axial position for different value of magnetic wiggler strength $\bar{\omega}_c$=0.794 (red), $\bar{\omega}_c$=0.294 (blue).

As shown in Fig.1, power increases exponentially at $0 \leq z \leq 3.3$, and at this range the growth rate has a constant value, This shows linear regime. Then power increases exponentially till z=33, which is the saturation point and radiation amplitude does not increase any more. This time, half of the electrons are confined in pondermotive potential wells, give energy to the wave and the other half receive energy from the wave. When both of them are equal, saturation is done. This then radiation amplitude does not increase and due to the reduced electrons energy, radiation amplitude decreases. A comparison between the two cases of FEL and TSFEL is shown in Fig. 2. As one can see, the Two-Stream (TS) instability causes the saturation length to decrease to about 30%. Effect of different strength value of magnetic wiggler on saturation length is shown in Fig.3, as one can see, saturation length is decrease as strength of wiggler pump increase.

CONCLUTIONS

In this paper we have made use of a self consistent nonlinear theory to describe the one dimensional evolution of a TSFEL. The system of equation we have derived describes the evolution of both the wave fields and the electron trajectories. The self field effects of the electron beam have been included by means of scalar potential rising from the current sources. The numerical investigation shows that the saturation length is affected by the TS instability. A comparison between the two cases of FEL and TSFEL is shown. The Two-Stream (TS) instability causes the saturation length to decrease to about 30%. Effect of different strength value of magnetic wiggler on saturation length is shown. Saturation length is decrease as strength of wiggler pump increase.

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