IMPLEMENTATION OF 2D-EMITTANCE COMPENSATION SCHEME IN THE BERLINPRO INJECTOR*

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Abstract

Helmholtz-Zentrum Berlin officially started Jan. 2011 the design and construction of the Berlin Energy Recovery Linac Project BERLinPro. The initial goal of this compact ERL is to develop the ERL accelerator physics and technology required to accelerate a high-current (100 mA) low emittance beam (1 mm·mrad normalized), as required for future ERL-based synchrotron light sources.

High power ERL-based FELs demand low emittance, high peak and average current beams. The injection energy in an ERL is usually rather low to decrease power consumption and avoid activation of the beam dump. Therefore, the space charge is the main reason of the emittance degradation in the injector. The implementation of an emittance compensation scheme in the injector is necessary to achieve a low emittance. Since injector's optics is axially non-symmetric, the 2D-emittance compensation scheme [1] should be used. The implementation of the 2D-emittance compensation scheme at BERLinPro injector is presented in this contribution. Other sources of emittance growth in ERL injectors are also discussed.

THEORY

The gun produces a beam with a low transverse emittance. The main request to the injector design is to keep the emittance low and allow bunching of the beam. The main source of emittance growth in the injector is transversal and longitudinal space charge forces and aberrations.

Transversal Space Charge

Consider effects of transversal space charge. In injector we have a space charge dominated beam [2]. If the longitudinal size in the beam frame is larger than the transversal one, we can divide the bunch into slices, neglect interaction between them and consider the motion of slices independently from each other [3].

The slices start at the cathode with the same radii and different current densities. Therefore, the motion of the slices in the phase space is different from each other. It leads to an increase of the phase space area filled by particles. As a result the emittance grows. The emittance compensation technique allows aligning slices again at some points of the trajectory. At these points emittance will be minimal.

In an axi-symmetrical system a solenoid is used to make "emittance compensation" at a certain point, usually in the booster. Estimations show, however, that the beam in the merger of the BERLinPro is still space charge dominated; therefore, it is necessary to have emittance compensation point in the main linac. For a system without axial symmetry the 2D-emittance compensation technique should be used to make both x- and y-emittances minimal at the middle of the linac [1].

To calculate and optimize beam parameters in this approximation a dedicated code was developed on C++. The code solves the system of equations (1) for each slice numerically by the 4th order Runge-Kutta method. This system is basically the Kapchinsky-Vladimirsky equations for a zero slice emittance complemented with equations of linear longitudinal motion to calculate the slice current.

$$\begin{aligned}
&\frac{\partial^2 x}{\partial s^2} = -k_x x + \frac{j}{x+y},\\
&\frac{\partial^2 y}{\partial s^2} = -k_y y + \frac{j}{x+y},\\
&j = \frac{I_{in}}{\beta_{in} I_0 \beta^2 \gamma^3}\cdot n,\\
&\frac{\partial n}{\partial s} = -n\cdot\delta'\left(\frac{1}{\beta^3\gamma^2} - \frac{D}{R}\right),\\
&\frac{\partial \delta'}{\partial s} = \frac{\varepsilon'(ct)}{E_0} - \delta'\frac{\varepsilon(ct)}{E_0} - \delta'^2\left(\frac{1}{\beta^3\gamma^2} - \frac{D}{R}\right),\\
&\frac{\partial ct}{\partial s} = \frac{\delta E}{E_0}\left(\frac{1}{\beta^3\gamma^2} - \frac{D}{R}\right),\\
&\frac{\partial \delta E}{\partial s} = \varepsilon(ct) - \varepsilon(0),
\end{aligned} \qquad (1)$$

here x and y are rms sizes, δE – energy deviation, ct – longitudinal coordinate of slices, δ' – s-derivative of the energy spread along the bunch, n – bunching level, I_{in} – initial current, $\varepsilon(ct)$ – cavity accelerating gradient, R – trajectory radius, $k_{x,y}$ – focusing strengths, D – dispersion, E_0 – beam energy, γ and β – relativistic factors, β_{in} – initial β.

One calculation of beam dynamics by this code from the gun end to the end of the main linac for BERLinPro injector takes about 1 second. For comparison ASTRA makes injector simulation for 2 minutes. This 100 time difference allows to make the injector optics optimization in a reasonable time.

The optimization is performed by random walk method. The found emittance minima are accepted, if they are lower than a target value. The target value is selected lower than a slice emittance after the gun or an emittance growth due to effects which are not included in the code (longitudinal space charge forces and aberrations).

*Work supported by grants of Helmholtz Association VH-NG-636 and HRJRG-214.
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ISBN 978-3-95450-117-5

Longitudinal Space Charge

In our case the longitudinal beam size in the beam's frame is much more than the transversal one. The longitudinal electrical field in the accelerator's frame can be estimate as:

$$E_z \approx \frac{Q}{\gamma 2\pi\varepsilon_0 rl}\ , \quad (2)$$

where Q is the bunch charge, r – transversal radius, l – bunch length, ε_0 – permeability of vacuum. In the injector after the booster longitudinal electrical field is about:

$$E_z \approx \frac{77\cdot 10^{-12}C}{14\cdot 2\pi\cdot 8.854\cdot 10^{-12}\cdot 0.002m\cdot 0.01m} \approx 5\,keV/m \quad (3)$$

The main impact of the longitudinal electrical space charge field on the beam dynamics is changing of the particles energy. The energy change δ_{sc} in the dispersion section can be the main reason of the emittance growth in the merger.

$$\delta_{sc} = \frac{\int E_z ds}{E_0} \approx \frac{E_z\cdot L}{E_0}\ , \quad (4)$$

where L – the length of a dispersion section. The particle offset and the additional angle at the end of a merger δx and $\delta x'$ are about:

$$\delta x = \int \frac{\partial\delta_{sc}}{\partial s} D\cdot ds \approx \delta_{sc}\cdot D,$$
$$\delta x' = \int \frac{\partial\delta_{sc}}{\partial s} D'\cdot ds \approx \delta_{sc}\cdot D'. \quad (5)$$

The offset of a slice centre at the end of the merger can be partly compensated by adjusting the dispersion at the end of the merger, if there is a correlated energy spread in the bunch (in this case the merger is not exactly achromatic).

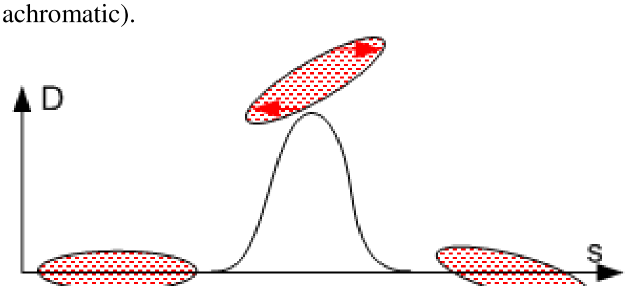

Figure 1: Transverse emittance grows due to longitudinal space charge in dispersion section.

The particle offset and the additional angle at the end of a merger leads to transversal emittance degradation in the dispersion section. The estimation of this effect is following:

$$\Delta\varepsilon^2 = \delta x^2\langle x'^2\rangle + \delta x'^2\langle x^2\rangle - \delta x\delta x'\langle x'x\rangle. \quad (6)$$

If there is no correlation between particles coordinate and angle, we can simplify.

$$\frac{\Delta\varepsilon}{\varepsilon} \approx \sqrt{\frac{\delta x^2}{x_{rms}^2} + \frac{\delta x'^2}{x_{rms}'^2}} \approx \sqrt{\left(\frac{\delta_{sc}\cdot D}{x_{rms}}\right)^2 + \left(\frac{\delta_{sc}\cdot D'}{x'_{rms}}\right)^2} \approx$$
$$\approx \frac{E_z L}{E_0}\sqrt{\left(\frac{D}{x_{rms}}\right)^2 + \left(\frac{D'}{x'_{rms}}\right)^2}. \quad (7)$$

where x_{rms} – the transversal bunch size, x'_{rms} – the transversal bunch angle.

Estimate this effect for BERLinPro injector parameters $L\approx 2$ m, $D\approx 0.5$ m, $D'\approx 0.3$, $x_{rms}\approx 0.001$ m, $x'_{rms}\approx 0.001$.

$$\frac{\Delta\varepsilon}{\varepsilon} \approx \frac{5keV/m\cdot 2m}{7MeV}\sqrt{\left(\frac{0.5m}{0.001m}\right)^2 + \left(\frac{0.3}{0.001}\right)^2} \approx 1. \quad (8)$$

Therefore emittance degradation due to this effect can be relatively strong and a merger design with the shortest dispersion section seems to be the only way to avoid this effect.

Aberrations

In the injector we have following aberrations: in the solenoid, chromatic aberrations and RF nonlinearity.

First, we estimate aberrations in a solenoid. The equation of motion in a solenoid to 3rd order terms is the following

$$r'' = -Kr + (K\frac{B''}{2B} - K^2)r^3 + Kr^2r'\frac{B'}{B} - Krr'^2,$$
$$K = \frac{e^2B^2}{4p^2c^2}. \quad (9)$$

In an approximation of a thin solenoid the angle which particles get is given by integration. The strongest of nonlinear terms is ~ r^3 and emittance growth due to it is:

$$\Delta\varepsilon \approx r_{rms}^4\int\left(\frac{e^2BB''}{8p^2c^2} - \left(\frac{eB}{2pc}\right)^4\right)ds. \quad (10)$$

If one increases the solenoid length keeping the strength constant, aberrations decrease. For our beam parameters, the emittance growth in solenoids with a magnetic length of 0.15 m is 0.2 mm·mrad. It seems to be acceptable and technically feasible.

Next, estimation of the emittance growth due to chromatic aberrations is given.

Particle offset due to the chromatic aberration is δx_2.

$$\delta x_2 = \eta_2\delta^2\ , \quad (11)$$

where η_2 is the second order dispersion. Emittance growth due to the chromatic aberration is given by

$$\frac{\Delta\varepsilon}{\varepsilon} \approx \frac{\langle\delta x_2\rangle}{x_{rms}} \approx \frac{\eta_2\delta_{rms}^2}{x_{rms}}. \quad (12)$$

In the merger we have $\eta_2\approx 0.5$ m^{-2} and $x_{rms}\approx 0.6$ mm. The acceptable relative emittance growth due to the chromatic aberration is 0.1, and rms energy spread in this case should be below 10^{-2}.

Further, the impact of the RF curvature is discussed.

The main impact of the RF nonlinearity on the beam dynamics is a possibility of an overbunching. Firstly, the slice motion in this case is not independent, and we can

ISBN 978-3-95450-117-5

not use emittance compensation techniques. Second, it leads to a significant increase of current in overbunching slices. As a result, their motion becomes totally different from the rest of the beam and the emittance increases dramatically.

To find the condition of overbunching, consider the longitudinal motion of two particles (1st and 2nd) in the bunch. For our estimation we assume that:

$$E(ct) \approx \tilde{E}\cos(ct\frac{2\pi}{\lambda}+\varphi_0)\,, \qquad (13)$$

where $E(ct)$ is the energy distribution in a bunch after the booster.

$$c\Delta t' = c\Delta t + R_{56}\frac{E(ct_2)-E(ct_1)}{E(0)},$$

$$c\Delta t' = c\Delta t + R_{56}\frac{\cos(ct_2\frac{2\pi}{\lambda}+\varphi_0)-\cos(ct_1\frac{2\pi}{\lambda}+\varphi_0)}{\cos(\varphi_0)}, \qquad (14)$$

where $c\ \ t$ and $c\ \ t'$ are distances between the particles before and after the merger, φ_0 is the phase of the reference particle.

Consider the case when 1st and 2nd particles are sufficiently close.

$$c\Delta t << \lambda/2\pi. \qquad (15)$$

The difference between phase of these particles and reference particle is $\ \ \varphi$.

$$\Delta\varphi = c(t_1+t_2)\cdot\frac{\pi}{\lambda}. \qquad (16)$$

To estimate the local bunching n_l, simplify (14).

$$c\Delta t' = c\Delta t - R_{56}c\Delta t\frac{2\pi}{\lambda}\frac{\sin(\Delta\varphi+\varphi_0)}{\cos(\varphi_0)},$$

$$n_l = \left(1-\frac{R_{56}}{\cos(\varphi_0)}\frac{2\pi}{\lambda}\sin(\varphi_0+\Delta\varphi)\right)^{-1}. \qquad (17)$$

If $n_l=\infty$ the overbunching is happened, the phase of the overbunching slice is $\ \ \varphi_{ovb}$.

$$1 = \frac{R_{56}}{\cos(\varphi_0)}\frac{2\pi}{\lambda}\sin(\varphi_0+\Delta\varphi_{ovb}),$$

$$\Delta\varphi_{ovb} = \arcsin\left(\frac{\cos(\varphi_0)}{R_{56}}\frac{\lambda}{2\pi}\right)-\varphi_0. \qquad (18)$$

Consider the case when 1st particle in the beginning and 2nd in the end of the bunch.

$$ct_1\,2\pi/\lambda = -ct_2\,2\pi/\lambda = \Delta\psi, \qquad (19)$$

where $\ \ \psi$ is differences between phase of reference particle and bunch head and tail. To estimate the global bunching factor n_g put (19) into (14),

$$c\Delta t' = c\Delta t - R_{56}2\sin(\Delta\psi)tg(\varphi_0),$$

$$n_g = \left(1-R_{56}\frac{2\pi}{\lambda}\frac{\sin(\Delta\psi)}{\Delta\psi}tg(\varphi_0)\right)^{-1}. \qquad (20)$$

Let's estimate the maximum possible bunching without overbunching n_g^*. Therefore consider case $\ \ \varphi_{ovb}=\ \ \psi$. For a short bunch $\ \ \psi<<1$, and small phase $\varphi_0<<1$ we can put (18) into (20) and simplify.

$$n_g^* \approx \left(1-R_{56}\frac{2\pi}{\lambda}tg(\varphi_0)\right)^{-1} =$$

$$= \left(1-R_{56}\frac{2\pi}{\lambda}(\arcsin\left(\frac{\cos(\varphi_0)}{R_{56}}\frac{\lambda}{2\pi}\right)-\Delta\psi)\right)^{-1} \approx \qquad (21)$$

$$\approx \left(1-R_{56}\frac{2\pi}{\lambda}(\frac{\lambda}{2\pi R_{56}}-\Delta\psi)\right)^{-1} = \frac{\lambda}{2\pi R_{56}\Delta\psi}.$$

So, small R_{56} and high φ_0 is preferable for bunching. To estimate the maximum possible φ_0, let's calculate full energy spread in bunch δ and rms one δ_{rms}. Full bunch length in the booster is $ct_{full}\approx 5$ mm, it corresponds to $\ \ \psi\approx 0.07\approx 4°$.

$$\delta = 2\sin(\Delta\psi)\cdot tg\varphi,$$

$$\delta_{rms} \approx \frac{\delta}{\sqrt{12}} \approx \frac{\Delta\psi\cdot tg\varphi_0}{\sqrt{3}}, \qquad (22)$$

$$\varphi_0 = arctg\left(\frac{\sqrt{3}\delta_{rms}}{\Delta\psi}\right) \approx arctg\left(\frac{\sqrt{3}\cdot 10^{-2}}{0.07}\right) \approx 15^0.$$

The maximal possible value of R_{56} at which overbunching does not happen R_{56}^* is following.

$$R_{56}^* = \frac{\cos(\varphi_0)}{\sin(\varphi_0+\Delta\psi)}\frac{\lambda}{2\pi} = \frac{\cos(15^0)}{\sin(15^0+4^0)}\frac{0.23m}{2\pi} \approx 0.11m. \qquad (23)$$

The bunching level in this case is:

$$n_g^* \approx \frac{\lambda}{2\pi R_{56}\Delta\varphi} \approx \frac{0.23m}{2\pi\cdot 0.11m\cdot 0.07} \approx 5\,. \qquad (24)$$

SIMULATION

We start ASTRA simulations with a particle distribution at the cathode. The beam is modeled with the uniform distribution in the transversal plane with radius 0.8 mm. This means the laser spot is with hard edges. Longitudinal distribution is defined by the form of the laser pulse and cathode response time. In the model we assume 15 ps flat-top laser pulse with 2 ps rise and decay time.

The phase of the gun cavity is adjusted to achieve small longitudinal decompression (15°). The phase influences the transversal focusing in the RF field and correlated energy spread after the gun. In our case the transversal beam size after the gun is large and aberrations in the solenoid give relatively large slice emittance which is hard to compensate further, therefore, low aberration solenoid design is necessary.

The dogleg merger (fig 2.) was chosen because it is the simplest variant with necessary parameters. It has $R_{56}\approx 11$ cm. As described in previous sections, this simplifies bunch compression. Phases in the booster cavities are set approximately 20° off crest to achieve necessary energy chirp. This phase is larger, than necessary for this bunch length. Therefore, nonlinearity of RF fields (cos-like dependence of the energy on the longitudinal coordinate s) does not limit the bunch compression.

ISBN 978-3-95450-117-5

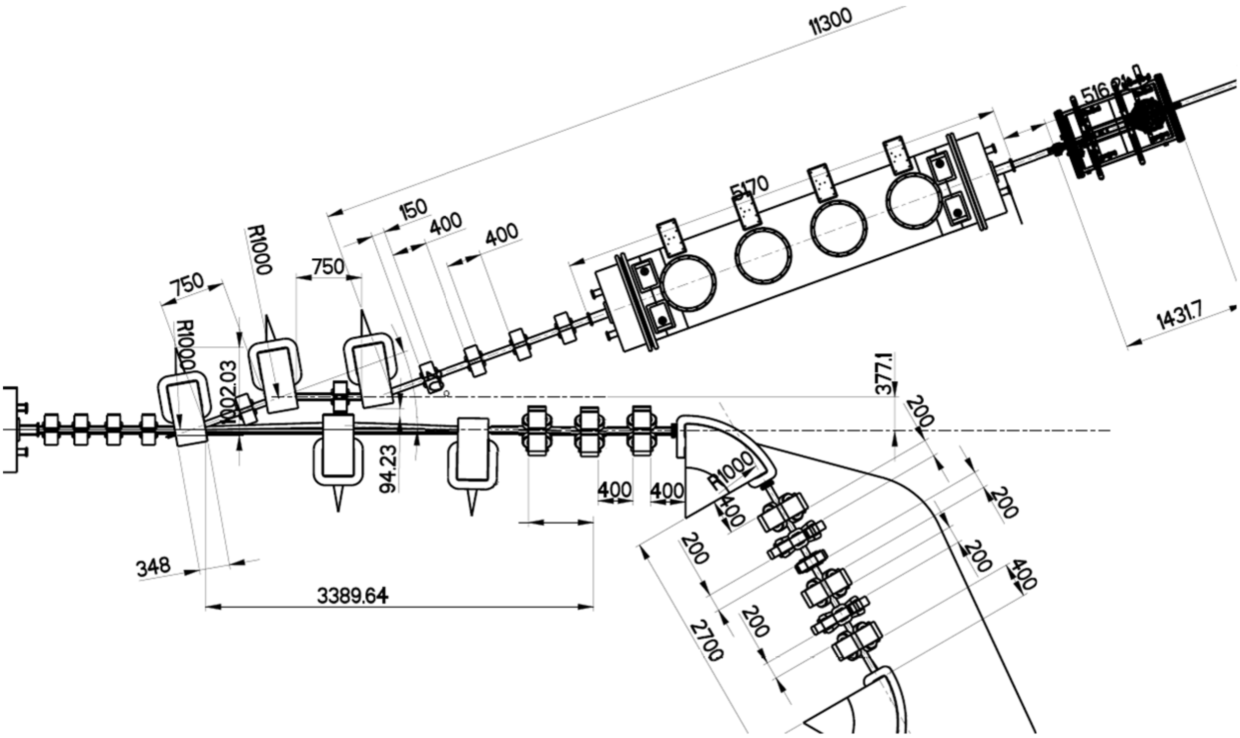

Figure 2: Draft of the injector and a part of the recirculator ring.

Final rms bunch length is set in this case to approx 0.55 mm. A higher compression demands the peak current of over 20 A, which is too high for a 7 MeV bunch. The bunch starts with 1 mm rms length after the gun and expands to 1.7 mm at the beginning of the booster due to positive correlated energy spread (energy at the head is higher) and a time-of-flight debunching in a free space. The acceleration in the booster reverses the sign of the correlated energy spread and the bunch is compressed slightly in the drift. The main compression is in the second dipole of the dogleg, but the bunch length decreases further up to the linac.

The strengths of 4 quadrupoles between the booster and the merger and 4 quadrupoles between the merger and the linac are optimized by the dedicated code to provide 2D-emittance compensation and minimize β-functions at the end of the linac (fig. 3).

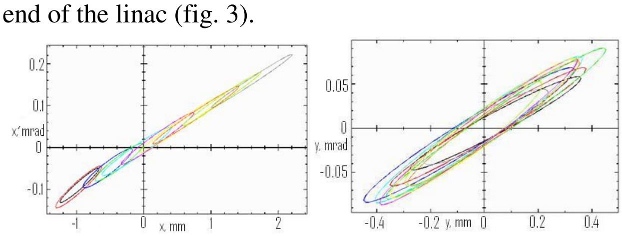

Figure 3: Slices transversal phase spaces after the linac without adjustment of the dispersion at the end of the merger.

Two quadrupoles inside the merger are adjusted to compensate the linear part of the particle offset due to the longitudinal space charge (fig. 4). The dispersion and its derivative at the end of the merger in this case are about 2 cm and $2 \cdot 10^{-2}$ correspondingly. Emittance growth due to this is negligible, if bunch to bunch energy fluctuation in SC booster is lower than 10^{-3}.

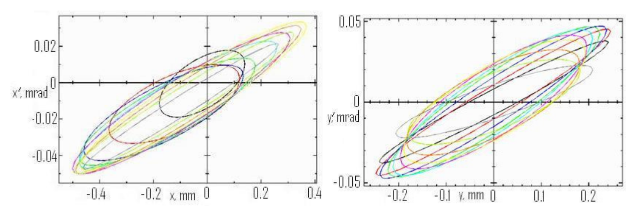

Figure 4: Slices transversal phase spaces after the linac with adjusted dispersion at the end of the merger.

Table 1: Beam parameters from the injector.

Beam energy	7 MeV
Max average beam current	100 mA
Bunch charge	77 pC
Longitudinal emittance	5 keV·mm
Bunch rms length	2 ps
Transversal normalized emittances, $\varepsilon_x/\varepsilon_y$	1/0.7 mm·mrad
Beam transversal output rms size	1 mm

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ISBN 978-3-95450-117-5