LONGITUDINAL STABILITY OF ERL WITH TWO ACCELERATING RF STRUCTURES

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Abstract

Modern ERL projects use superconductive accelerating RF structures. Their RF quality is typically very high. Therefore, the RF voltage induced by electron beam is also high. In ERL the RF voltage induced by the accelerating beam is almost canceled by the RF voltage induced by the decelerating beam. But, a small variation of the RF voltage may cause the deviations of the accelerating phases. These deviations then may cause further voltage variation. Thus, the system may be unstable. The stability conditions for ERL with one accelerating structure are well known [1, 2]. The ERL with split RF structure was discussed recently [3, 4]. The stability conditions for such ERLs are discussed in this paper.

INTRODUCTION

The scheme of an ERL with two accelerating structures is shown in Fig. 1.

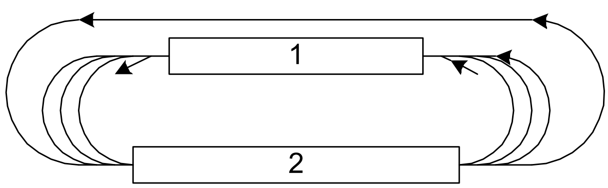

Figure 1: Scheme of ERL with two linacs.

Electrons are injected to the linac 1. After two passes through linac 1 and linac 2 they are used, for example, in undulators. After that electrons are decelerated.

There are four electron beams in each linac simultaneously. Each beam induced large voltage in the linac, but the sum is not so large. If the phases of the beams vary, the sum voltage also vary, and initially small phase deviation may increase due to the dependence of flight times through arcs on the particle energy. This longitudinal instability is considered in our paper.

THE VOLTAGE EQUATIONS

To simplify the picture, consider each linac as one RF cavity. Its equivalent circuit is shown in Fig. 2.

The gap voltage expression $U = L\,d\left(I_b + I_g - C\,dU/dt - U/R\right)/dt$, I_b and I_g are the currents of the beam and of the RF generator, leads to

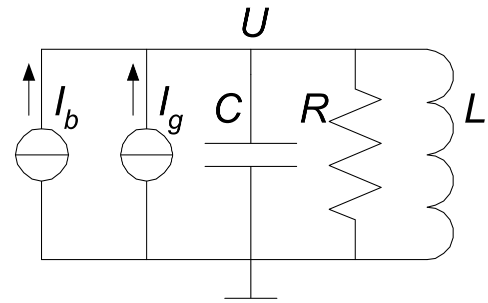

Figure 2: Equivalent circuit of the RF cavity.

the standard equation

$$\frac{d^2U}{dt^2} + \frac{1}{RC}\frac{dU}{dt} + \frac{1}{LC}U = \frac{1}{C}\frac{d}{dt}\left(I_b + I_g\right) \qquad (1)$$

Taking the effective voltage on the linac with number α in the form $\mathrm{Re}(U_\alpha e^{-i\omega t})$ (ω is the frequency of the RF generator), one obtains:

$$\frac{2}{\omega}\frac{dU_\alpha}{dt} = \frac{i\xi_\alpha - 1}{Q_\alpha}U_\alpha + \rho_\alpha(I_{b\alpha} + I_{g\alpha}). \qquad (2)$$

where $\omega_\alpha = \frac{1}{\sqrt{L_\alpha C_\alpha}} = (1 - \frac{\xi_\alpha}{2Q_\alpha})\omega$ is the resonant frequency, $Q_\alpha = R_\alpha \big/ \sqrt{L_\alpha / C_\alpha} >> 1$ is the loaded quality of the cavity, $\rho_\alpha = R_\alpha / Q_\alpha = \sqrt{L/C}$ and R_α are the characteristic and the loaded shunt impedances for the fundamental (TM_{010}) mode, and $I_{b\alpha}$ and $I_{g\alpha}$ are the complex amplitudes of the beam and (reduced to the gap) generator currents correspondingly. We are interested in the case of constant $I_{g\alpha}$. The beam currents $I_{b\alpha}$ depends on all U_α due to phase motion. Linearization of Eq. (2) near the stationary solution

$$U_{0\alpha} = \frac{R_\alpha}{1 - i\xi_\alpha}\left[I_{b\alpha}(\mathbf{U}_0) + I_{g\alpha}\right] \qquad (3)$$

gives:

$$\frac{2}{\omega}\frac{d\delta U_\alpha}{dt} = \frac{i\xi_\alpha - 1}{Q_\alpha}\delta U_\alpha + \qquad (4)$$

ISBN 978-3-95450-117-5

$$+\rho_\alpha \sum_\beta \left(\frac{\partial I_{b\alpha}}{\partial \operatorname{Re} U_\beta} \operatorname{Re} \delta U_\beta + \frac{\partial I_{b\alpha}}{\partial \operatorname{Im} U_\beta} \operatorname{Im} \delta U_\beta \right)$$

Strictly speaking, I_b depends on the values of U at previous moments of time, so Eq. (4) is valid only if the damping times Q_α/ω is much longer than the time of flight through the ERL.

THE STABILITY CONDITIONS

Considering the exponential solutions $\exp(\omega\lambda t/2)$ of system of linear differential equations Eq. (4), one can find the stability conditions. Indeed, the system Eq. (4) corresponds to the system of the linear homogeneous equations $\lambda\delta\mathbf{U} = \mathbf{M}\delta\mathbf{U}$ with the consistency condition $|\mathbf{M} - \lambda\mathbf{E}| = 0$. Re($\lambda$) < 0 for all roots of this equation (i. e., eigenvalues of the matrix **M**) is the stability condition.

The stability condition for ERL with one linac was derived in paper [2]. In this case

$$\mathbf{M} = \begin{pmatrix} -\frac{1}{Q} + \rho\frac{\partial \operatorname{Re} I_b}{\partial \operatorname{Re} U} & -\frac{\xi}{Q} + \rho\frac{\partial \operatorname{Re} I_b}{\partial \operatorname{Im} U} \\ \frac{\xi}{Q} + \rho\frac{\partial \operatorname{Im} I_b}{\partial \operatorname{Re} U} & -\frac{1}{Q} + \rho\frac{\partial \operatorname{Im} I_b}{\partial \operatorname{Im} U} \end{pmatrix} \qquad (5)$$

and the characteristic equation is

$$\lambda^2 - \lambda Tr(\mathbf{M}) + |\mathbf{M}| = 0 \qquad (6)$$

According to Eq. (5) the stability condition is

$$Tr(\mathbf{M}) = \rho\left(\frac{\partial \operatorname{Re} I_b}{\partial \operatorname{Re} U} + \frac{\partial \operatorname{Im} I_b}{\partial \operatorname{Im} U} \right) - \frac{2}{Q} < 0 . \qquad (7)$$

One can say, that the beam "active conductivity" $(\partial \operatorname{Re} I_b/\partial \operatorname{Re} U + \partial \operatorname{Im} I_b/\partial \operatorname{Im} U)/2$ has not to exceed the linac active conductivity $(\rho Q)^{-1}$.

For the ERL with two linacs

$$\mathbf{M} = \begin{pmatrix} \rho_1\frac{\partial \operatorname{Re} I_{b1}}{\partial \operatorname{Re} U_1} - \frac{1}{Q_1} & \rho_1\frac{\partial \operatorname{Re} I_{b1}}{\partial \operatorname{Im} U_1} - \frac{\xi_1}{Q_1} & \rho_1\frac{\partial \operatorname{Re} I_{b1}}{\partial \operatorname{Re} U_2} & \rho_1\frac{\partial \operatorname{Re} I_{b1}}{\partial \operatorname{Im} U_2} \\ \rho_1\frac{\partial \operatorname{Im} I_{b1}}{\partial \operatorname{Re} U_1} + \frac{\xi_1}{Q_1} & \rho_1\frac{\partial \operatorname{Im} I_{b1}}{\partial \operatorname{Im} U_1} - \frac{1}{Q_1} & \rho_1\frac{\partial \operatorname{Im} I_{b1}}{\partial \operatorname{Re} U_2} & \rho_1\frac{\partial \operatorname{Im} I_{b1}}{\partial \operatorname{Im} U_2} \\ \rho_2\frac{\partial \operatorname{Re} I_{b2}}{\partial \operatorname{Re} U_1} & \rho_2\frac{\partial \operatorname{Re} I_{b2}}{\partial \operatorname{Im} U_1} & \rho_2\frac{\partial \operatorname{Re} I_{b2}}{\partial \operatorname{Re} U_2} - \frac{1}{Q_2} & \rho_2\frac{\partial \operatorname{Re} I_{b2}}{\partial \operatorname{Im} U_2} - \frac{\xi_2}{Q_2} \\ \rho_2\frac{\partial \operatorname{Im} I_{b2}}{\partial \operatorname{Re} U_1} & \rho_2\frac{\partial \operatorname{Im} I_{b2}}{\partial \operatorname{Im} U_1} & \rho_2\frac{\partial \operatorname{Im} I_{b2}}{\partial \operatorname{Re} U_2} + \frac{\xi_2}{Q_2} & \rho_2\frac{\partial \operatorname{Im} I_{b2}}{\partial \operatorname{Im} U_2} - \frac{1}{Q_2} \end{pmatrix} \qquad (8)$$

and the characteristic equation is (see, e. g., [5])

$$\lambda^4 - S_1\lambda^3 + S_2\lambda^2 - S_3\lambda + S_4 = 0 , \qquad (9)$$

where $S_1 = \sum_{1\le k\le 4} A\begin{pmatrix} k \\ k \end{pmatrix} = \sum_{1\le k\le 4} M_{kk} = Tr(\mathbf{M})$,

$$S_2 = \sum_{1\le k<l\le 4} A\begin{pmatrix} k & l \\ k & l \end{pmatrix}, \quad S_3 = \sum_{1\le k<l<m\le 4} A\begin{pmatrix} k & l & m \\ k & l & m \end{pmatrix},$$

and $S_4 = A\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix} = |\mathbf{M}|$ are the sums of main minors of the matrix **M**. The necessary conditions for stability (Re(λ) < 0 for all four roots of Eq. (9)) is positivity of all the coefficients of the polynomial Eq. (9). In particular, the only independent on detunings ξ_1 and ξ_2 condition $S_1 < 0$ gives

$$\rho_1\left(\frac{\partial \operatorname{Re} I_{b1}}{\partial \operatorname{Re} U_1} + \frac{\partial \operatorname{Im} I_{b1}}{\partial \operatorname{Im} U_1} \right) + \rho_2\left(\frac{\partial \operatorname{Re} I_{b2}}{\partial \operatorname{Re} U_2} + \frac{\partial \operatorname{Im} I_{b2}}{\partial \operatorname{Im} U_2} \right) < \frac{2}{Q_1} + \frac{2}{Q_2} \qquad (10)$$

The sufficient conditions are given by the Liénard-Chipart criterion [5]. It requires the positivity of of all the coefficients of the polynomial Eq. (9) and the third Hurwitz minor

$$S_1 < 0,\ S_2 > 0,\ S_4 > 0,$$

$$\Delta_3 = \begin{vmatrix} -S_1 & -S_3 & 0 \\ 1 & S_2 & S_4 \\ 0 & -S_1 & -S_3 \end{vmatrix} = S_1(S_2S_3 - S_1S_4) - S_3^2 > 0 \qquad (11)$$

Coefficient S_2 depends on detunings as

$$S_2(\xi_1,\xi_2) = \sum_{1\le k<l\le 4} A\begin{pmatrix} k & l \\ k & l \end{pmatrix} =$$

$$= \frac{\xi_1^2}{Q_1^2} + \frac{\xi_2^2}{Q_2^2} + \frac{\xi_1}{Q_1}\rho_1\left(\frac{\partial \operatorname{Im} I_{b1}}{\partial \operatorname{Re} U_1} - \frac{\partial \operatorname{Re} I_{b1}}{\partial \operatorname{Im} U_1} \right) + \qquad (12)$$

$$+ \frac{\xi_2}{Q_2}\rho_2\left(\frac{\partial \operatorname{Im} I_{b2}}{\partial \operatorname{Re} U_2} - \frac{\partial \operatorname{Re} I_{b2}}{\partial \operatorname{Im} U_2} \right) + S_2(0,0)$$

Therefore the condition $S_2 > 0$ is satisfied for large enough detunings

$$\left[\frac{\xi_1}{Q_1} + \frac{\rho_1}{2}\left(\frac{\partial \operatorname{Im} I_{b1}}{\partial \operatorname{Re} U_1} - \frac{\partial \operatorname{Re} I_{b1}}{\partial \operatorname{Im} U_1} \right) \right]^2 + \left[\frac{\xi_2}{Q_2} + \frac{\rho_2}{2}\left(\frac{\partial \operatorname{Im} I_{b2}}{\partial \operatorname{Re} U_2} - \frac{\partial \operatorname{Re} I_{b2}}{\partial \operatorname{Im} U_2} \right) \right]^2 > \cdot$$

$$S_2(0,0) - \frac{\rho_1^2}{4}\left(\frac{\partial \operatorname{Im} I_{b1}}{\partial \operatorname{Re} U_1} - \frac{\partial \operatorname{Re} I_{b1}}{\partial \operatorname{Im} U_1} \right)^2 - \frac{\rho_2^2}{4}\left(\frac{\partial \operatorname{Im} I_{b2}}{\partial \operatorname{Re} U_2} - \frac{\partial \operatorname{Re} I_{b2}}{\partial \operatorname{Im} U_2} \right)^2 \qquad (13)$$

As

$$S_4(\xi_1,\xi_2) = |\mathbf{M}(\xi_1,\xi_2)| = |\mathbf{M}(0,0)| + \left(\frac{\xi_1}{Q_1} \right)^2 \left(\frac{\xi_2}{Q_2} \right)^2 + \ldots, \qquad (14)$$

the condition $S_4 > 0$ is also satisfied for large enough detunings.

In the simplest case of the isochronous ERL arcs the conductivity matrix is zero. Then it is easy to proof, that all stability conditions are satisfied.

As the qualities of the superconducting cavities are very large, it is interesting to consider the opposite limiting case, neglecting small terms $1/Q_{1,2}$ in the matrix Eq. (8). Then all stability conditions do not depend on the beam current. They depend only on the ratio ρ_1/ρ_2 and the beam conductivity matrix, which is fully defined by the ERL magnetic system.

THE CONDUCTIVITY MATRIX

To proceed further, we have to specify the elements of the beam conductivity matrix in the stability conditions.

ISBN 978-3-95450-117-5

The complex amplitude of the beam current I_b may be written in the form

$$I_{b\alpha} = -2I\sum_{n=0}^{N-1}\left(e^{i\varphi_{2n+\alpha-1}+i\psi_{2n+\alpha-1}} + e^{i\varphi_{4N-2n-\alpha}+i\psi_{4N-2n-\alpha}}\right) \approx$$
$$\approx I_{b\alpha}(\mathbf{U}_0) - 2iI\sum_{n=0}^{N-1}\left(e^{i\varphi_{2n+\alpha-1}}\psi_{2n+\alpha-1} + e^{i\varphi_{4N-2n-\alpha}}\psi_{4N-2n-\alpha}\right) \quad (15)$$

where I is the average beam current, $\varphi_{2n+\alpha-1}$ is the equilibrium phase for the n-th pass through the resonator with the number α (α = 1, 2), and N is the number of orbits for acceleration. The small energy and phase deviations ε_n and ψ_n obey the linear equations:

$$\varepsilon_{n+1} = \varepsilon_n + e\,\mathrm{Im}\left[U_{0\alpha(n)}e^{-i\varphi_n}\right]\psi_n + e\,\mathrm{Re}\left[\delta U_{\alpha(n)}e^{-i\varphi_n}\right], \quad (16)$$

Where $\alpha(2n)=1, \alpha(2n+1)=2 \ for\ 0 \le n \le N-1$ and $\alpha(2n)=2, \alpha(2n+1)=1\ for\ N \le n \le 2N-1$,

$$\psi_{n+1} = \psi_n + \omega\left(\frac{dt}{dE}\right)_{n+1}\varepsilon_{n+1}, \quad (17)$$

where $\left(\frac{dt}{dE}\right)_n$ is the longitudinal dispersion of the n-th 180-degree bend. The initial conditions for the system of Eqs. (16) and (17) are, certainly, ε_0=0 and ψ_0=0, if we have no special devices to control them for the sake of the beam stabilization, or other purposes. The solution of Eq. (16) and Eq. (17) may be written using the longitudinal sine-like trajectory S_{nk} and its "derivative" S'_{nk} (elements 56 and 66 of the transport matrix). These functions are the solutions of the homogenous system

$$S'_{n+1,k} = S'_{n,k} + e\,\mathrm{Im}\left[U_{0\alpha(n)}e^{-i\varphi_n}\right]S_{n,k}, \quad (18)$$

$$S_{n+1,k} = S_{n,k} + \omega\left(\frac{dt}{dE}\right)_{n+1}S'_{n+1,k}, \quad (19)$$

with the initial conditions $S_{k,k}$ = 0, $S'_{k,k}$ = 1. Then

$$\psi_n = e\sum_{k=0}^{n-1}S_{nk}\,\mathrm{Re}\left[\delta U_{\alpha(k)}e^{-i\varphi_k}\right], \quad (20)$$

$$\varepsilon_n = e\sum_{k=0}^{n-1}S'_{nk}\,\mathrm{Re}\left[\delta U_{\alpha(k)}e^{-i\varphi_k}\right]. \quad (21)$$

Substitution of Eq. (20) to Eq. (15) gives

$$\delta I_{b\alpha} = -2ieI\sum_{n=0}^{N-1}\{e^{i\varphi_{2n+\alpha-1}}\sum_{k=0}^{2n+\alpha-2}S_{2n+\alpha-1,k}\,\mathrm{Re}\left[\delta U_{\alpha(k)}e^{-i\varphi_k}\right]+ e^{i\varphi_{4N-2n-\alpha}}\sum_{k=0}^{4N-2n-\alpha-1}S_{4N-2n-\alpha,k}\,\mathrm{Re}\left[\delta U_{\alpha(k)}e^{-i\varphi_k}\right]\} \quad (22)$$

Then the necessary condition of the stability Eq. (10) may be written as

$$e\rho_1 I\sum_{n=0}^{N-1}[\sum_{k=0}^{n-1}S_{2n,2k}\sin(\varphi_{2n}-\varphi_{2k}) + \sum_{k=0}^{N-1}S_{4N-2n-1,2k}\sin(\varphi_{4N-2n-1}-\varphi_{2k}) + \sum_{k=N+1}^{2N-n-1}S_{4N-2n-1,2k-1}\sin(\varphi_{4N-2n-1}-\varphi_{2k-1})] +$$
$$e\rho_2 I\sum_{n=0}^{N-1}[\sum_{k=1}^{n}S_{2n+1,2k-1}\sin(\varphi_{2n+1}-\varphi_{2k-1}) + \sum_{k=1}^{N}S_{4N-2n-2,2k-1}\sin(\varphi_{4N-2n-2}-\varphi_{2k-1}) + \sum_{k=N}^{2N-n-2}S_{4N-2n-2,2k}\sin(\varphi_{4N-2n-2}-\varphi_{2k})] < \frac{1}{Q_1}+\frac{1}{Q_2} \quad (23)$$

For an ERL it needs to satisfy (at least approximately) the recuperation condition

$$\mathrm{Re}\left[U_{01}\sum_{n=0}^{N-1}\left(e^{-i\varphi_{2n}} + e^{-i\varphi_{4N-2n-1}}\right)\right] = 0$$
$$\mathrm{Re}\left[U_{02}\sum_{n=0}^{N-1}\left(e^{-i\varphi_{2n+1}} + e^{-i\varphi_{4N-2n-2}}\right)\right] = 0 \quad (24)$$

For the longitudinal stability it also needs to have longitudinal focusing for most of passes through the linac (see Eq. (15, 16)):

$$e\,\mathrm{Im}\left[U_{0\alpha(n)}e^{-\varphi_n}\right] < 0 \quad (25)$$

(if all $(dt/dE)_n > 0$). Conditions Eq. (24) and Eq. (25) may be satisfied simultaneously, if

$$\arg(eU_{0\alpha(n)}e^{-i\varphi_n}) + \arg(eU_{0\alpha(4N-n-1)}e^{-i\varphi_{4N-n-1}}) = -\pi, \quad (26)$$

$0 \le n \le 2N-1$,

which leads to

$$\varphi_{4N-n-1} = \pi - \varphi_n + 2\arg\left(eU_{0\alpha(n)}\right) \quad \text{for}\ 0 \le n \le 2N-1 \quad (27)$$

ISBN 978-3-95450-117-5

Conditions Eq. (27) affords equality of beam energies after *n*-th and (4*N*-*n*)-th passes through a linac.

To make the stability condition Eq. (23) more explicit, consider a simple example. Assume that equilibrium phases are equal during acceleration. In this simplest case $\varphi_{2n} - \arg(eU_{01}) = \Phi_1, \varphi_{2n+1} - \arg(eU_{02}) = \Phi_2$ for $0 \le n \le N-1$. Eq. (27) defines the equilibrium phases for deceleration. Then Eq. (23) gives

$$e\rho_1 I \sin(2\Phi_1) \sum_{n=0}^{N-1}\sum_{k=0}^{N-1} S_{4N-2n-1,2k} + e\rho_2 I \sin(2\Phi_2) \sum_{n=0}^{N-1}\sum_{k=0}^{N-1} S_{4N-2n-2,2k+1} < \frac{1}{Q_1} + \frac{1}{Q_2}. \quad (28)$$

SIMULATIONS

Numerical calculations were made for proposed scheme of ERL with two accelerating structures (the simplest scheme is shown in Fig. 1). Parameters of accelerating structures: $Q_1 = Q_2 = 10^6$, $\rho_1 = 40\text{M}\Omega$, $\rho_2 = 90\text{M}\Omega$, $\omega = 2\pi \cdot 1.3 \cdot 10^9$ Hz, $I = 10$ mA, $U_1 = 0.8$ GV, $U_2 = 1.8$ GV. Considering the magnetic structure with acceptable growth of the horizontal emittance [6, 7], one can check the stability conditions Eq. (11). This work is in progress yet.

CONCLUSION

In this paper we derived the criterion of the longitudinal stability for the ERL with two accelerating structures. Further numerical investigations of this criterion are planned.

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